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TRI-LINEAR MODEL FOR THE OUT-OF-PLANE SEISMIC ASSESSMENT OF UNREINFORCED MASONRY WALLS

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ABSTRACT

A new model describing the response of vertically-spanning unreinforced masonry walls subjected to out-of-plane loading is presented. The model approximates the real behaviour of the walls by a tri-linear force-displacement relationship, which is suitable for engineering practice. Differently from the existing tri-linear models, which are mainly based on experimentally calibrated parameters, the present one relies on principles of mechanics. Its fully analytical formulation provides the potential for using it in the assessment of unreinforced masonry for a wide range of wall configurations and boundary conditions. The comparison of model simulations with experimental results from shake table tests documented in the literature validates the model for cantilever and parapet walls.

Keywords: unreinforced masonry; out-of-plane behaviour; seismic assessment; pushover curve

1. INTRODUCTION

Out-of-plane failure modes are often responsible for the partial collapse of unreinforced masonry structures. The seismic behaviour of vertically-spanning unreinforced masonry walls undergoing large out-of-plane deflections can be described by simplified bi-linear and tri-linear models, see Figure 1 (Doherty et al., 2002; Griffith et al., 2003). Bi-linear models are derived by modelling masonry as rigid blocks separated by fully cracked cross-sections and obtained by non-linear kinematic analysis of the walls. Tri-linear models are derived when the deformability or the limited compressive strength of the masonry are taken into account, which may highly affect the out-of-plane force capacity of the wall. Considering their relative simplicity and exiguous computational cost, these simplified models have recently gained attention and are nowadays recommended by building codes for the out-of-plane seismic assessment of existing unreinforced masonry structures. Assessment of the out-of-plane capacity of URM walls according to today's codes is based on the use of bi-linear models (NTC 2008; NZSEE 2006). Tri-linear models are required for predicting the displacement demand of the walls, which is obtained from an equivalent linear elastic single-degree-of-freedom system with a stiffness equal to the secant stiffness K_2 (Derakhshan et al., 2017; Sorrentino et al., 2016), see Figure 1. The computational cost demanded by the tri-linear models is very small. For this reason, such models lend themselves ideally to non-linear time-history analyses (Al Shawa et al., 2012; Sorrentino et al., 2016).

Tri-linear models are defined by three displacement parameters Δ_1, Δ_2, Δ_U, and one force parameter F_1, representing the maximum force that the wall can sustain. The tri-linear models that are nowadays available in the literature make use of parameters that have been calibrated based on experimental tests (Derakhshan et al., 2013a; Doherty et al., 2002; Griffith et al., 2003; Al Shawa et al., 2012). A new

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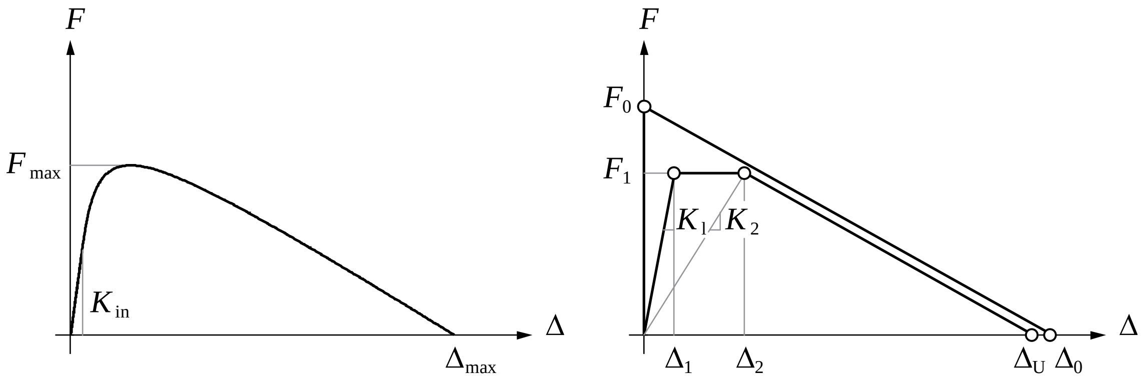


Figure 1 Left: typical pushover curve of vertically-spanning URM walls. Right: bi- and tri-linear idealisations. From (Godio and Beyer, 2018).

formulation of the tri-linear model is developed in (Godio and Beyer, 2018) and herein presented. The formulation is an engineering approximation of the exact solution derived in (Godio and Beyer, 2017), where the analytical expression for the pushover curve of out-of-plane loaded vertically-spanning unreinforced masonry walls was found. The formulation obtained for the tri-linear model is therefore also analytical. It is based on a limited number of parameters, namely the wall slenderness ratio, the masonry elastic modulus and the ratio between the axial load and the Euler's critical load. The tri-linear model is benchmarked against results from experimental data. Its validation for URM walls that span vertically between two supports was presented in (Godio and Beyer, 2018). Herein, the validation is extended to cantilever or parapet walls.

2. TRI-LINEAR MODEL FOR URM PARAPETS

Figure 2 illustrates a cantilever or parapet URM wall. These walls are usually represented as a rigid macroblock undergoing rocking. In this paper, the wall is represented as a deformable body in which, when the deflection increases, diffuse cracking occurs and progressively spreads in the regions of maximum bending moment.

The unreinforced masonry parapet is subjected to a vertical load O, denoting the overburden, and to an out-of-plane ground acceleration $a_g(t)$ applied at the base of the parapet (Figure 2). The top displacement Δ represents the displacement of the control point, at which the tri-linear single-degree-of-freedom system herein developed for the parapet wall refers to. A triangular inertia force distribution is assumed along the wall height (Figure 2). This assumption has been corroborated by experimental observations from laboratory shake table tests (Giaretton et al., 2016; Graziotti et al., 2016; Griffith et al., 2004) and is justified for walls undergoing significant rocking (Doherty et al., 2002; Griffith et al., 2003). Its application to the tri-linear model herein presented has been validated in (Godio and Beyer, 2018) for URM walls that span vertically between two supports and is validated here for cantilever or parapet walls.

Writing D'Alembert's principle for the configuration depicted in Figure 2 leads to the following equation of motion for the parapet wall (Griffith et al., 2003):

$$\ddot{\Delta}(t) + \frac{C}{M}\dot{\Delta}(t) + \frac{3}{2}\frac{F\big(\Delta(t)\big)}{M} = \frac{3}{2}a_g(t), \tag{1}$$

where $\Delta(t)$, $\dot{\Delta}(t)$ and $\ddot{\Delta}(t)$ are respectively the displacement, velocity and acceleration measured at the wall top, M is the total mass of the wall and C is the equivalent viscous damping factor. The tri-linear model makes use of an equivalent viscous damping with damping ratio c, resulting in the damping coefficient $C = \sqrt{6MK_1}c$ (Griffith et al., 2003). The response $F\big(\Delta(t)\big)$ represents the force-displacement relationship of the URM parapet, when this latter is subjected to a horizontal uniformly

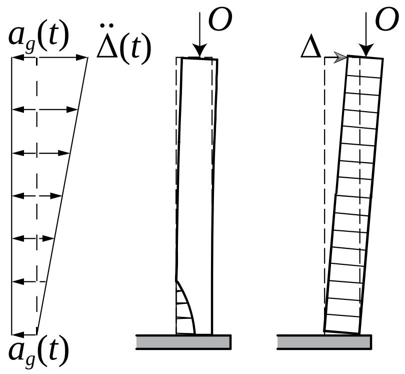


Figure 2 URM cantilever or parapet walls, from (Godio and Beyer, 2018).

distributed load (Griffith et al., 2003). Its expression is herein given by the analytical relationship of the developed tri-linear model.

3. FORMULATION

The scheme used for the derivation of the tri-linear model is outlined. Equations refer to a wall with height H_w, length L_w and thickness t_w, subjected to a vertical load O and to its self-weight W. The elastic modulus of the masonry is E_m and its mass density is ρ. The moment of inertia of the uncracked cross-section of the parapet is $I_\mathrm{w} = L_\mathrm{w} t_\mathrm{w}^3/12$.

1.1 Bi-linear model parameters F_0 and Δ_0

The non-linear kinematic analysis gives the parameters F_0 and Δ_0 of the bi-linear model that describes the rigid-body mechanism depicted in Figure 2:

$$F_0 = \frac{t_\mathrm{w}}{H_\mathrm{w}}(O + W), \tag{2}$$

and

$$\Delta_0 = \frac{t_\mathrm{w}}{2}\frac{O+W}{O+W/2}. \tag{3}$$

The crack typically forms at the base of the cantilever, except for the presence of intermediate restraints located at higher levels, changes in the wall thickness or presence of weak mortar layers (Giaretton et al., 2016).

1.2 Force parameter F_1

The plateau force F_1 of the tri-linear model is equal to the maximum force F_max supported by the cantilever wall when this latter is subjected to a uniformly distributed load. This latter force can be expressed in terms of the F_max/F_0 ratio (Godio and Beyer, 2017) and leads to the expression:

$$\frac{F_1}{F_0} = 1 - \left(\frac{P}{P_\mathrm{E}}\right)^{0.4}. \tag{4}$$

In the above equation:

$$P = O + W \tag{5}$$

and

$$P_{\mathrm{E}} = \frac{\pi^2 E_{\mathrm{m}} I_{\mathrm{w}}}{4H_{\mathrm{w}}^2}, \tag{6}$$

denote respectively the effective axial load (Godio and Beyer, 2017) and the Euler's critical load of the wall.

1.3 Initial stiffness K_1

The initial stiffness K_1 of the first branch of the tri-linear model is set equal to:

$$K_1 = 0.7K_{\mathrm{in}}, \tag{7}$$

where K_{in} is the initial stiffness of the wall derived from a geometrically non-linear Euler-Bernoulli beam with uncracked cross-sections (Godio and Beyer, 2018). In particular, K_{in} can be written as the product between the stiffness of a geometrically linear beam, K_{lin}, and the Ψ_K, a function embodying 2nd order effects (Godio and Beyer, 2018):

$$K_{\mathrm{in}} = K_{\mathrm{lin}} \Psi_K. \tag{8}$$

The stiffness K_{lin} is:

$$K_{\mathrm{lin}} = \frac{8E_{\mathrm{m}} I_{\mathrm{w}}}{H_{\mathrm{w}}^3}, \tag{9}$$

and Ψ_K function can be approximated by:

$$\Psi_K = 1 - \frac{P}{P_{\mathrm{E}}}, \tag{10}$$

as demonstrated in (Godio and Beyer, 2018).

1.4 Ultimate displacement Δ_U

The ultimate displacement Δ_{U} is expressed as a ratio of the rigid-body displacement Δ_0 (Godio and Beyer, 2018):

$$\frac{\Delta_{\mathrm{U}}}{\Delta_0} = \tau. \tag{11}$$

Reduction of the ultimate displacement in URM walls can be due to different factors, as for instance the rounding of the unit corners due to local deformation (Lagomarsino, 2015), the unit or mortar crushing (Derakhshan et al., 2013a) and the reduced depth of the mortar layer due to mortar pointing or the dropping out of mortar occurred during rocking (Derakhshan et al., 2013b; Doherty et al., 2002). This reduction leads to a shift of the descending branch of the bi-linear model. It corresponds to considering an effective wall thickness $t_{\mathrm{w,eff}} = \tau t_{\mathrm{w}}$ into the expressions for F_0 and Δ_0.

1.5 Displacement parameters Δ_1 and Δ_2

The parameters Δ_1 and Δ_2 of the tri-linear model are (Godio and Beyer, 2018):

$$\frac{\Delta_1}{\Delta_0} = \left[1 - \left(\frac{P}{P_{\mathrm{E}}}\right)^{0.4}\right] \frac{F_0}{\Delta_0} \frac{1}{0.7K_{\mathrm{in}}}, \tag{12}$$

and:

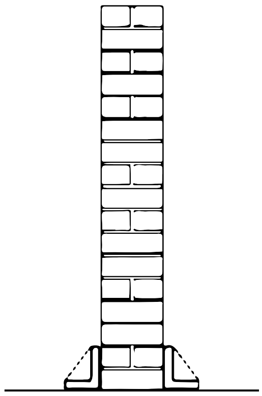


Figure 3 Cross-section of the walls tested on the shake table by (Giaretton et al., 2016).

$$\frac{\Delta_2}{\Delta_0} = \left(\frac{P}{P_E}\right)^{0.4}. \tag{13}$$

They are derived respectively as $\Delta_1 = F_1/K_1$ and $\Delta_2 = (1 - F_1/F_0)\Delta_0$ (Sorrentino et al., 2016).

1.6 Failure condition

The tri-linear model is integrated numerically as a single-degree-of-freedom system with non-linear elastic behaviour following the tri-linear $F - \Delta$ relationship of Figure 2. The following condition:

$$|\Delta| \geq \Delta_U, \tag{14}$$

is used in the simulations as failure condition for the wall.

4. VALIDATION

In (Godio and Beyer, 2018), the tri-linear model was compared to results from shake table tests involving URM walls spanning vertically between two supports. The tri-linear model is validated here by the comparison with experimental results from shake table tests carried out on URM parapet walls. The series of tests carried out at the University of Auckland and presented in (Giaretton et al., 2016) is used as benchmark.

Two unreinforced brick masonry walls of height 1180 mm, width 1200 mm and nominal thickness 230 mm are subjected to an out-of-plane base motion. The walls are indicated with P4-B and P7-C (Giaretton et al., 2016). The wall P6-B was also tested, but it is not considered for the validation as it was inclined of 45° on the shake table and was consequently subjected to a combined in-plane and out-plane excitation (Giaretton et al., 2016). Figure 3 illustrates the boundary conditions adopted for the parapets: the first two brick courses are restrained by two angular steel profiles into which the bricks are casted. The walls are free of overburden, O = 0 kN. The mass density of the masonry is ρ =1750 kg/m^3.

The ground motion recorded during the Christchurch earthquake (New Zealand, February 2011) was applied through the shake table (Giaretton et al., 2016). During the tests, the ground motion was scaled at increasing amplitudes of 10% (~ 0.05 g) until cracking was initiated into the walls. The tests were then repeated on the pre-cracked walls until rocking was initiated. The measurements recorded on the pre-cracked walls at the 30% of the ground motion are used for comparison with the tri-linear model. The acceleration history measured at the wall bottom (i.e. just above the support system) is used as input for the tri-linear model. For the simulations, a damping ratio of 5% is used. This value constitutes a lower bound of the values observed during free-rocking tests not only in this test campaign (Giaretton et al., 2016) but also in others (Graziotti et al., 2016; Griffith et al., 2004).

Since no indications are provided about the elastic modulus of the masonry and due to the difficulty of predicting the state of degradation of the walls following the repeated shakes, the values of E_{m} are derived from the experimental $F - \Delta$ curves shown in Figure 4. In these curves, F is the force derived by multiplying the absolute acceleration measured at the centre of mass of the parapet by the total wall mass; assuming a triangular distribution of the relative acceleration along the wall height (Figure 2) results in:

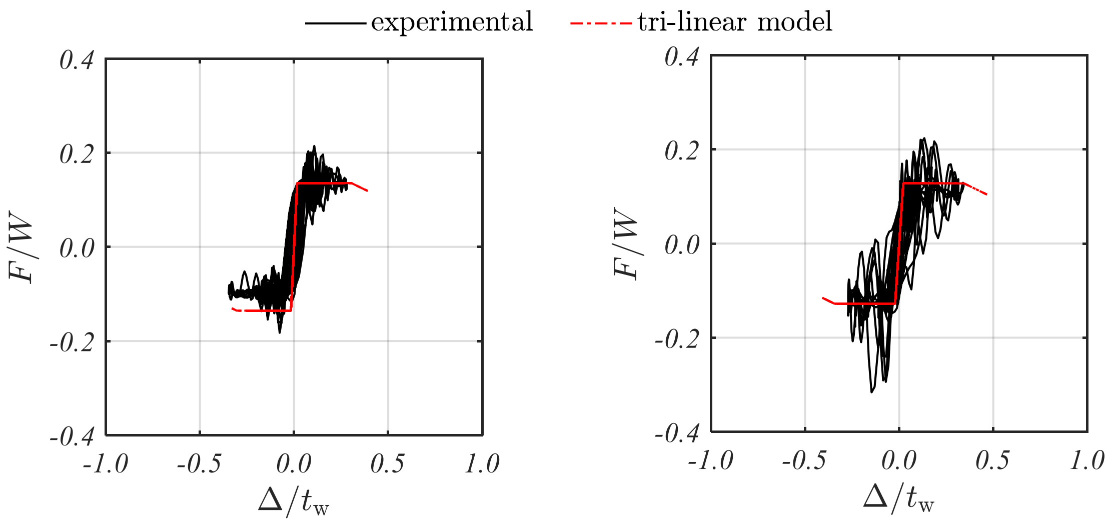
$$F = M\frac{\ddot{\Delta}}{2}, \tag{15}$$

where $\ddot{\Delta}$ is the acceleration measured at the wall top. Based on these curves, K_{lin} can be directly estimated as $K_{\mathrm{lin}} = F/\Delta$, and E_{m} deduced. The experimental acceleration is filtered by means of a low-pass filter at 20 Hz to avoid spikes.

Table 1 gives the parameters of the tri-linear model used for simulating the shake table tests. Figure 4 shows how the tri-linear model satisfactorily predicts the peak of the experimental force-displacement curves. Figure 5 and Figure 6 compare the tri-linear model with the experimental results in terms of displacement and acceleration histories, respectively, and show how the tri-linear model is able to seize the peak displacements and accelerations measured at the top of the wall, as well as the frequency content of the experimental response. Table 2 quantifies the error committed by the tri-linear model in reproducing the displacement histories through the ratio between the tri-linear (TRL) and experimental (EXP) peak wall displacements Δ_{max}, and three error estimators RMS, WME and WME(20), whose definition can be found in (Godio and Beyer, 2018; Al Shawa et al., 2012). The mean values are in line with those found in the previous validation carried out on the model for walls spanning between two supports (Godio and Beyer, 2018).

Table 1 Parameters of the tri-linear model used for simulating the shake table tests carried out at the University of Auckland.

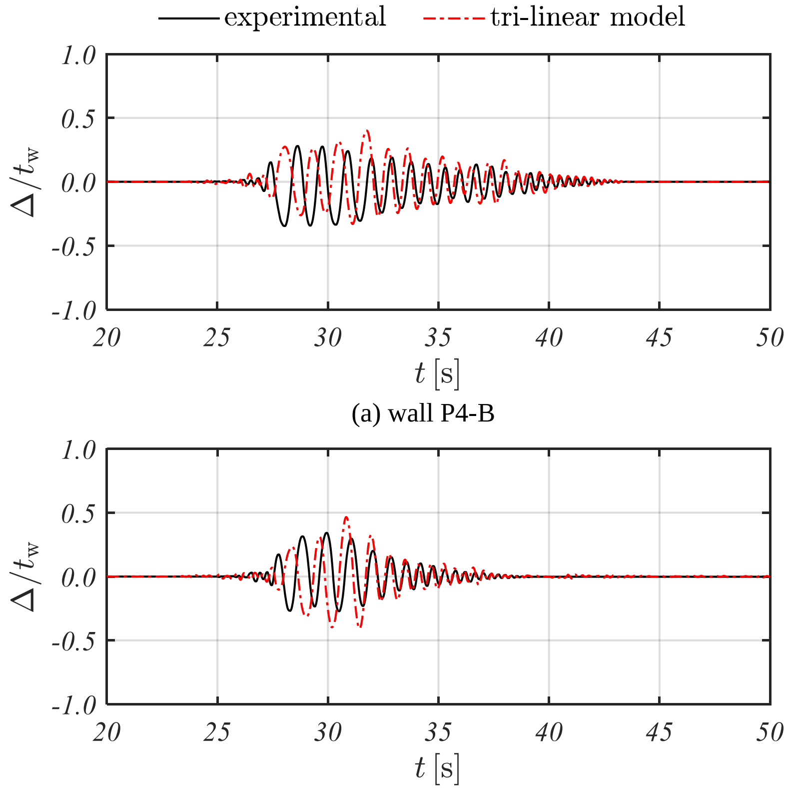
Wall	$\boldsymbol{\Delta_1/\Delta_0}$	$\boldsymbol{\Delta_2/\Delta_0}$	$\boldsymbol{F_1/F_0}$
(a) wall P4-B	0.017	0.305	0.695
(b) wall P7-C	0.022	0.345	0.655



(a) wall P4-B

(b) wall P7-C

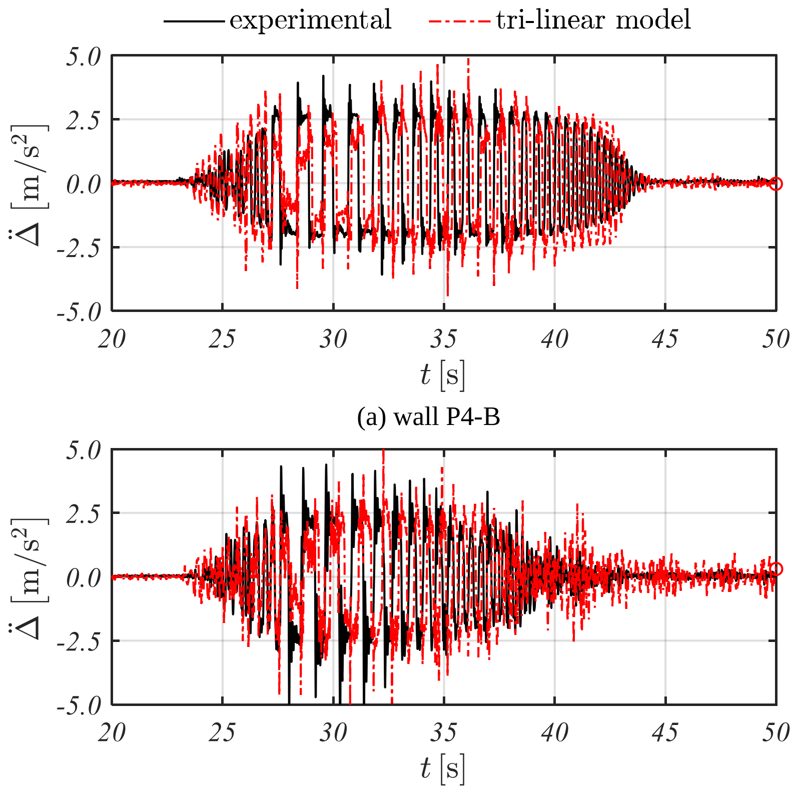
Figure 4 Shake table tests carried out at the University of Auckland (Giaretton et al., 2016): simulation by means of the proposed tri-linear model. Force-displacement response.



(a) wall P4-B

(b) wall P7-C

Figure 5 Shake table tests carried out at the University of Auckland (Giaretton et al., 2016): simulation by means of the proposed tri-linear model. Displacement histories.



(a) wall P4-B

(b) wall P7-C

Figure 6 Shake table tests carried out at the University of Auckland (Giaretton et al., 2016): simulation by means of the proposed tri-linear model. Acceleration histories.

Table 2 Error committed by the tri-linear model in simulating the shake table tests carried out at the University of Auckland.

Wall	$\Delta_{\text{max}}^{\text{TRL}}/\Delta_{\text{max}}^{\text{EPX}}$	**RMS**	**WME**	**WME(20)**
(a) wall P4-B	1.160	0.055	1.028	1.190
(b) wall P7-C	1.364	0.243	1.091	1.243
mean	1.262	0.149	1.059	1.217

5. CONCLUSIONS

A new tri-linear model for vertically-spanning URM walls was presented and validated for cantilever or parapet walls, based on results from shake table tests documented in the literature. The herein presented validation complements the one previously carried out for walls spanning vertically between two supports (Godio and Beyer, 2018). Use of the tri-linear model can be envisaged for carrying out non-linear time history analyses, and for computing the displacement demand of the walls, through an equivalent linear elastic single-degree-of-freedom system with a stiffness equal to the secant stiffness K_2 given by the tri-linear model. A possible extension of the model may involve the introduction of the diaphragm deformability at the top and bottom supports of the wall in the formulation. Examples of this kind can be found in (Derakhshan et al., 2015; Landi et al., 2015).

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