16TH EUROPEAN CONFERENCE ON
EARTHQUAKE ENGINEERING THESSALONIKI
18 - 21 JUNE 2018

EFFECTS OF THE VARIATION OF GROUNDWATER LEVEL ON EARTHQUAKE GROUND MOTIONS

Weihua LI[1], Chenggang ZHAO[2], Bing BAI[3]

ABSTRACT

Based on Biot's dynamic theory for saturated porous media, a study is presented about the effects of the variation of groundwater level on earthquake ground motions through the analyses of wave propagations in layered foundation. In this analysis, a layered foundation model is built, the saturated soil layer below the groundwater level is assumed to be water-saturated soils, and the soil layer over the groundwater level is modeled as air-saturated soils. Numerical results show that, under both the incident P and SV wave, the amplitudes of ground surface displacement are mainly relative to the variation of groundwater level if the relative stiffness of soil skeleton is small. With the decline of the groundwater level, the response peaks of ground surface displacement increase and the frequencies of the resonance peaks decrease. But when the relative stiffness of soil skeleton becomes large, the variation of groundwater level has little effects on earthquake ground motions.

Keywords: groundwater level; earthquake ground motions; fluid saturated porous media; layered foundation model

1. INTRODUCTION

Soil site conditions can cause changes in the amplitudes, spectral content, and duration of strong earthquake ground motion and ultimately result in damage to man-made structures. So the seismic response analysis of soil deposits becomes an important part of earthquake engineering.

Over the past decades, a lot of researches have been worked on that. References to the studies have appeared in many accounts of earthquake effects (Sánchez-Sesma and Crouse 2015; Trifunac 2016). However, in the existing studies, the effects of the change of the groundwater level on the earthquake ground motion are seldom considered. In the interpretation of the causes of the variations of site intensities, Medvede(1965) cites depth to the underground water table and its role in influencing in reported observations of intensities. In perusal of the liquefaction susceptibility criteria, Tinsley et al. (1985) shows that the ultimate categorical variables such as "very high" or "moderate" are derived on the basis of multiple site characteristics, including water content and the depth-to-water table.

To analysis the effects of the change of the groundwater level on the earthquake ground motion, the dynamic interaction between the solid skeleton and pore fluid in soil should be considered. The dynamic theory of fluid-saturated porous medium developed by Biot (1956) provides theoretical supports for this. After Biot's theory, many researchers began to pay special attention to seismic ground response analyses of saturated and partially saturated soils (Schanz,2009). Lin et al. (2005) discussed surface displacements, surface strain, rocking, and energy partitioning during reflection-of-plane waves in a fluid saturated poroelastic half-space. By treating the soil as a partially

[1] Associate professor, School of Civil Engineering, Beijing Jiaotong University, Beijing, China, whli@bjtu.edu.cn
[2] Professor, School of Civil Engineering, Beijing Jiaotong University, Beijing, China, cgzhao@bjtu.edu.cn
[3] Professor, School of Civil Engineering, Beijing Jiaotong University, Beijing, China, bbai@bjtu.edu.cn

water-saturated porous medium, Yang(2001, 2002) studied the effects of saturation on the seismic ground motions at the free surface of a half space and multi-layered soil-bedrock system. According to these studies, even a slight decrease in the full saturation of the overlying soil may cause an appreciable difference in the displacement amplitudes of the ground motions at the ground surface. Zhou (2006) studied the seismic reflection and transmission coefficients at the interface between air-saturated and water-saturated porous media. The analysis showed that the propagation characteristic of the wave in air-saturated soils is great different from that of wave in water-saturated soils. All the studies suggest us that the effects of the change of the groundwater level on the earthquake ground motion may need to be carefully taken into account.

To gain a better understanding of the effects of the underground water table on the nature of strong motions, the present paper examines a detailed site subjected to excitation by plane P and SV waves. A layered foundation model with soil layer overlying the bedrock half-space is considered. The saturated soil layer below the groundwater level is assumed to be water-saturated poroelastic medium, the soil layer over the groundwater level is modeled as air-saturated poroelastic medium, and the bedrock is taken to be linear elastic solid. The elastic wave propagation in a layered foundation model excited by plane P waves and SV waves is solved using an analytical approach. Numerical results are presented to illustrate the effects of the variation of groundwater level on the surface displacements under different incident angles and wave frequencies.

2. ANALYSIS MODEL

A cross-section of the 2D model we analyze is shown in Fig. 1. A cover soil layer with thickness H overlies the bedrock. The cover layer is divided into two layers: the saturated soil layer below the groundwater level is seemed as be water-saturated soils (water saturation S_r=100%), and the dry soil layer over the groundwater level is taken as air-saturated soils(water saturation S_r=0). The groundwater depth is h_1 below the ground surface. Using a Cartesian system, $z_1 = 0$ represents the ground surface. z_3 =0 represents the interface separating the underlying elastic bedrock and cover soil layer. The in-plane waves of the free field excited by P (SV) wave incidence are also shown in Fig. 1.

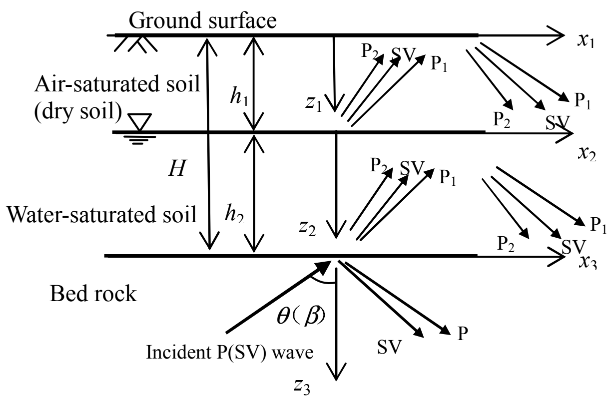

Figure 1. Analysis model

The bed rock is modeled by elastic medium. The cover soil is considered as poroelastic media that follows Biot (1956)'s theory. The saturated soil layer below the groundwater level is seemed as be water-saturated poroelastic media, and the dry soil layer over the groundwater level is taken as air-saturated poroelastic media.

2.1. *Poroelastic medium*

Following Biot's theory, the motion equations of poroelastic medium in which the solid frame is isotropic elastic and the pore fluid is allowed only dilatational deformation can be expressed in terms of displacement vectors of solid skeleton and pore fluid as,

$$N\nabla^2\boldsymbol{u}^{\mathrm{s}}+\nabla\left[(A+N)e+Q\varepsilon\right]=\left(\rho_{11}\frac{\partial^2\boldsymbol{u}^{\mathrm{s}}}{\partial t^2}+\rho_{12}\frac{\partial^2\boldsymbol{u}^{\mathrm{f}}}{\partial t^2}\right)+b\left(\frac{\partial\boldsymbol{u}^{\mathrm{s}}}{\partial t}-\frac{\partial\boldsymbol{u}^{\mathrm{f}}}{\partial t}\right) \tag{1a}$$

$$\nabla\left[Qe+R\varepsilon\right]=\left(\rho_{12}\frac{\partial^2\boldsymbol{u}^{\mathrm{s}}}{\partial t^2}+\rho_{22}\frac{\partial^2\boldsymbol{u}^{\mathrm{f}}}{\partial t^2}\right)-b\left(\frac{\partial\boldsymbol{u}^{\mathrm{s}}}{\partial t}-\frac{\partial\boldsymbol{u}^{\mathrm{f}}}{\partial t}\right) \tag{1b}$$

Where $\boldsymbol{u}^{\mathrm{s}}$ and $\boldsymbol{u}^{\mathrm{f}}$ denote the displacement vectors of the solid frame and the pore fluid, respectively; $e=\nabla\cdot\boldsymbol{u}^{\mathrm{s}}$, $\varepsilon=\nabla\cdot\boldsymbol{u}^{\mathrm{f}}$; ρ_{11} and ρ_{12} are ρ_{22} are the dynamic mass coefficients, $\rho_{11}=(1-n)\rho_{\mathrm{s}}+\rho$, $\rho_{12}=-\rho$, $\rho_{22}=n\rho_{\mathrm{f}}+\rho$, ρ_{s} is the density of the solid material, ρ_{f} is the density of fluid, ρ is the inertia density induced by solid-fluid interaction; $b=n^2\eta/k_{\mathrm{p}}$ is the dissipative coefficient, n is the porosity, η is the coefficient of viscosity, k_{p} is the dynamic permeability coefficient in m^2. A , N , R and Q are the elastic moduli for the poroelastic medium. According to Biot and Willis (1957), the elastic moduli can be expressed by,

$$A=(K_{\mathrm{b}}-\frac{2}{3}\mu)+(\frac{K_{\mathrm{s}}^2}{K_{\mathrm{d}}-K_{\mathrm{b}}})\left[\left(1-\frac{K_{\mathrm{b}}}{K_{\mathrm{s}}}\right)-n\right]^2,\ N=\mu,\ R=\frac{n^2K_{\mathrm{s}}^2}{K_{\mathrm{d}}-K_{\mathrm{b}}},\ Q=n(1-\frac{K_{\mathrm{b}}}{K_{\mathrm{s}}}-n)\frac{K_{\mathrm{s}}^2}{K_{\mathrm{d}}-K_{\mathrm{b}}} \tag{2}$$

Where, $K_{\mathrm{d}}=K_{\mathrm{s}}\left[1+n\left(\frac{K_{\mathrm{s}}}{K_{\mathrm{f}}}-1\right)\right]$, K_{s} is the bulk modulus of the solid grain, K_{b} is the bulk modulus of the dry frame, K_{f} is the bulk modulus of fluid; λ and μ are the Lame constants of the dry frame. Based on the poroelastic model proposed by Biot, the isotropic stress-strain relation in the saturated poroelastic medium is

$$\sigma_{ij}^{\ \mathrm{s}}=2N\varepsilon_{ij}+\delta_{ij}\left(Ae+Q\varepsilon\right),\ \sigma^{\mathrm{f}}=Qe+R\varepsilon \tag{3}$$

Where $\sigma_{ij}^{\ \mathrm{s}}$ is the stress of the solid skeleton, σ^{f} is the stress of pore fluid, and $\varepsilon_{ij}=\dfrac{u_{i,j}+u_{j,i}}{2}$.

Based on Helmholtz theorem, the displacement vector $\boldsymbol{u}^{\mathrm{s}}$ for the solid frame depends on the scalar and vector displacement potentials, i.e. ϕ_{s} and $\boldsymbol{\psi}_{\mathrm{s}}$, as

$$\boldsymbol{u}^{\mathrm{s}}=\nabla\phi_{\mathrm{s}}+\nabla\times\boldsymbol{\psi}_{\mathrm{s}} \tag{4a}$$

and for the pore fluid, $\boldsymbol{u}^{\mathrm{f}}$ depends on ϕ_{f} nd $\boldsymbol{\psi}_{\mathrm{f}}$, similarly

$$\boldsymbol{u}^{\mathrm{f}}=\nabla\phi_{\mathrm{f}}+\nabla\times\boldsymbol{\psi}_{\mathrm{f}} \tag{4b}$$

Note that the soil skeleton of the saturated soil layer below the groundwater level is the same as that of the dry soil layer over the groundwater level. As a result, the parameters ρ_{s} , K_{s} , λ and μ are same separately for the soil layers below and over the groundwater. For the saturated soil layer below the groundwater level, the bulk modulus of fluid $K_{\mathrm{f}}=K_{\mathrm{w}}$ (K_{w} is the bulk modulus of water), the density of fluid $\rho_{\mathrm{f}}=\rho_{\mathrm{w}}$ (ρ_{w} is the density of water）; while for the dry soil layer over the groundwater level, $K_{\mathrm{f}}=K_{\mathrm{a}}$ (K_{a} is the bulk modulus of air)，$\rho_{\mathrm{f}}=\rho_{\mathrm{a}}$ (ρ_{a} is the density of air). It is assumed that the change of groundwater level does not cause the change of soil porosity in this analysis.

2.2. *Elastic medium*

The governing equation for linear elastic rock foundation is of the form (Eringen, 1975)

$$\left(\lambda_{\mathrm{j}}+\mu_{\mathrm{j}}\right)\mathrm{grad}\,\theta+\mu_{\mathrm{j}}\nabla^2\boldsymbol{u}^{(\mathrm{j})}=\rho_{\mathrm{j}}\frac{\partial^2\boldsymbol{u}^{(\mathrm{j})}}{\partial t^2} \tag{5}$$

where $\theta=\mathrm{div}\boldsymbol{u}'$, λ_{j} and μ_{j} are Lame's constants, ρ_{j} denotes the solid density, $\boldsymbol{u}^{(\mathrm{j})}$ is the displacement vectors and may be written as

$$\boldsymbol{u}^{(j)} = \nabla\phi_j + \nabla\times\boldsymbol{\psi}_j \tag{6}$$

where ϕ_j and $\boldsymbol{\psi}_j$ are scalar and vector displacement potentials, respectively.

3. FORMULATION OF THE SYSTEM

We now consider an incident plane P wave (or SV wave) with angular frequency ω, angle of incidence θ (β for SV wave) and amplitude A_j^i (B_j^i for an SV wave) crosses the elastic-poroelastic medium interface from the bed-rock half-space, As shown in Figure 1. The displacement and propagation vectors are situated in the *x-z* plane. In the bed-rock region where $z_3 \ge 0$, a reflected P wave with amplitude A_j^r and an SV wave with B_j^r are generated. In the saturated soil region where $0 \le z_2 \le h_2$, transmitted P$_1$, P$_2$, and SV waves with amplitudes $A_{s1}^{(w)u}$, $A_{s2}^{(w)u}$ and $B_s^{(w)u}$ as well as reflected P$_1$, P$_2$, and S waves with amplitudes $A_{s1}^{(w)d}$, $A_{s2}^{(w)d}$ and $B_s^{(w)d}$ are generated. In the dry soil region where $0 \le z_1 \le h_1$, transmitted P$_1$, P$_2$, and SV waves with amplitudes $A_{s1}^{(a)u}$, $A_{s2}^{(a)u}$ and $B_s^{(a)u}$ as well as reflected P$_1$, P$_2$, and S waves with amplitudes $A_{s1}^{(a)d}$, $A_{s2}^{(a)d}$ and $B_s^{(a)d}$ are generated. The displacement potentials of all of these waves can be defined as follows.

In the bed-rock region where $z_3 \ge 0$,

For an incident P wave,

$$\phi_j^i = A_j^i \exp\left[i\left(\omega t - k_{jp}x_3\sin\theta + k_{jp}z_3\cos\theta\right)\right] \tag{7a}$$

For an incident SV wave,

$$\psi_j^i = B_j^i \exp\left[i\left(\omega t - k_{js}x_3\sin\beta + k_{js}z_3\cos\beta\right)\right] \tag{7b}$$

The reflected P wave, $\phi_j^r = A_j^r \exp\left[i\left(\omega t - k_{jp}x_3\sin\theta - k_{jp}z_3\cos\theta\right)\right]$ (8a)

The reflected SV wave, $\psi_j^r = B_j^r \exp\left[i\left(\omega t - k_{js}x_3\sin\beta - k_{js}z_3\cos\beta\right)\right]$ (8b)

In which k_{jp} and k_{js} are the wave numbers of the incident P wave and SV wave.

In the saturated soil region where $0 \le z_2 \le h_2$,

For the solid skeleton,

$$\begin{aligned}\phi_s^{(w)} &= A_{s1}^{(w)u}\exp\left[i\left(\omega t - k_{p1x}^{(w)}x_1 + k_{p1z}^{(w)}z_1\right)\right] + A_{s1}^{(w)d}\exp\left[i\left(\omega t - k_{p1x}^{(w)}x_1 - k_{p1z}^{(w)}z_1\right)\right]\\ &+ A_{s2}^{(w)u}\exp\left[i\left(\omega t - k_{p2x}^{(w)}x_1 + k_{p2z}^{(w)}z_1\right)\right] + A_{s2}^{(w)d}\exp\left[i\left(\omega t - k_{p2x}^{(w)}x_1 - k_{p2z}^{(w)}z_1\right)\right]\end{aligned} \tag{9a}$$

$$\psi_s^{(w)} = B_s^{(w)u}\exp\left[i\left(\omega t - k_{sx}^{(w)}x_1 + k_{sz}^{(w)}z_1\right)\right] + B_s^{(w)d}\exp\left[i\left(\omega t - k_{sx}^{(w)}x_1 + k_{sz}^{(w)}z_1\right)\right] \tag{9b}$$

For the pore fluid,

$$\begin{aligned}\phi_f^{(w)} &= \xi_1^{(w)}A_{s1}^{(w)u}\exp\left[i\left(\omega t - k_{p1x}^{(w)}x_1 + k_{p1z}^{(w)}z_1\right)\right] + \xi_1^{(w)}A_{s1}^{(w)d}\exp\left[i\left(\omega t - k_{p1x}^{(w)}x_1 - k_{p1z}^{(w)}z_1\right)\right]\\ &+ \xi_2^{(w)}A_{s2}^{(w)u}\exp\left[i\left(\omega t - k_{p2x}^{(w)}x_1 + k_{p2z}^{(w)}z_1\right)\right] + \xi_2^{(w)}A_{s2}^{(w)d}\exp\left[i\left(\omega t - k_{p2x}^{(w)}x_1 - k_{p2z}^{(w)}z_1\right)\right]\end{aligned} \tag{9c}$$

$$\psi_f^{(w)} = \xi_3^{(w)}B_s^{(w)u}\exp\left[i\left(\omega t - k_{sx}^{(w)}x_1 + k_{sz}^{(w)}z_1\right)\right] + \xi_3^{(w)}B_s^{(w)d}\exp\left[i\left(\omega t - k_{sx}^{(w)}x_1 + k_{sz}^{(w)}z_1\right)\right] \tag{9d}$$

In the dry soil region where $0 \le z_1 \le h_1$,

For the solid skeleton,

$$\begin{aligned}\phi_s^{(a)} &= A_{s1}^{(a)u}\exp\left[i\left(\omega t - k_{p1x}^{(a)}x_1 + k_{p1z}^{(a)}z_1\right)\right] + A_{s1}^{(a)d}\exp\left[i\left(\omega t - k_{p1x}^{(a)}x_1 + k_{p1z}^{(a)}z_1\right)\right]\\ &+ A_{s2}^{(a)u}\exp\left[i\left(\omega t - k_{p1x}^{(a)}x_1 + k_{p1z}^{(a)}z_1\right)\right] + A_{s2}^{(a)d}\exp\left[i\left(\omega t - k_{p1x}^{(a)}x_1 + k_{p1z}^{(a)}z_1\right)\right]\end{aligned} \tag{10a}$$

$$\psi_s^{(w)} = B_s^{(a)u}\exp\left[i\left(\omega t - k_{sx}^{(a)}x_1 + k_{sz}^{(a)}z_1\right)\right] + B_s^{(a)d}\exp\left[i\left(\omega t - k_{sx}^{(a)}x_1 + k_{sz}^{(a)}z_1\right)\right] \tag{10b}$$

For the pore fluid,

$$\phi_{\mathrm{f}}^{(\mathrm{a})} = \xi_1^{(\mathrm{a})} A_{\mathrm{s1}}^{(\mathrm{a})\mathrm{u}} \exp\left[i\left(\omega t - k_{\mathrm{p1}x}^{(\mathrm{a})} x_1 + k_{\mathrm{p1}z}^{(\mathrm{a})} z_1\right)\right] + \xi_1^{(\mathrm{a})} A_{\mathrm{s1}}^{(\mathrm{a})\mathrm{d}} \exp\left[i\left(\omega t - k_{\mathrm{p1}x}^{(\mathrm{a})} x_1 + k_{\mathrm{p1}z}^{(\mathrm{a})} z_1\right)\right]$$
$$+\xi_2^{(\mathrm{a})} A_{\mathrm{s2}}^{(\mathrm{a})\mathrm{u}} \exp\left[i\left(\omega t - k_{\mathrm{p1}x}^{(\mathrm{a})} x_1 + k_{\mathrm{p1}z}^{(\mathrm{a})} z_1\right)\right] + \xi_2^{(\mathrm{a})} A_{\mathrm{s2}}^{(\mathrm{a})\mathrm{d}} \exp\left[i\left(\omega t - k_{\mathrm{p1}x}^{(\mathrm{a})} x_1 + k_{\mathrm{p1}z}^{(\mathrm{a})} z_1\right)\right] \tag{10c}$$

$$\psi_{\mathrm{f}}^{(\mathrm{a})} = \xi_3^{(\mathrm{a})} \exp\left[i\left(\omega t - k_{\mathrm{s}x}^{(\mathrm{a})} x_1 + k_{\mathrm{s}z}^{(\mathrm{a})} z_1\right)\right] + \xi_3^{(\mathrm{a})} B_{\mathrm{s}}^{(\mathrm{a})\mathrm{d}} \exp\left[i\left(\omega t - k_{\mathrm{s}x}^{(\mathrm{a})} x_1 + k_{\mathrm{s}z}^{(\mathrm{a})} z_1\right)\right] \tag{10d}$$

In equations (9) and (10), superscript 'u' and 'd' represent the transmitted and the reflected waves, '(a)' and '(w)' represent the air-saturated and water-saturated soil respectively; $\xi_1^{(\alpha)}$, $\xi_2^{(\alpha)}$, and $\xi_3^{(\alpha)}$ ($\alpha = \mathrm{a, w}$) denote the amplitude ratio of the solid skeleton potential to the pore fluid potential for P_1 wave, P_2 wave, and SV wave respectively; $k_{\mathrm{p1}x}^{(\alpha)}$, $k_{\mathrm{p2}x}^{(\alpha)}$, and $k_{\mathrm{s}x}^{(\alpha)}$ ($\alpha = \mathrm{a, w}$) are the apparent wave numbers in the x direction, $k_{\mathrm{p1}z}^{(\alpha)}$, $k_{\mathrm{p2}z}^{(\alpha)}$, and $k_{\mathrm{s}z}^{(\alpha)}$ ($\alpha = \mathrm{a, w}$) the apparent wave numbers in the z direction of P_1 wave, P_2 wave, and SV wave respectively, and

$$k_{\mathrm{p1}x}^{(\alpha)\,2} + k_{\mathrm{p1z}}^{(\alpha)\,2} = \left(\frac{\omega}{c_{\mathrm{p1}}^{(\alpha)}}\right)^2, \quad k_{\mathrm{p2}x}^{(\alpha)\,2} + k_{\mathrm{p2z}}^{(\alpha)\,2} = \left(\frac{\omega}{c_{\mathrm{p2}}^{(\alpha)}}\right)^2, \quad k_{\mathrm{s}x}^{(\alpha)2} + k_{\mathrm{s}z}^{(\alpha)2} = \left(\frac{\omega}{c_{\mathrm{s}}^{(\alpha)}}\right)^2 \tag{11}$$

where $c_{\mathrm{p1}}^{(\alpha)}$, $c_{\mathrm{p2}}^{(\alpha)}$, and $c_{\mathrm{s}}^{(\alpha)}$ ($\alpha = \mathrm{a, w}$) are the wave velocities of P_1 wave, P_2 wave, and SV wave respectively, and can be calculated according to Ref (Deresiewicz, 1962).
All these waves have the equal wave numbers in the x direction by Snell's law, that is
For an incident P wave,

$$k_{\mathrm{p1}x}^{(\mathrm{w})} = k_{\mathrm{p2}x}^{(\mathrm{w})} = k_{\mathrm{s}x}^{(\mathrm{w})} = k_{\mathrm{p1}x}^{(\mathrm{a})} = k_{\mathrm{p2}x}^{(\mathrm{a})} = k_{\mathrm{s}x}^{(\mathrm{a})} = k_{\mathrm{js}} \sin\beta = k_{\mathrm{jp}} \sin\theta \tag{12a}$$

For an incident SV wave,

$$k_{\mathrm{p1}x}^{(\mathrm{w})} = k_{\mathrm{p2}x}^{(\mathrm{w})} = k_{\mathrm{s}x}^{(\mathrm{w})} = k_{\mathrm{p1}x}^{(\mathrm{a})} = k_{\mathrm{p2}x}^{(\mathrm{a})} = k_{\mathrm{s}x}^{(\mathrm{a})} = k_{\mathrm{jp}} \sin\theta = k_{\mathrm{js}} \sin\beta \tag{12b}$$

3. BOUNDARY CONDITIONS AND SOLUTIONS

Assuming there is no energy exchange between air and water, the boundary conditions at the interface between the air saturated poroelastic medium and the water saturated poroelastic medium can be expressed as (Deresiewicz and Skalak, 1963),
Continuity of the normal and tangential stresses,

$$\left(\sigma_{zz}^{\mathrm{s(a)}} + \sigma^{\mathrm{f(a)}}\right)\Big|_{z_1 = h_1} = \left(\sigma_{zz}^{\mathrm{s(w)}} + \sigma^{\mathrm{f(w)}}\right)\Big|_{z_2 = 0}, \quad \sigma_{xz}^{\mathrm{s(a)}}\Big|_{z_1 = h_1} = \sigma_{xz}^{\mathrm{s(w)}}\Big|_{z_2 = 0} \tag{13a}$$

Continuity of the pore fluid pressures,

$$\sigma^{\mathrm{f(a)}}\Big|_{z_1 = h_1} = \sigma^{\mathrm{f(w)}}\Big|_{z_2 = 0} \tag{13b}$$

Continuity of the normal and tangential displacements of the solid skeleton,

$$u_z^{\mathrm{s(a)}}\Big|_{z_1 = h_1} = u_z^{\mathrm{s(w)}}\Big|_{z_2 = 0}, \quad u_x^{\mathrm{s(a)}}\Big|_{z_1 = h_1} = u_x^{\mathrm{s(w)}}\Big|_{z_2 = 0} \tag{13c}$$

Continuity of the normal displacements of fluid phase,

$$u_z^{\mathrm{f(a)}}\Big|_{z_1 = h_1} = u_z^{\mathrm{f(w)}}\Big|_{z_2 = 0} \tag{13d}$$

Considering the interface between the bed rock and the water saturated poroelastic medium is impermeable, the boundary conditions can be expressed as,
Continuity of the total normal and tangential stresses,

$$\sigma_{zz}^{(\mathrm{j})}\Big|_{z_3 = 0} = \sigma_{zz}^{\mathrm{s(w)}}\Big|_{z_2 = h_2} + \sigma^{\mathrm{f(w)}}\Big|_{z_2 = h_2}, \quad \sigma_{xz}^{\mathrm{j}}\Big|_{z_3 = 0} = \sigma_{xz}^{\mathrm{s(w)}}\Big|_{z_2 = h_2} \tag{14a}$$

Continuity of the normal and tangential displacements of the bed rock and the solid skeleton of the water saturated proelastic medium,

$$u_x^{(\mathrm{j})}\Big|_{z_3 = 0} = u_x^{\mathrm{s(w)}}\Big|_{z_2 = h_2}, \quad u_z^{(\mathrm{j})}\Big|_{z_3 = 0} = u_z^{\mathrm{s(w)}}\Big|_{z_2 = h_2} \tag{14b}$$

Continuity of the normal displacements of the bed rock and the fluid phase of the water saturated proelastic medium,

$$u_z^{\mathrm{f(w)}}\Big|_{z_2 = h_2} = u_z^{\mathrm{s(w)}}\Big|_{z_2 = h_2} \tag{14c}$$

The traction-free boundary condition at the surface of the dry soil layer can be written as,

$$\sigma_{zz}^{\mathrm{s(a)}}\Big|_{z_1=0} = \sigma_{xz}^{\mathrm{s(a)}}\Big|_{z_1=0} = \sigma^{\mathrm{f(a)}}\Big|_{z_1=0} = 0 \tag{15}$$

Where superscript '(j)' represents the bed rock, 's' and 's' represent the solid skeleton and fluid phase of the saturated soil respectively.

Substituting Eqs.(7)-(10) into the boundary condition in Eqs.(13)-(15), where the stress and displacement components are given by Eqs.(3), (4), and (6), the following expression for the unknown amplitudes are obtained.

For an incident P wave,

$$\boldsymbol{A}\boldsymbol{X}^{\mathrm{T}} = \boldsymbol{B}^{\mathrm{T}} \tag{16}$$

where, $\boldsymbol{A} = \lfloor a_{ij} \rfloor$($i$, j=1, 2, …, 14), the elements of the matrixes $\boldsymbol{A}$ can be found in the Appendix,

$$\mathbf{X} = \left[A_{\mathrm{j}}^{\mathrm{r}} \;\; B_{\mathrm{j}}^{\mathrm{r}} \;\; A_{\mathrm{s1}}^{\mathrm{(w)u}} \;\; A_{\mathrm{s1}}^{\mathrm{(w)d}} \;\; A_{\mathrm{s2}}^{\mathrm{(w)u}} \;\; A_{\mathrm{s2}}^{\mathrm{(w)d}} \;\; B_{\mathrm{s}}^{\mathrm{(w)u}} \;\; B_{\mathrm{s}}^{\mathrm{(w)d}} \;\; A_{\mathrm{s1}}^{\mathrm{(a)u}} \;\; A_{\mathrm{s1}}^{\mathrm{(a)d}} \;\; A_{\mathrm{s2}}^{\mathrm{(a)u}} \;\; A_{\mathrm{s2}}^{\mathrm{(a)d}} \; B_{\mathrm{s}}^{\mathrm{(a)u}} \;\; B_{\mathrm{s}}^{\mathrm{(a)d}} \right],$$

$$\boldsymbol{B} = A_{\mathrm{j}}^{\mathrm{i}} \left[0 \;\; 0 \;\; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; -a_{10,1} \;\; a_{11,1} \;\; 0 \;\; a_{13,1} \;\; -a_{14,1} \right].$$

For an incident SV wave,

$$\mathbf{A}\mathbf{X}^{\mathrm{T}} = \mathbf{C}^{\mathrm{T}} \tag{17}$$

where, $\mathbf{C} = B_{\mathrm{j}}^{\mathrm{i}} \left[0 \;\; 0 \;\; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \;\; a_{10,2} \;\; -a_{11,2} \;\; 0 \; -a_{13,2} \;\; a_{14,2} \right].$

Solving equations (16) and (17), the unknown amplitudes $A_{\mathrm{j}}^{\mathrm{r}}$, $B_{\mathrm{j}}^{\mathrm{r}}$, $A_{\mathrm{s1}}^{\mathrm{(w)u}}$, $A_{\mathrm{s1}}^{\mathrm{(w)d}}$, $A_{\mathrm{s2}}^{\mathrm{(w)u}}$, $A_{\mathrm{s2}}^{\mathrm{(w)d}}$, $B_{\mathrm{s}}^{\mathrm{(w)u}}$, $B_{\mathrm{s}}^{\mathrm{(w)d}}$, $A_{\mathrm{s1}}^{\mathrm{(a)u}}$, $A_{\mathrm{s1}}^{\mathrm{(a)d}}$, $A_{\mathrm{s2}}^{\mathrm{(a)u}}$, $A_{\mathrm{s2}}^{\mathrm{(a)d}}$, $B_{\mathrm{s}}^{\mathrm{(a)u}}$ and $B_{\mathrm{s}}^{\mathrm{(a)d}}$ can be known, the wave potential functions can be determined accordingly.

4. EFFECTS OF THE VARIATION OF GROUNDWATER LEVEL ON EARTHQUAKE GROUND MOTIONS

From the viewpoints of earthquake engineering and strong motion seismology, an important aspect of the previous analysis is the description of the displacement amplitudes on the ground surface. Substituting Eq. (10) into Eq. (4), the horizontal displacement amplitude u_x and the vertical displacement amplitude u_z on the ground surface can be expressed as,

$$u_x = \left| -k_{\mathrm{p1}x}^{\mathrm{(a)}} A_{s1}^{\mathrm{(a)u}} - k_{\mathrm{p1}x}^{\mathrm{(a)}} A_{s1}^{\mathrm{(a)d}} - k_{\mathrm{p2}x}^{\mathrm{(a)}} A_{s2}^{\mathrm{(a)u}} - k_{\mathrm{p2}x}^{\mathrm{(a)}} A_{s2}^{\mathrm{(a)d}} + k_{\mathrm{s}z}^{\mathrm{(a)}} B_{\mathrm{s}}^{\mathrm{(a)u}} - k_{\mathrm{s}z}^{\mathrm{(a)}} B_{\mathrm{s}}^{\mathrm{(a)d}} \right| \tag{18a}$$

$$u_z = \left| k_{\mathrm{p1}z}^{\mathrm{(a)}} A_{s1}^{\mathrm{(a)u}} - k_{\mathrm{p1}z}^{\mathrm{(a)}} A_{s1}^{\mathrm{(a)d}} + k_{\mathrm{p2}z}^{\mathrm{(a)}} A_{s2}^{\mathrm{(a)u}} - k_{\mathrm{p2}z}^{\mathrm{(a)}} A_{s2}^{\mathrm{(a)d}} + k_{\mathrm{s}x}^{\mathrm{(a)}} B_{\mathrm{s}}^{\mathrm{(a)u}} + k_{\mathrm{s}x}^{\mathrm{(a)}} B_{\mathrm{s}}^{\mathrm{(a)d}} \right| \tag{18b}$$

In the analysis, the displacements are normalized by the displacement intensity of the incident wave. The dimensionless frequency ω/ω_1 (where ω_1 is the first natural frequency of the layer soil when the groundwater level is on the ground surface) is defined and used

The thickness of the cover soil layer H = 100m, the properties of the bed rock and the cover soil layer are as follows. Lame's constants of the bed rock are $\lambda_{\mathrm{j}} = \mu_{\mathrm{j}} = 5.3\times10^{9}\,\mathrm{Pa}$, the density of the bed rock is $\rho_{\mathrm{j}} = 2650\mathrm{kg/m^3}$. The density of the solid skeleton for the cover soil layer is ρ_{s} = 2700 kg/m^3, the inertia density induced by solid-fluid interaction ρ =0, other properties of the cover soil layer are listed in Table 1. Two groups of material constants for the cover soil are considered. For group 1, the cover soil is soft, the shear modulus of the solid skeleton with respect to the water bulk modulus is μ/K_{w} =0.03; for group 2, the cover soil is stiff, μ/K_{w}=0.5. The phase velocities of the cover soil for a frequency of 10Hz are also presented in the table.

Incident P wave condition is firstly considered. Figure 2 shows the horizontal and vertical displacement amplitudes on the ground surface (z_1=0) for the cover soil with the 1st group parameters versus the incident angle θ and for different depths of groundwater table (h_1=0.1H, 0.2H, 0.3H, and 0.4H) corresponding to three frequencies, ω/ω_1= 1.0, ω/ω_1= 2.0, and ω/ω_1= 5.0. It can be seen that the change of groundwater table has great influence on the displacement amplitudes when the cover soil is soft (i.e. the relative stiffness of soil skeleton μ/K_{w} is relatively small). And the effects strongly depend on the angle and frequency of incidence. At small frequencies (ω/ω_1=1.0), the effect of the change of groundwater table on the horizontal displacement amplitudes is obvious, and the effect first increases and then decreases with the incident angle increasing from 0° to 90°. At large frequencies (ω/ω_1=5.0), both the horizontal and vertical displacement amplitudes change obviously with the change of groundwater table. With the decline of the groundwater level, the vertical displacement amplitudes increase due to a vertical P wave incidence (θ =0°). The vertical displacement amplitudes for h_1=0.4H is about four times as big as for h_1=0.1H.

Table 1 The material parameters of the cover soil layer

Groups		ρ_f /(kg·m^{-3})	μ/Pa	K_s/Pa	K_b/Pa	K_f/Pa	n	k/m^2	η /Pa·s	c_{p1} /m/s	c_{p2} /m/s	c_s /m/s
1	Water-saturated	1000	0.66×10^8	1.1×10^8	3.6×10^{10}	2.2×10^9	0.47	1.0	1.0×10^{-3}	1558.3	48.6	187.7
	Air-saturated	1.2	0.66×10^8	1.1×10^8	3.6×10^{10}	1.0×10^{-5}	0.47	1.0	1.87×10^{-5}	375.6	12.0	216.7
2	Water-saturated	1000	11.0×10^8	16.3×10^8	3.6×10^{10}	2.2×10^9	0.47	1.0	1.0×10^{-3}	1969.2	157.7	766.1
	Air-saturated	1.2	11.0×10^8	16.3×10^8	3.6×10^{10}	1.0×10^{-5}	0.47	1.0	1.87×10^{-5}	1532.6	11.9	884.8

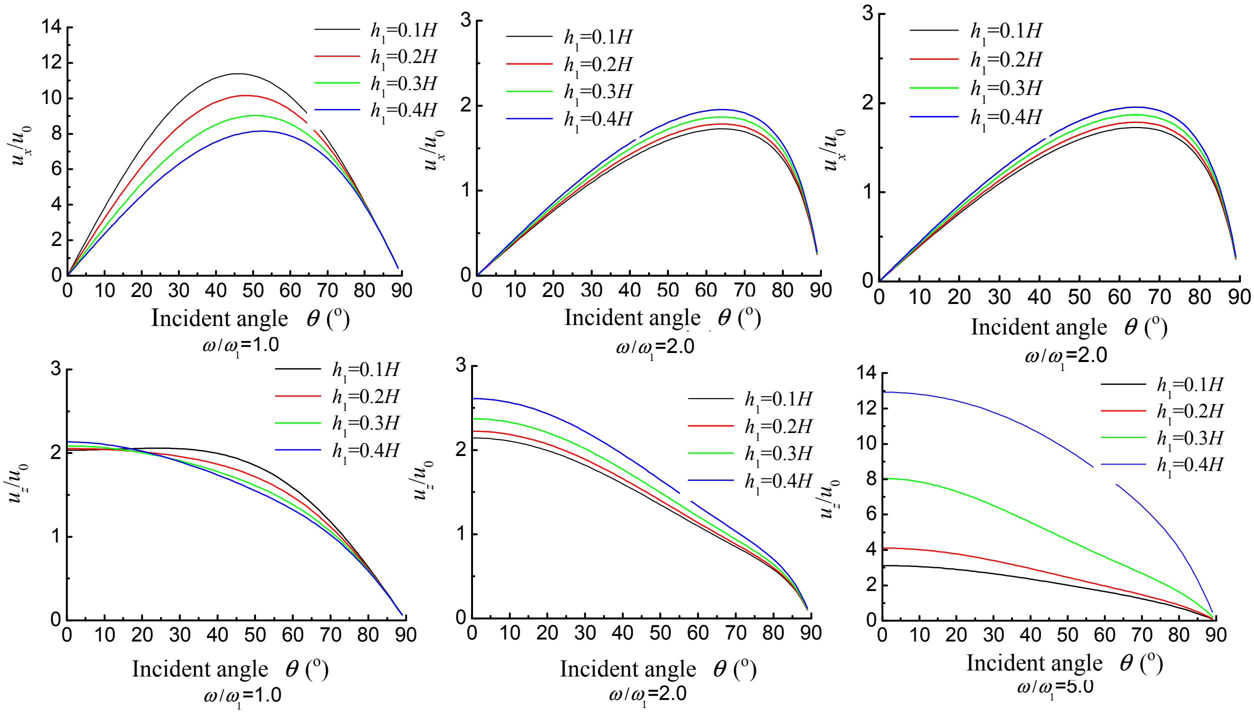

Figure 2. Displacement amplitude versus the incident angle under different groundwater levels for group 1 for an incident P wave

In order to better explain the influence of the frequency of incidence, Fig. 3 shows the horizontal and vertical displacement amplitudes on the ground surface for the cover soil with the 1st group parameters versus the dimensionless frequency ω/ω_1 for different depths of groundwater table with angles of incidence $\theta = 0°$ and $\theta = 30°$. As shown in the figures, the changes in the depths of groundwater table can cause substantial changes in the amplitude and frequency of the resonance peak. For vertical incidence $\theta = 0°$, the decline of the groundwater level produces a decrease in the natural frequencies and an increase in the response peaks of the vertical displacement amplitudes. For oblique incidence $\theta = 30°$, the response peaks of the vertical displacement amplitudes increase and the natural frequencies decrease with the decline of the groundwater level just as the condition $\theta = 0°$. While the changes in the depths of groundwater table have a slightly effect on the horizontal displacement amplitudes, the horizontal displacement amplitudes increase with the decline of the groundwater level, but the increase is small.

The changes should result from the difference of the phase velocity of the P_1 wave between the water saturated poroelastic and the air saturated poroelastic. With the decline of the groundwater level, the cover soil changes from water saturated into air saturated. When the cover soil is soft, for water saturated condition, the stiffness of the solid skeleton is smaller than the pore fluid (water). While for air saturated condition, the stiffness of the solid skeleton is stronger than the pore fluid (air). This difference will cause the change of the phase velocity of the P_1 wave. As shown in Table 1, the phase velocity of the P_1 wave in air saturated condition is obviously smaller than in water saturated condition for group 1.

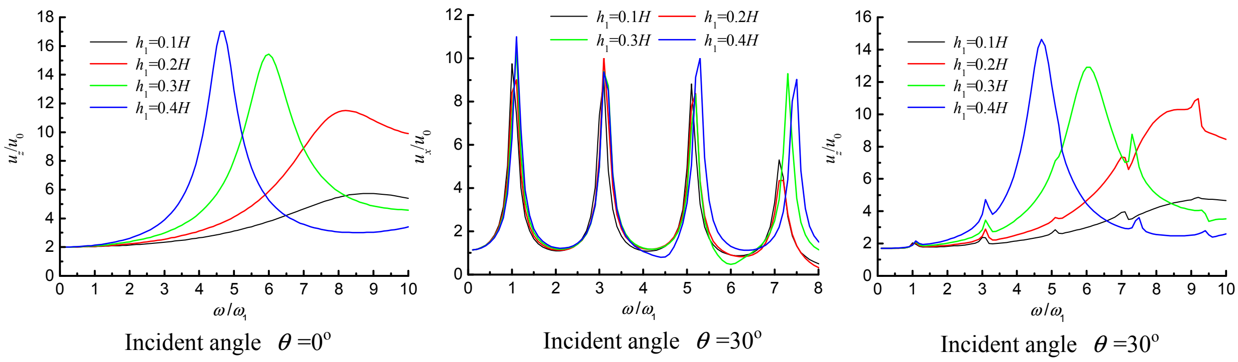

Figure 3. Displacement amplitude versus the relative frequency under different groundwater levels for group 1 for an incident P wave

To illustrate the effect of the soil skeleton on the above results, Figure 4 shows the horizontal and vertical displacement amplitudes on the ground surface (z_1=0) for the cover soil with the 2st group parameters versus the incident angle θ and for different depths of groundwater table corresponding to three frequencies, $\omega/\omega_1 = 1.0$, $\omega/\omega_1 = 2.0$, and $\omega/\omega_1 = 5.0$. The μ/K_w ratio of the soil with the 2st group parameters is relatively large. At this condition, the stiffness of the soil skeleton is in the same rank as the stiffness of water, the cover soil is solid skeleton dominated both for water saturated and air saturated. From the figure, it can be seen that, the change of groundwater table has little influence on the displacement amplitudes for large μ/K_w ratio. The phenomena that the displacement amplitudes on the ground surface for large μ/K_w ratio are smaller than that for small μ/K_w ratio also can be seen from the figure. For solid skeleton dominated case, the phase velocity of the P_1 wave in water saturated condition is similar with that in the air saturated condition which can be seen in Table 1.

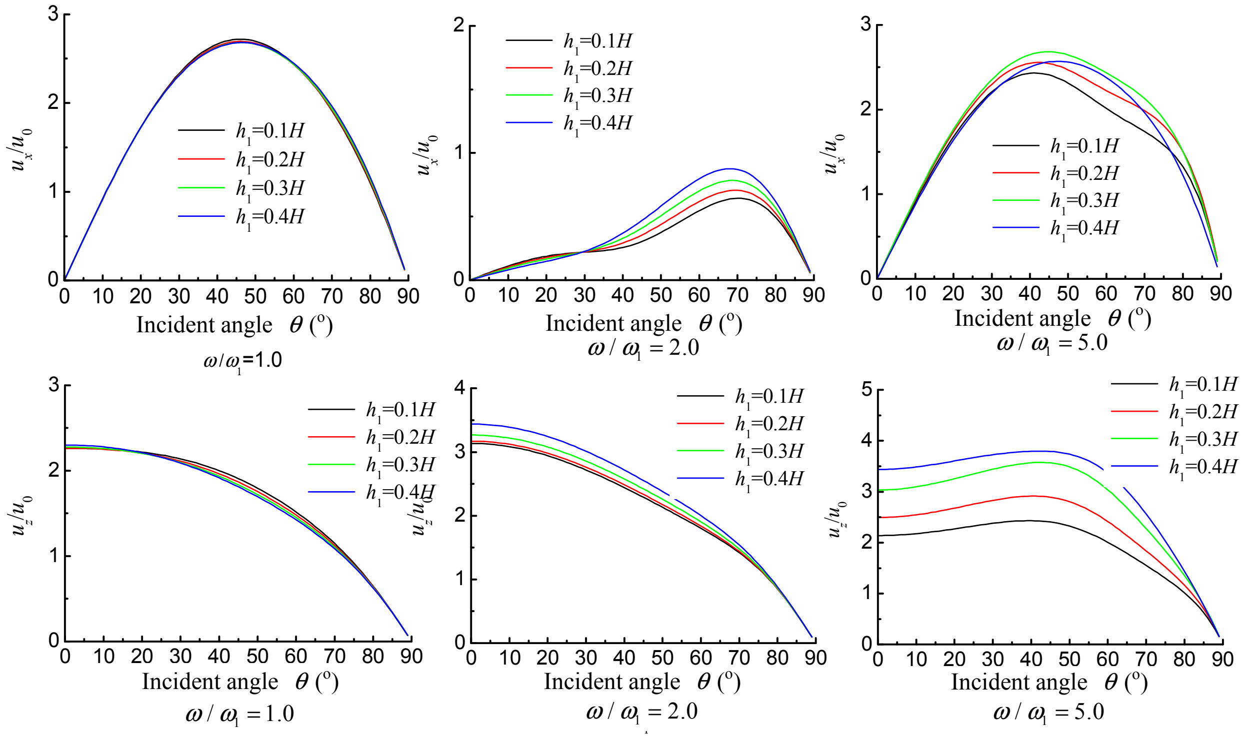

Figure 4. Displacement amplitude versus the incident angle under different groundwater levels for group 2 for an incident P wave

For incident SV wave condition, Figure 5 shows the horizontal and vertical displacement amplitudes on the ground surface (z_1=0) for the cover soil with the 1st group parameters versus the incident angle and for different depths of groundwater table ($h_1 = 0.1H$, $0.2H$, $0.3H$, and $0.4H$) corresponding to three frequencies, ω/ω_1 = 1.0, ω/ω_1 = 2.0, and ω/ω_1 = 5.0. It can be seen that the change of groundwater table also has great influence on the displacement amplitudes when the cover soil is soft. And the effects strongly depend on the angle and frequency of incidence.

5. CONCLUSIONS

The wave propagation in gas saturated soil is quite different from that in water saturated soil. Therefore, it can be preliminarily deduced that the change of groundwater table should have an effect on earthquake ground motions. Based on Biot's dynamic theory for saturated porous media, a layered foundation model is built in the paper, the saturated soil layer below the groundwater level is assumed to be water-saturated poroelastic medium, and the soil layer under the groundwater level is modeled as poroelastic medium. Analytical solutions of incident plane harmonic waves propagating in layered saturated poroelastic medium are established, and a study about the effects of the variation of groundwater level on earthquake ground motions is presented. Numerical results show that, the amplitudes of ground surface displacement are mainly relative to the variation of groundwater level if the relative stiffness of soil skeleton is small. With the decline of the groundwater level, the response peaks of ground surface displacement increase and the frequencies of the resonance peaks decrease. But when the relative stiffness of soil skeleton becomes large, the variation of groundwater level has little effects on earthquake ground motions. That is to say, for soft cover soil, the influence of the change of the groundwater level on the of earthquake ground motions should paid more attention.

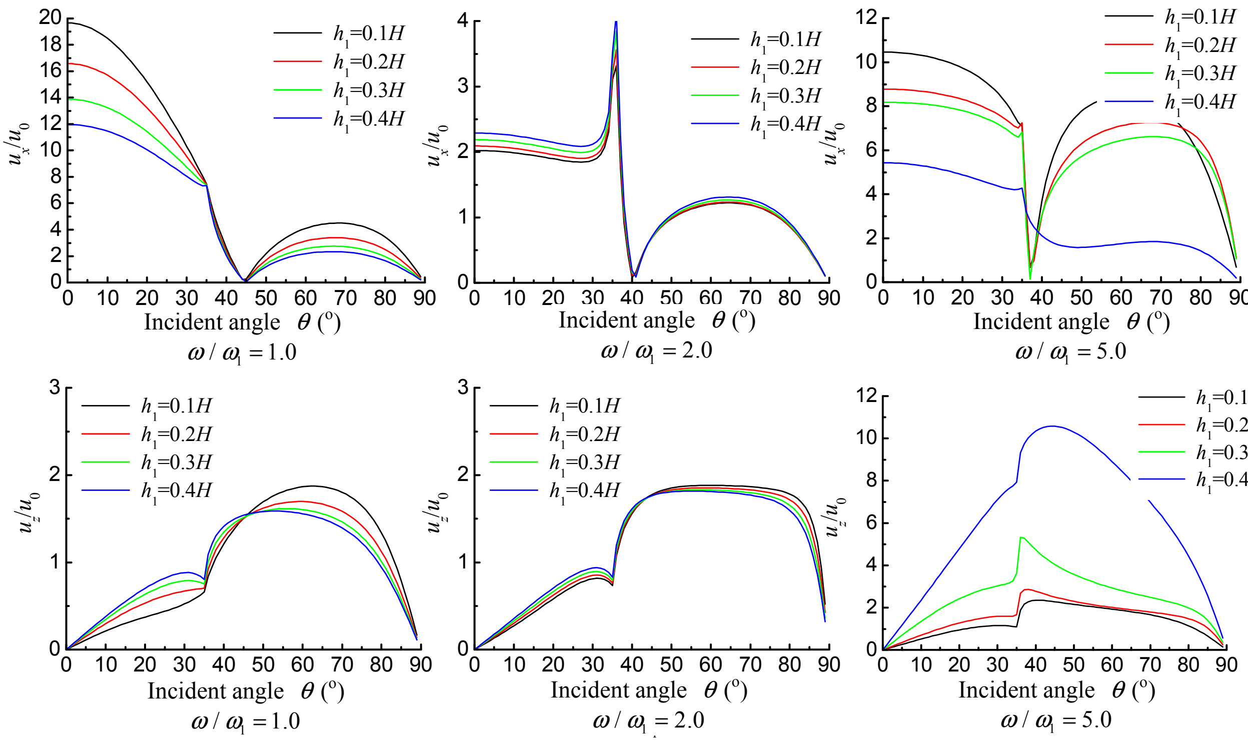

Figure 5. Displacement amplitude versus the incident angle under different groundwater levels for group 1 for an incident SV wave

ACKNOWLEDGEMENTS

The paper was edited by Elsevier Language Editing Services. The first author is grateful for the support of the National Natural Science Foundation of China (Grant No.51378058) and the National Basic Research Program of China (973 Program) (Grant No.2015CB057800).

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APPENDIX

The elements of the matrix $[a]$ in Equations (16) and (17) are as follows:

$a_{11}=a_{12}=a_{13}=a_{14}=a_{15}=a_{16}=a_{17}=a_{18}=0$, $a_{19}=a_{1,10}=\left(A^{(a)}+Q^{(a)}\xi_1^{(a)}\right)\left(k_{p1x}^{(a)\,2}+k_{p1z}^{(a)\,2}\right)+2Nk_{p1z}^{(a)\,2}$,

$a_{1,10}=a_{1,11}=\left(A^{(a)}+Q^{(a)}\xi_2^{(a)}\right)\left(k_{p2x}^{(a)\,2}+k_{p2z}^{(a)\,2}\right)+2Nk_{p2z}^{(a)\,2}$, $a_{1,13}=-a_{1,14}=2Nk_{sx}^{(a)}k_{sz}^{(a)}$;

$a_{21}=a_{22}=a_{23}=a_{24}=a_{25}=a_{26}=a_{27}=a_{28}=0$, $a_{29}=-a_{2,10}=2Nk_{p1x}^{(a)}k_{p1z}^{(a)}$, $a_{2,11}=-a_{2,12}=2Nk_{p2x}^{(a)}k_{p2z}^{(a)}$,

$a_{2,13}=a_{2,14}=N\left(k_{sx}^{(a)\,2}-k_{sz}^{(a)\,2}\right)$;

$a_{31}=a_{32}=a_{33}=a_{34}=a_{35}=a_{36}=a_{37}=a_{38}=0$, $a_{39}=a_{3,10}=\left(Q^{(a)}+R^{(a)}\xi_1^{(a)}\right)\left(k_{p1x}^{(a)\,2}+k_{p1z}^{(a)\,2}\right)$,

$a_{3,11}=a_{3,12}=\left(Q^{(a)}+R^{(a)}\xi_2^{(a)}\right)\left(k_{p2x}^{(a)\,2}+k_{p2z}^{(a)\,2}\right)$, $a_{3,13}=a_{3,14}=0$;

$a_{41}=a_{42}=0$, $a_{43}=a_{44}=k_{p1x}^{(w)}$, $a_{45}=a_{46}=k_{p2x}^{(w)}$, $a_{47}=-a_{48}=-k_{sz}^{(w)}$, $a_{49}=-k_{p1x}^{(a)}\exp\left(ik_{p1x}^{(a)}h_1\right)$, $a_{4,10}=-k_{p1x}^{(a)}\exp\left(-ik_{p1x}^{(a)}h_1\right)$,

$a_{4,11}=-k_{p2x}^{(a)}\exp\left(ik_{p2x}^{(a)}h_1\right)$, $a_{4,12}=-k_{p2x}^{(a)}\exp\left(-ik_{p2x}^{(a)}h_1\right)$, $a_{4,13}=k_{sz}^{(a)}\exp\left(ik_{sz}^{(a)}h_1\right)$, $a_{4,14}=-k_{sz}^{(a)}\exp\left(-ik_{sz}^{(a)}h_1\right)$;

$a_{51}=a_{52}=0$, $a_{53}=-a_{54}=k_{p1z}^{(w)}$, $a_{55}=-a_{56}=k_{p2z}^{(w)}$, $a_{57}=a_{58}=k_{sx}^{(w)}$, $a_{59}=-k_{p1z}^{(a)}\exp\left(ik_{p1z}^{(a)}h_1\right)$, $a_{5,10}=k_{p1z}^{(a)}\exp\left(-ik_{p1z}^{(a)}h_1\right)$,

$a_{5,11}=-k_{p2z}^{(a)}\exp\left(ik_{p2z}^{(a)}h_1\right)$, $a_{5,12}=k_{p2z}^{(a)}\exp\left(-ik_{p2z}^{(a)}h_1\right)$, $a_{5,13}=-k_{sx}^{(a)}\exp\left(ik_{sx}^{(a)}h_1\right)$, $a_{5,14}=-k_{sx}^{(a)}\exp\left(-ik_{sx}^{(a)}h_1\right)$;

$a_{61}=a_{62}=0$, $a_{63}=-a_{64}=\xi_1^{(w)}k_{p1z}^{(w)}$, $a_{65}=-a_{66}=\xi_2^{(w)}k_{p2z}^{(w)}$, $a_{67}=a_{68}=\xi_3^{(w)}k_{sx}^{(w)}$,

$a_{69}=-\xi_1^{(w)}k_{p1z}^{(a)}\exp\left(ik_{p1z}^{(a)}h_1\right)$, $a_{6,10}=\xi_1^{(w)}k_{p1z}^{(a)}\exp\left(-ik_{p1z}^{(a)}h_1\right)$, $a_{6,11}=-\xi_2^{(w)}k_{p2z}^{(a)}\exp\left(ik_{p2z}^{(a)}h_1\right)$,

$a_{6,12}=\xi_2^{(w)}k_{p2z}^{(a)}\exp\left(-ik_{p2z}^{(a)}h_1\right)$, $a_{6,13}=-\xi_3^{(w)}k_{sx}^{(a)}\exp\left(ik_{sx}^{(a)}h_1\right)$, $a_{6,14}=-\xi_3^{(w)}k_{sx}^{(a)}\exp\left(-ik_{sx}^{(a)}h_1\right)$;

$a_{71}=a_{72}=0$, $a_{77}=a_{78}=0$, $a_{73}=a_{74}=\left(Q^{(w)}+R^{(w)}\xi_1^{(w)}\right)\left(k_{p1x}^{(w)\,2}+k_{p1z}^{(w)\,2}\right)$, $a_{75}=a_{76}=\left(Q^{(w)}+R^{(w)}\xi_2^{(w)}\right)\left(k_{p2x}^{(w)\,2}+k_{p2z}^{(w)\,2}\right)$,

$a_{79}=-\left(Q^{(a)}+R^{(a)}\xi_1^{(a)}\right)\left(k_{p1x}^{(a)\,2}+k_{p1z}^{(a)\,2}\right)\exp\left(ik_{p1z}^{(a)}h_1\right)$, $a_{7,10}=-\left(Q^{(a)}+R^{(a)}\xi_1^{(a)}\right)\left(k_{p1x}^{(a)\,2}+k_{p1z}^{(a)\,2}\right)\exp\left(-ik_{p1z}^{(a)}h_1\right)$,

$a_{7,11}=-\left(Q^{(a)}+R^{(a)}\xi_2^{(a)}\right)\left(k_{p2x}^{(a)\,2}+k_{p2z}^{(a)\,2}\right)\exp\left(ik_{p2z}^{(a)}h_1\right)$, $a_{7,12}=-\left(Q^{(a)}+R^{(a)}\xi_2^{(a)}\right)\left(k_{p2x}^{(a)\,2}+k_{p2z}^{(a)\,2}\right)\exp\left(-ik_{p2z}^{(a)}h_1\right)$,

$a_{7,13}=a_{7,14}=0$;

$a_{81}=a_{82}=0$, $a_{83}=a_{84}=\left(A^{(w)}+Q^{(w)}\xi_1^{(w)}\right)\left(k_{p1x}^{(w)\,2}+k_{p1z}^{(w)\,2}\right)+2Nk_{p1z}^{(w)\,2}$,

$a_{85}=a_{86}=\left(A^{(w)}+Q^{(w)}\xi_2^{(w)}\right)\left(k_{p2x}^{(w)\,2}+k_{p2z}^{(w)\,2}\right)+2Nk_{p2z}^{(w)\,2}$, $a_{87}=-a_{88}=2Nk_{sx}^{(w)}k_{sz}^{(w)}$;

$a_{89}=-\left[\left(A^{(a)}+Q^{(a)}\xi_1^{(a)}\right)\left(k_{p1x}^{(a)\,2}+k_{p1z}^{(a)\,2}\right)+2Nk_{p1z}^{(a)\,2}\right]\exp\left(ik_{p1z}^{(a)}h_1\right)$, $a_{8,10}=-\left[\left(A^{(a)}+Q^{(a)}\xi_1^{(a)}\right)\left(k_{p1x}^{(a)\,2}+k_{p1z}^{(a)\,2}\right)+2Nk_{p1z}^{(a)\,2}\right]\exp\left(-ik_{p1z}^{(a)}h_1\right)$,

$a_{8,11}=-\left[\left(A^{(a)}+Q^{(a)}\xi_2^{(a)}\right)\left(k_{p2x}^{(a)\,2}+k_{p2z}^{(a)\,2}\right)+2Nk_{p2z}^{(a)\,2}\right]\exp\left(ik_{p2z}^{(a)}h_1\right)$, $a_{8,12}=-\left[\left(A^{(a)}+Q^{(a)}\xi_2^{(a)}\right)\left(k_{p2x}^{(a)\,2}+k_{p2z}^{(a)\,2}\right)+2Nk_{p2z}^{(a)\,2}\right]\exp\left(-ik_{p2z}^{(a)}h_1\right)$,

$a_{8,13}=-2Nk_{sx}^{(a)}k_{sz}^{(a)}\exp\left(ik_{sz}^{(a)}h_1\right)$, $a_{8,14}=2Nk_{sx}^{(a)}k_{sz}^{(a)}\exp\left(-ik_{sz}^{(a)}h_1\right)$;

$a_{91}=a_{92}=0$, $a_{93}=-a_{94}=2Nk_{p1x}^{(w)}k_{p1z}^{(w)}$, $a_{95}=-a_{96}=2Nk_{p2x}^{(w)}k_{p2z}^{(w)}$, $a_{97}=a_{98}=N\left(k_{sx}^{(w)\,2}-k_{sz}^{(w)\,2}\right)$,

$a_{99}=-2Nk_{p1x}^{(a)}k_{p1z}^{(a)}\exp\left(ik_{p1z}^{(a)}h_1\right)$, $a_{9,10}=2Nk_{p1x}^{(a)}k_{p1z}^{(a)}\exp\left(-ik_{p1z}^{(a)}h_1\right)$, $a_{9,11}=-2Nk_{p2x}^{(a)}k_{p2z}^{(a)}\exp\left(ik_{p2z}^{(a)}h_1\right)$,

$a_{9,12}=2Nk_{p2x}^{(a)}k_{p2z}^{(a)}\exp\left(-ik_{p2z}^{(a)}h_1\right)$, $a_{9,13}=-N\left(k_{sx}^{(a)\,2}-k_{sz}^{(a)\,2}\right)\exp\left(ik_{sz}^{(a)}h_1\right)$, $a_{9,14}=-N\left(k_{sx}^{(a)\,2}-k_{sz}^{(a)\,2}\right)\exp\left(-ik_{sz}^{(a)}h_1\right)$;

$a_{10,1}=-k_{jp}\sin\theta$, $a_{10,2}=-k_{js}\cos\beta$, $a_{10,3}=k_{p1x}^{(w)}\exp\left(ik_{p1z}^{(w)}h_2\right)$, $a_{10,4}=k_{p1x}^{(w)}\exp\left(-ik_{p1z}^{(w)}h_2\right)$,

$a_{10,5}=k_{p2x}^{(w)}\exp\left(ik_{p2z}^{(w)}h_2\right)$, $a_{10,6}=k_{p2x}^{(w)}\exp\left(-ik_{p2z}^{(w)}h_2\right)$, $a_{10,7}=-k_{sz}^{(w)}\exp\left(ik_{sz}^{(w)}h_2\right)$, $a_{10,8}=k_{sz}^{(w)}\exp\left(-ik_{sz}^{(w)}h_2\right)$,

$a_{10,9}=a_{10,10}=a_{10,11}=a_{10,12}=a_{10,13}=a_{10,14}=0$;

$a_{11,1}=k_{jp}\cos\theta$, $a_{11,2}=-k_{js}\sin\beta$, $a_{11,3}=k_{p1z}^{(w)}\exp\left(ik_{p1z}^{(w)}h_2\right)$, $a_{11,4}=-k_{p1z}^{(w)}\exp\left(-ik_{p1z}^{(w)}h_2\right)$, $a_{11,5}=k_{p2z}^{(w)}\exp\left(ik_{p2z}^{(w)}h_2\right)$,

$a_{11,6}=-k_{p2z}^{(w)}\exp\left(-ik_{p2z}^{(w)}h_2\right)$, $a_{11,7}=k_{sx}^{(w)}\exp\left(ik_{sz}^{(w)}h_2\right)$, $a_{11,8}=k_{sx}^{(w)}\exp\left(-ik_{sz}^{(w)}h_2\right)$,

$a_{11,9}=a_{11,10}=a_{11,11}=a_{11,12}=a_{11,13}=a_{11,14}=0$;

$a_{12,1}=a_{12,2}=0$, $a_{12,3}=\left(1-\xi_1^{(w)}\right)k_{p1z}^{(w)}\exp\left(ik_{p1z}^{(w)}h_2\right)$, $a_{12,4}=-\left(1-\xi_1^{(w)}\right)k_{p1z}^{(w)}\exp\left(-ik_{p1z}^{(w)}h_2\right)$,

$a_{12,5}=\left(1-\xi_2^{(w)}\right)k_{p2z}^{(w)}\exp\left(ik_{p2z}^{(w)}h_2\right)$, $a_{12,6}=-\left(1-\xi_2^{(w)}\right)k_{p2z}^{(w)}\exp\left(-ik_{p2z}^{(w)}h_2\right)$, $a_{12,7}=\left(1-\xi_3^{(w)}\right)k_{sx}^{(w)}\exp\left(ik_{sz}^{(w)}h_2\right)$,

$a_{12,8}=\left(1-\xi_3^{(w)}\right)k_{sx}^{(w)}\exp\left(-ik_{sz}^{(w)}h_2\right)$, $a_{12,9}=a_{12,10}=a_{12,11}=a_{12,12}=a_{12,13}=a_{12,14}=0$;

$a_{13,1}=2\mu_j k_{jp}^2\sin\theta\cos\theta$, $a_{13,2}=\mu_j k_{js}^2\left(\cos^2\beta-\sin^2\beta\right)$, $a_{13,3}=2Nk_{p1x}^{(w)}k_{p1z}^{(w)}\exp\left(ik_{p1z}^{(w)}h_2\right)$, $a_{13,4}=-2Nk_{p1x}^{(w)}k_{p1z}^{(w)}\exp\left(-ik_{p1z}^{(w)}h_2\right)$,

$a_{13,5}=2Nk_{p2x}^{(w)}k_{p2z}^{(w)}\exp\left(ik_{p2z}^{(w)}h_2\right)$, $a_{13,6}=-2Nk_{p2x}^{(w)}k_{p2z}^{(w)}\exp\left(-ik_{p2z}^{(w)}h_2\right)$, $a_{13,7}=-N\left(k_{sz}^{(w)\,2}-k_{sx}^{(w)\,2}\right)\exp\left(ik_{sz}^{(w)}h_2\right)$,

$a_{13,8}=-N\left(k_{sz}^{(w)\,2}-k_{sx}^{(w)\,2}\right)\exp\left(-ik_{sz}^{(w)}h_2\right)$, $a_{13,9}=a_{13,10}=a_{13,11}=a_{13,12}=a_{13,13}=a_{13,14}=0$;

$a_{14,1}=-\left[\lambda_j k_{jp}^2\sin^2\theta+\left(\lambda_j+2\mu_j\right)k_{jp}^2\cos^2\theta\right]$, $a_{14,2}=2\mu_j k_{js}^2\sin\beta\cos\beta$,

$a_{14,3}=\left[\left(A^{(w)}+Q^{(w)}+\left(Q^{(w)}+R^{(w)}\right)\xi_1^{(w)}\right)\left(k_{p1x}^{(w)2}+k_{p1z}^{(w)2}\right)+2Nk_{p1z}^{(w)2}\right]\exp\left(ik_{p1z}^{(w)}h_2\right)$,

$a_{14,4}=\left[\left(A^{(w)}+Q^{(w)}+\left(Q^{(w)}+R^{(w)}\right)\xi_1^{(w)}\right)\left(k_{p1x}^{(w)2}+k_{p1z}^{(w)2}\right)+2Nk_{p1z}^{(w)2}\right]\exp\left(-ik_{p1z}^{(w)}h_2\right)$,

$a_{14,5}=\left[\left(A^{(w)}+Q^{(w)}+\left(Q^{(w)}+R^{(w)}\right)\xi_1^{(w)}\right)\left(k_{p2x}^{(w)2}+k_{p2z}^{(w)2}\right)+2Nk_{p2z}^{(w)2}\right]\exp\left(ik_{p2z}^{(w)}h_2\right)$,

$a_{14,6}=\left[\left(A^{(w)}+Q^{(w)}+\left(Q^{(w)}+R^{(w)}\right)\xi_2^{(w)}\right)\left(k_{p2x}^{(w)2}+k_{p2z}^{(w)2}\right)+2Nk_{p2z}^{(w)2}\right]\exp\left(-ik_{p2z}^{(w)}h_2\right)$,

$a_{14,7}=2Nk_{sx}^{(w)}k_{sz}^{(w)}\exp\left(ik_{sz}^{(w)}h_2\right)$, $a_{14,8}=-2Nk_{sx}^{(w)}k_{sz}^{(w)}\exp\left(-ik_{sz}^{(w)}h_2\right)$,

$a_{14,9}=a_{14,10}=a_{14,11}=a_{14,12}=a_{14,13}=a_{14,14}=0$.