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VIBRATION SUPPRESSION USING A PASSIVE REACTION MASS INCORPORATING INERTERS IN A MULTI-STOREY BUILDING

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ABSTRACT

This paper investigates the use of a vibration suppression system consisting of a reaction mass connected to the structure at a single point via a combination of springs, dampers and inerters. The classical tuned mass damper (TMD) configuration is used as a benchmark. Specifically, a three-storey building model, with the device connected at the top floor, subjected to base excitation is analysed. In order to identify optimum absorber configurations, the structure-immittance approach is utilized, for minimising the maximum inter-storey drift. Using this approach, a full set of networks with pre-determined number of elements can be analysed systematically and the element parameters can be constrained or fixed at the same time. It is shown that with one damper, one inerter and no more than two springs, several absorber layouts can be identified with superior performance than a traditional TMD.

Keywords: Vibration suppression; Passive control; Inerter; Structure-immittance approach

1. INTRODUCTION

It is important to control civil engineering structures to improve their motion performance under dynamic loadings, especially for those subjected to seismic excitation. For decades, a large variety of passive devices have been proposed to control the structural motion and vibration. Currently, the most widely used passive strategy is based on energy dissipation mechanism (Soong et al. 1997), such as the traditional Tuned Mass Damper (TMD). TMD is a classical engineering device consisting of one mass, one spring and one viscous damper, which was first introduced by Frahm (1911) as a "dynamic vibration absorber". TMD is used to reduce the steady state response of the system by tuning the natural frequency of the TMD to the vicinity of the natural frequency of the system, which causes TMD to vibrate in resonance and dissipate the vibration energy of the system (Den Hartog 1956). Performance of the TMD will be improved as the mass of the TMD increasing. However, in most applications, the amount of mass that is acceptable to add to the original system is no more than 10% of mass of the targeted vibration mode (Lazar et al. 2014).

Recently, a novel passive mechanical element, the "inerter", was introduced by Smith (2002) and has received a large amount of attention. It is a two-terminal device with the property that the generated force is proportional to the relative acceleration across its two terminals. By using gearing mechanism, one significant advantage of inerter is that it can provide higher inertance than its real mass. Several inerter-incorporated base isolation absorber layouts were identified by Wang et al. (2010). Another seismic control device called tuned viscous mass damper (TVMD) consisting of a inerter-like ball screw mechanism was proposed by Ikago et al. (2012), which can generate magnificent inertance by using a flywheel with far smaller mass. Ball screw inertial damper was also studied by Takewaki et al. (2012) on multi-storey shear building to demonstrate its performance when subjected to different inertial damper allocations and earthquake ground motions. Later, a tuned inerter damper (TID) was proposed by Lazar et al. (2014). It is shown that the performance of a TID mounted at the bottom of a building is potentially better than a TMD mounted on the top of the structure.

With the invention of the inerter, the force-current analogy (Firestone 1933) between mechanical and electrical networks can be fully achieved with mechanical elements springs, dampers and inerters corresponding to the electrical elements inductors, resistors and capacitors, respectively. For passive electrical networks, using Bott, Duffin theorem (1949), it has been shown that any positive-real functions can be realized as the driving-point immittance of a network consisting of only resistors, capacitors and inductors. Hence, mechanical networks

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consisting of dampers, inerters and springs can also be obtained by synthesizing the positive-real functions. In general, two approaches are applied to identify beneficial passive vibration absorber, the structure-based approach and the immittance-based approach. For the structure-based approach, such as TID (Lazar et al. 2014) and TMDI (Marian et al. 2014), network layouts are first proposed. Parameter values for each of the elements are then selected based on performance criteria. With this approach, the complexity and topology of the absorber can be pre-determined. However, only one network layout can be considered at a time even though there are large number of possible layouts, which inevitably limits the achievable performance of the proposed absorber. On the other hand, with the immittance-based approach, such as the application proposed by Wang et al. (2012) and Li et al. (2016), a positive-real immittance function which is capable of providing the optimum performance of the system can be obtained firstly. Then, the corresponding network layout and element values can be identified using network synthesis theory. However, without pre-restricted complexity and topology of the network, undesirable networks with very complicated layouts may occur. To overcome the disadvantages of structure-based and immittance-based approaches, a novel structure-immittance approach was proposed by Zhang et al. (2017) for passive vibration absorber identification. With this approach, a new class of structural-immittance functions can be obtained, which cover a full set of network layouts with explicit information of all topology possibilities; meanwhile, the number of each element type can be pre-determined, and the element values can be fixed or constrained.

In this paper, a vibration suppression system consisting of a reaction mass connected to the top of the structure at a single point via a combination of springs, dampers and inerters is investigated. Structure-immittance approach is used to obtain the optimal absorber configurations. This paper is organized as follows: In section 2, the structure-immittance approach is demonstrated. In section 3, two cases with different number of elements are studied by utilizing the structure-immittance method to identify the optimal layouts, which are applied on a multi-storey building model to show the performance superiority compared with the traditional TMD. Finally, conclusions are drawn in section 4.

2. STRUCTURE-IMMITTANCE METHODOLOGY

By utilizing the force-current analogy, the structure-immittance method can be applied both on mechanical networks as well as on electrical networks. Therefore, the transfer function of the mechanical network is defined as

$$\mathrm{Y}(s)=\frac{\mathrm{F}(s)}{\mathrm{V}(s)} \tag{1}$$

where F(s) and V(s) are the force and relative velocity across the two terminals in Laplace domain, respectively. s is the complex frequency parameter of the Laplace transform. Mapping it to electrical system, the transfer function corresponds to the admittance of electrical networks, which is

$$\mathrm{Y}(s)=\frac{\mathrm{I}(s)}{\mathrm{U}(s)} \tag{2}$$

where I(s) and U(s) are the current and voltage across the two terminals in Laplace domain, respectively. One-port (two-terminal) mechanical networks are taken into consideration to develop structure-immittance method in this paper. The definition of structural-immittances, which include all the possible arrangements of a set number of elements, are given as the transfer functions from force to velocity. In this section, two mechanical systems as well as their structural-immittances are demonstrated, which are denoted as one spring case (including one inerter, one damper and one spring) and two springs case (including one inerter, one damper and two springs).

2.1 Networks of one inerter, one damper and one spring case

For the case with one inerter, one damper and one spring (i.e. one spring case), there are totally 8 absorber layouts. By using the structure-immittance method, two networks termed as Q_{11} and Q_{12}, which contain the possibilities of all the 8 layouts arrangements, can be obtained. There are 4 steps (Zhang et al. 2017) to find Q_{11} and Q_{12}. It should be noticed that the networks obtained in each step must satisfy the condition that at most one spring is present.

Step 1: Two generic sub-networks M_{11} and M_{12} are constructed as shown in step 1 of Figure 1. One is inerter based sub-network and the other is damper-based sub-network.

Step 2: Two sub-networks M_{11} and M_{12} are connected to each other both in parallel and in series to form two new networks N_{11} and N_{12}, as shown in step 2 of Figure 1.

Step 3: Since in networks N_{11} and N_{12}, some of the springs are redundant and can produce the same effect on the network, redundancy of the springs in networks N_{11} and N_{12} need to be checked and removed. For example, in network N_{11}, considering $0 < 1/k_1 < \infty$, $k_2 = 0$, $1/k_3 = 0$ and $k_4 = 0$, one layout with inerter, damper and spring in series connection can be obtained; considering $0 < 1/k_3 < \infty$, $k_2 = 0$, $1/k_1 = 0$ and $k_4 = 0$, same layout can be obtained. Hence, k_1 is redundant and can be removed. All of the springs in N_{11} and N_{12} should be checked one by one to guarantee that there is no redundancy left in the networks. As a result, two simplified networks P_{11} and P_{12} can be obtained, as shown in step 3 of Figure 1.

Step 4: Springs will be added to the network in parallel and in series again to form the final networks. The adding rule is: a spring will be first added in series and the redundancy will be checked for this added spring. Then, another spring will be added in parallel and the redundancy will also be checked. Repeat the same procedure until no new spring is needed (Note: it is the same when a spring is added in parallel first). Thus, the final networks can be constructed. Follow the adding rule, final networks of the one spring case can be obtained as Q_{11} and Q_{12} as shown in step 4 of Figure 1.

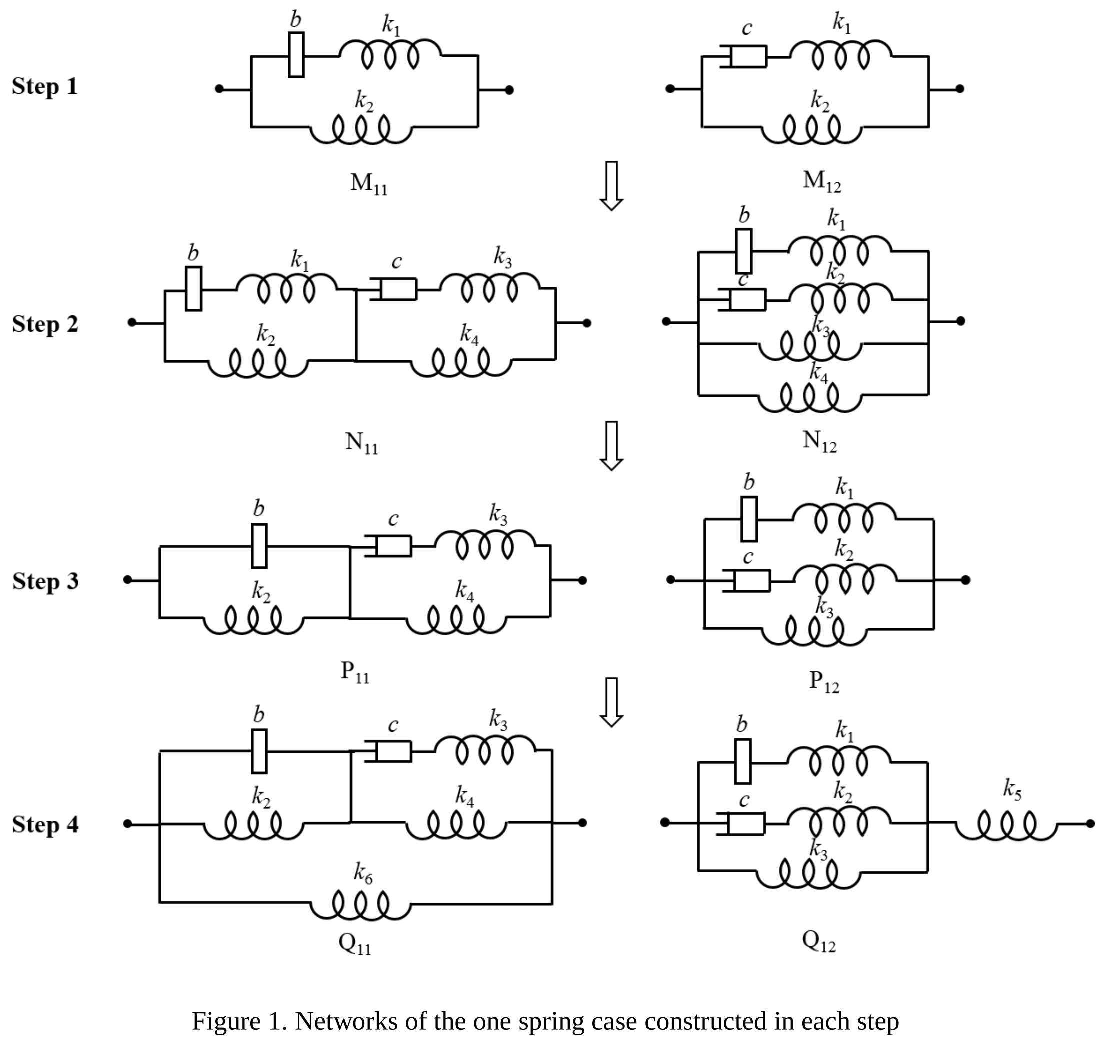


Figure 1. Networks of the one spring case constructed in each step

The structural-immittances of the networks Q_{11} and Q_{12} are respectively

$$Y_{11}(s)=\frac{bcs^2+b(k_4+k_6)s+c(k_2+k_6)}{bc(\frac{1}{k_3})s^3+bs^2+cs+k_2+k_4},$$

$$Y_{12}(s)=\frac{bc(\frac{1}{k_1}+\frac{1}{k_2})s^3+bs^2+cs+k_3}{b(\frac{1}{k_1}+\frac{1}{k_5})s^3+c(\frac{1}{k_2}+\frac{1}{k_5})s^2+s}. \tag{3}$$

For $Y_{11}(s)$, only one of k_2, $1/k_3$, k_4 and k_6 is positive and all the others should be equal to zero. Similarly, for $Y_{12}(s)$, only one of $1/k_1$, $1/k_2$, k_3 and $1/k_5$ is positive and all the others are equal to zero. In this way, $Y_{11}(s)$ and $Y_{12}(s)$ include all the transfer function expressions of the one inerter, one damper and one spring combinations. These transfer functions can be used, along with the constraints listed here to find the optimal one spring, one inerter, one damper layout and configuration for a given system and objective function.

2.2 Networks of one inerter, one damper and two springs case

For the case with one inerter, one damper and two springs (i.e. two springs case), there are totally 18 absorber layouts. By using the structure-immittance method, two networks termed as Q_{21} and Q_{22}, which contain the possibilities of all the 18 layouts arrangements, can be obtained. Similarly, 4 steps are needed to obtain the final networks Q_{21} and Q_{22}. It should also be noted that at most two springs are present for the networks obtained in each step.

Networks Q_{21} and Q_{22} are shown in Figure 2. Steps used to obtain Q_{21} and Q_{22} are similar to those used to obtain Q_{11} and Q_{12} in section 2.1, so they are not listed here.

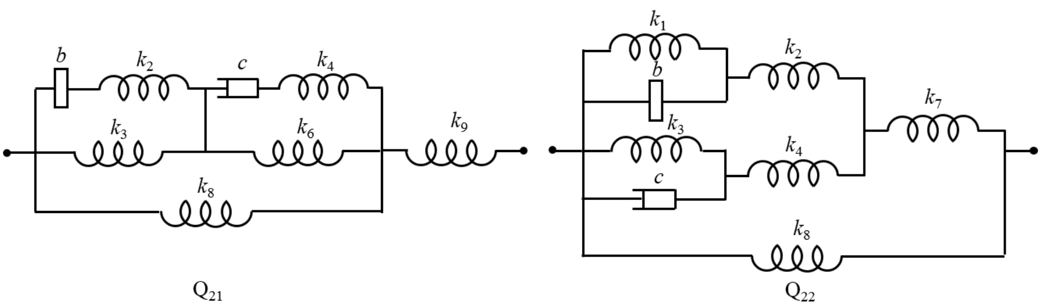


Figure 2. Networks Q_{21} and Q_{22} of the two springs case

Structural-immittances of the networks Q_{21} and Q_{22} are obtained as:

$$Y_{21}(s)=\frac{bc(\frac{k_3}{k_2}+\frac{k_8}{k_2}+\frac{k_6}{k_4}+\frac{k_8}{k_4}+1)s^3+b(k_6+k_8)s^2+c(k_3+k_8)s+k_3k_6+k_3k_8+k_6k_8}{s\left[bc(\frac{1}{k_2}+\frac{1}{k_4}+\frac{1}{k_9})s^3+b(\frac{k_3}{k_2}+\frac{k_6}{k_2}+\frac{k_6}{k_9}+\frac{k_8}{k_9}+1)s^2+c(\frac{k_3}{k_4}+\frac{k_3}{k_9}+\frac{k_6}{k_4}+\frac{k_8}{k_9}+1)s+k_3+k_6\right]},$$

$$Y_{22}(s)=\frac{bc(\frac{1}{k_2}+\frac{1}{k_4})s^3+b(\frac{k_3}{k_2}+\frac{k_3}{k_4}+\frac{k_8}{k_2}+\frac{k_8}{k_7}+1)s^2+c(\frac{k_1}{k_2}+\frac{k_8}{k_4}+\frac{k_1}{k_4}+\frac{k_8}{k_7}+1)s+k_1+k_3+k_8}{s\left[bc(\frac{1}{k_2k_4}+\frac{1}{k_2k_7}+\frac{1}{k_4k_7})s^3+b(\frac{1}{k_2}+\frac{1}{k_7})s^2+c(\frac{1}{k_4}+\frac{1}{k_7})s+\frac{k_1}{k_2}+\frac{k_1}{k_7}+\frac{k_3}{k_4}+\frac{k_3}{k_7}+1\right]}. \tag{4}$$

$Y_{21}(s)$ and $Y_{22}(s)$ include all the possibilities of the one inerter, one damper and two springs combinations. For $Y_{21}(s)$, only two of $1/k_2$, k_3, $1/k_4$, k_6, k_8 and $1/k_9$ are positive and all the others are equal to zero. For $Y_{22}(s)$, only two of k_1, $1/k_2$, k_3, $1/k_4$, $1/k_7$ and k_8 are positive and all the others are equal to zero.

3. APPLICATION TO A MULTI-STOREY BUILDING

The proposed vibration suppression system considered in this section involves connecting a reaction mass to the top of the main structure at a single point via a combination of springs, dampers and inerters. The vibration absorber is applied on an undamped three-storey building model subjected to the base excitation, where the building and vibration absorber model are shown in Figure 3. The floor mass and inter-storey stiffness are denoted as M_1, M_2, M_3 and k_{s1}, k_{s2}, k_{s3}, respectively. Dynamics of the combination of inerters, dampers and springs is represented by the transfer function $Y(s)$. The reaction mass is represented by m. $r(t)$, $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$ represent the absolute displacement of the ground, the 1st floor, the 2nd floor, the 3rd floor and the reaction mass in time domain, respectively. In this paper, floor mass is taken as $M_1 = M_2 = M_3 = 1000$ kg, inter-storey stiffness is taken as $k_{s1} = k_{s2} = k_{s3} = 1500$ kN/m, reaction mass m is taken to be equal to 150 kg for consisting with Lazar et al. (2014).

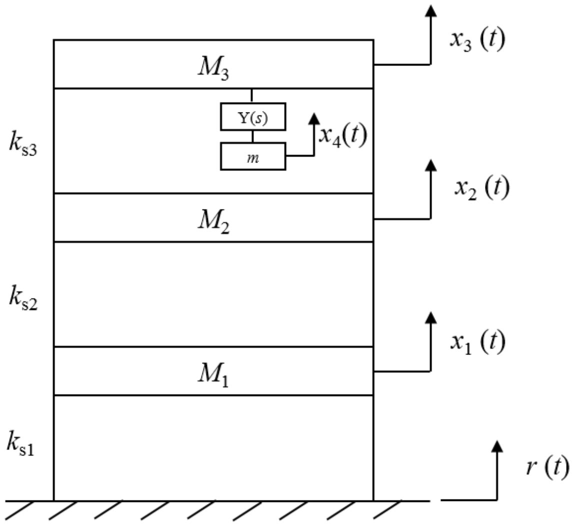


Figure 3. Three-storey building model with the vibration absorber mounted on the top

The equation of motion of the absorber in Laplace domain is written in absolute coordinates as

$$ms^2X_4(s) = -Y(s)s[X_4(s) - X_3(s)] \tag{5}$$

And the equation of motion of the three-storey building in Laplace domain is

$$\begin{aligned} M_1s^2X_1(s) &= k_{s1}[R(s) - X_1(s)] - k_{s2}[X_1(s) - X_2(s)] \\ M_2s^2X_2(s) &= k_{s2}[X_1(s) - X_2(s)] - k_{s3}[X_2(s) - X_3(s)] \\ M_3s^2X_3(s) &= k_{s3}[X_2(s) - X_3(s)] + Y(s)s[X_4(s) - X_3(s)] \end{aligned} \tag{6}$$

where $X_1(s)$, $X_2(s)$, $X_3(s)$ and $R(s)$ are the displacements $x_1(t)$, $x_2(t)$, $x_3(t)$ and $r(t)$ in Laplace domain.
Define inter-storey drift $X_{1\text{-}r}(s)$, $X_{2\text{-}1}(s)$ and $X_{3\text{-}2}(s)$ as:

$$\begin{aligned} X_{1-r}(s) &= X_1(s) - R(s) \\ X_{2-1}(s) &= X_2(s) - X_1(s) \\ X_{3-2}(s) &= X_3(s) - X_2(s) \end{aligned} \tag{7}$$

where $X_{1\text{-}r}(s)$, $X_{2\text{-}1}(s)$ and $X_{3\text{-}2}(s)$ represent the displacements of the 1st floor relative to the ground, the 2nd floor relative to the 1st floor and the 3rd floor relative to the 2nd floor in Laplace domain, respectively. Substituting Equations 5 and 7 into Equation 6 to eliminate $X_1(s)$, $X_2(s)$, $X_3(s)$ and $X_4(s)$ yields

$$\begin{aligned} &(M_1 s^2 + k_{s1})X_{1-r}(s) - k_{s2}X_{2-1}(s) = -M_1 s^2 R(s) \\ &M_2 s^2 X_{1-r}(s) + (k_{s2} + M_2 s^2)X_{2-1}(s) - k_{s3}X_{3-2}(s) = -M_2 s^2 R(s) \\ &aX_{1-r}(s) + aX_{2-1}(s) + (a + k_{s3})X_{3-2}(s) = -aR(s) \end{aligned} \tag{8}$$

where

$$a = M_3 s^2 + Y(s)s - \frac{Y(s)^2 s^2}{ms^2 + Y(s)s} \tag{9}$$

In this paper, the inter-storey drift relative to the base acceleration is considered as the performance index. Therefore, the objective function is defined as

$$J_\infty = \max\left(\left\| T_{s^2R \to X_{i-(i-1)}}(j\omega) \right\|_\infty \right),\ i = 1, 2, 3 \tag{10}$$

where $T_{s^2R \to X_{i-(i-1)}}$ represents the transfer function with the ground acceleration as input and inter-storey drift between *i*th floor and (*i*-1)th floor as output (Note: when $i = 1$, $X_{i-1}(s)$ refers to the ground displacement $R(s)$). $\left\| T_{s^2R \to X_{i-(i-1)}}(j\omega) \right\|_\infty$ is the standard H_∞-norm, which represents the maximum magnitude of $T_{s^2R \to X_{i-(i-1)}}$ across the whole frequency domain.

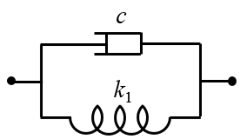
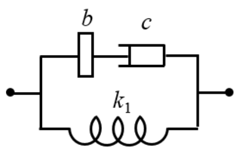
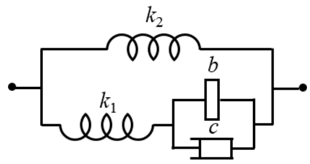
In order to identify the absorber's optimal layout, objective function J_∞ is optimized using a combination of patternsearch and fminsearch in Matlab with fminsearch refining the results obtained via patternsearch. The optimal layouts and their corresponding optimal parameter values for the one spring case and the two springs case are shown in Table 1. The traditional TMD is also optimized as a benchmark.

For the TMD, $Y(s)$ is taken to be the transfer function of a spring and a damper connected in parallel. The minimum value of the objective function is $J_\infty = 0.0172$ with $c = 1.71$ kNs/m and $k = 38.76$ kN/m. Based on the tuning rule proposed by Den Hartog (1956), there are two peaks in the vicinity of the 1st mode natural frequency. However, they can hardly be observed from Figure 4. This is because the mass *m* of the absorber is small compared with the mass of the main structure, thus these two peaks are very close together. For example, in Figure 4(a), the frequencies of these two peaks are 2.54 Hz and 2.63 Hz, respectively.

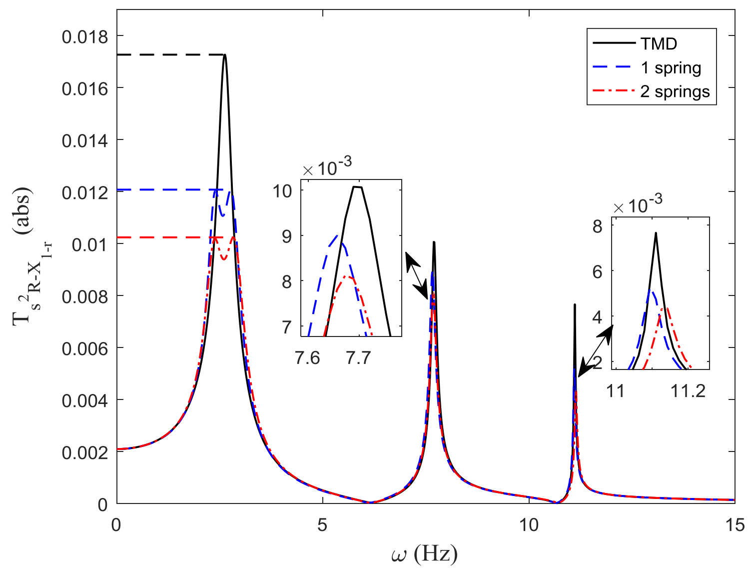
For the one spring case, before the optimization procedure, it might be thought that the optimal layout consists of a damper connected to a spring in parallel and then connected to an inerter in series, based on the idea that this is similar to TMD with an added inerter to provide larger inertance. However, the optimal performance of this layout occurs when *b* is infinite. The reason is that the inerter is connected with the reaction mass in series, which actually reduce the effective reaction mass. This situation can be understood physically as follow: when *b* is equal to zero, the reaction mass is disconnected with the spring and damper, therefore, there will be no force applied on the main structure at all. As *b* becomes larger, the connection of the reaction mass with spring and damper is increased. When *b* is infinite, the reaction mass is connected to the spring and damper rigidly, which is equivalent to the layout of TMD. After the optimization procedure for the one spring case (transfer function given in Equation 3), the optimal layout of the absorber is obtained, where an inerter is connected to a damper in series and then they are connected to a spring in parallel. The minimum value of the objective function is $J_\infty = 0.0120$ with $b = 145.6$ kg, $c = 3.64$ kNs/m and $k_1 = 65.30$ kN/m. It can be observed that, for the one spring case, performance of the absorber with optimal layout is improved by 30.2% compared with the traditional TMD absorber.

For the two springs case (transfer function given in Equation 4), the minimum value of the objective function is $J_\infty = 0.0101$ with $b = 23.91$ kg, $c = 0.81$ kNs/m, $k_1 = 89.14$ kN/m and $k_2 = 41.86$ kN/m. It can be observed that, for this case, performance of the absorber with optimal layout is improved by 41.3% compared with the traditional TMD absorber. With the absence of the inerter, an absorber consisting of one damper and two springs is optimized as a comparison. The minimum value of the objective function is J_∞ = 0.0169 (c = 1.95 kNs/m, k_1 = 39.79 kN/m and k_2 = 590.1 kN/m), which is only slightly improved compared with the TMD. Hence, the performance superiority of the absorber in the two springs case is due to the presence of inerter rather than merely due to the presence of two springs.

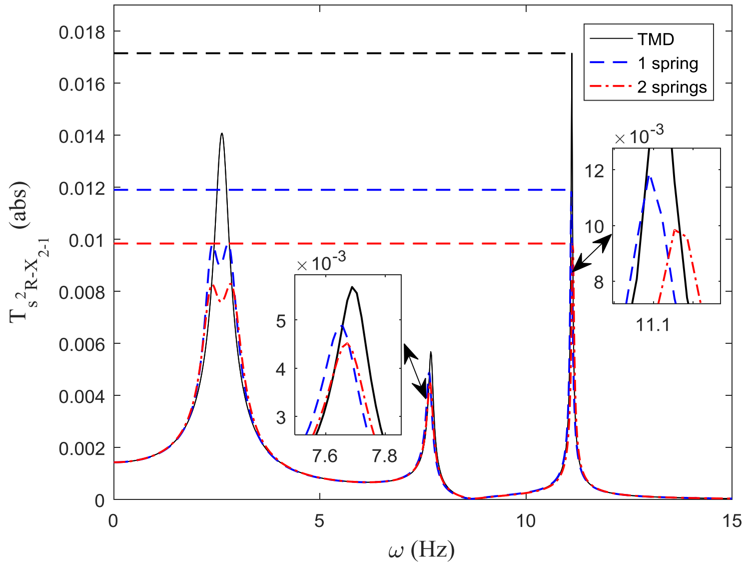
Table 1. Optimal parameter values and layouts of the one spring case, the two springs case and the TMD case

Optimal parameter values and layouts	**Suppression system**		
	TMD	**One spring case**	**Two springs case**
J_∞	0.0172	0.0120	0.0101
Performance improvement	/	30.2%	41.3%
Layouts			
b (kg)	/	145.6	23.91
c (kNs/m)	1.71	3.64	0.81
k_1 (kN/m)	38.76	65.30	89.14
k_2 (kN/m)	/	/	41.86

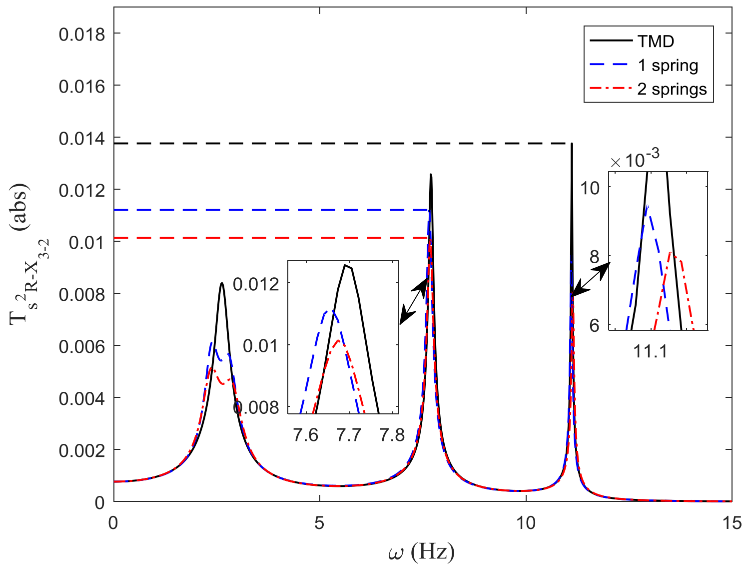
Frequency responses of $T_{s^2R\to X_{1-r}}$, $T_{s^2R\to X_{2-1}}$ and $T_{s^2R\to X_{3-2}}$ for the one spring case, the two springs case and the TMD case are shown in Figure 4. The horizonal lines highlight the objective function values for each inter-storey drift in turn. It can be observed that, for one spring case and two springs case, frequency responses of $T_{s^2R\to X_{1-r}}$, $T_{s^2R\to X_{2-1}}$ and $T_{s^2R\to X_{3-2}}$ are reduced at each natural frequency of the main structure compared with the traditional TMD. Moreover, performance of the absorber with two springs is superior than the absorber with one spring.



(a)



(b)



(c)

Figure 4. Frequency response comparison for the one spring case, the two springs case and the TMD case of inter-storey drift between: (a) the 1st floor and the ground; (b) the 2nd floor and the 1st floor; (c) the 3rd floor and the 2nd floor subjected to the ground acceleration

4. CONCLUSIONS

In this paper, a vibration suppression system consisting of a reaction mass connected to the structure at a single point via a combination of springs, dampers and inerters is investigated. The structure-immittance method is used to identify the optimal absorber layouts with a pre-determined number of elements. Two cases, one with one inerter, one damper and one spring, the other with one inerter, one damper and two springs are analysed systematically. An undamped three-storey building model subjected to base excitation is used to illustrate the efficiency of the proposed system. It can be observed that the maximum inter-storey drift is reduced significantly by applying this vibration suppression system compared with the traditional TMD absorber. Moreover, performance of the proposed system is improved as the number of elements used increasing. This paper enlarges the range of the passive vibration suppression systems as well as demonstrates the performance superiority of the proposed vibration suppression absorbers.

5. ACKNOWLEDGMENTS

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