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DESIGN PRINCIPLES OF SEISMIC METASURFACES TO CONTROL LOVE WAVES

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ABSTRACT

Metasurfaces, consisting in an array of resonant inclusions or resonant elements placed at a waveguide free surface, can be designed to interact with elastic waves and used to redirect, steer or absorb the elastic wave energy. Metasurfaces in half spaces are capable to support the propagation of shear horizontally polarized waves and to open band gaps in the spectrum of vertically polarized Rayleigh waves. In this work, we discuss the possibility of designing a metasurface to achieve control of seismic Love waves.
We analytically derive the dispersion law of Love waves existing below a resonant metasurface and observe how surface resonators can significantly modify the mode shapes and phase velocity of Love waves. Love waves can be controlled at the seismic scale by designing a metabarrier of meter-scale resonators using steel masses restrained by elastic connectors. We guide the design of such metabarrier using the derived analytical dispersion relation and show its ability to manipulate Love waves in the frequency range relevant for seismic isolation.

Keywords: Metasurfaces, Seismic Metabarrier; Love wave; Resonant Metamaterials; Dispersion Relation

1. INTRODUCTION

Artificially engineered materials have been recently proposed as innovative isolation devices for the protection of large urbanized areas from seismic hazards, due to their ability to control elastic wave propagation at low frequency. These materials, originally introduced as seismic metamaterials, are inspired by physical concepts well established in the fields of phononic crystals and resonant metamaterials.
Phononic crystals are materials or structural systems characterized by some form of spatial periodicity, either in the arrangement of the different material phases or in their geometry (Deymier 2013). In periodic media strong wave dispersion effects are caused by destructive interference between transmitted and reflected waves, scattered by the periodicity of the media. Such phenomenon, referred as Bragg scattering, creates band gaps (BGs), frequency ranges within the dispersion relation where waves cannot propagate. In the context of earthquake engineering, large-scale experiments showed that phononic crystals made of cylindrical holes in sedimentary soil can significantly attenuate the ground accelerations at a frequencies around 50 Hz (Brûlé et al. 2014). However, the proposed idea requires large structures to function at the low frequencies characteristic of seismic events (1–10 Hz).
Conversely, resonant metamaterials are artificial composites with local resonant particles or structures of subwavelength dimensions hosted in an elastic medium or structure (Kadic et al. 2013). In a medium which contains resonant inclusions, elastic waves hybridize with the local resonances, leading to band gaps at subwavelength scales. For seismic applications, where the waves wavelengths can be of several meters or decameters, the subwavelength behavior of metamaterials is of great advantage and allows for the design of viable devices (i.e. of smaller and feasible spatial dimensions) capable to interact with seismic waves (below 10 Hz). Sub-wavelength structures have been proposed to protect buildings from incoming seismic longitudinal and shear bulk waves as well as vertically polarized surface waves (i.e.

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Rayleigh waves), in the form of resonant foundations (Finocchio et al. 2014; La Salandra et al. 2017) or resonant metabarriers (Palermo et al. 2016; Dertimanis et al. 2016).
The latter consists of an array of vertically resonant units each of them realized with a steel mass and elastic supports encased in cylindrical concrete shell buried below the surface ground. Such an array of resonant units constitutes a metasurface, i.e. a planar metamaterial placed at the free surface of a medium to control the propagation of surface waves. In particular, the designed metabarrier exploits the coupling between Rayleigh waves and surface resonators to steer part of their energy into the soil bulk and isolate critical infrastructures or entire blocks.
Indeed, in layered soils, Rayleigh waves are accompanied by horizontally polarized surface waves, the Love waves, generated by the constructive interference of shear waves trapped in a superficial soft layer (e.g. a basin) over a rigid substrate (e.g. the bedrock).
Love waves are dispersive, have multi-modal nature (i.e. there exist an infinity number of Love solution supported by a soft layer) and travel with a lower speed in comparison with primary and secondary bulk waves. Due to their polarization, Love waves are often responsible of large damages in structures and infrastructures, also at significant distance from the epicenter.
For this reason, here we investigate the possibility of designing a metabarrier for Love waves control. We study analytically the interaction of Love waves with a metasurface of horizontal resonators, used as simplified model of the engineered metabarrier. Starting from the classical dispersion relation for the Love waves, we consider the effect of the metasurface on the Love wave propagation in terms of shear stress exchanged by the resonators over the free surface. We derive the dispersion relation of Love waves interacting with this array of resonators and highlights how the Love wave mode shapes and phase velocity can be affected by changing the resonators mechanical parameters (mass and stiffness).

2. DISPERSION LAW FOR LOVE WAVES COUPLED WITH A METASURFACE

2.1 Analytical derivation

We considered a layered medium which consists of an isotropic and homogeneous layer with a thickness h_1 laying on an isotropic and homogeneous half-space. The contact between the two layers is considered to be perfect. As shown in Figure 1, the soft layer has a shear velocity β_1, and a density ρ_1, while the bottom half-space has a shear velocity β_2, and a density ρ_2, with $\beta_1 < \beta_2$. The related shear moduli are given as:

$$\begin{cases} \mu_1 = \rho_1 \cdot \beta_1^2 \\ \mu_2 = \rho_2 \cdot \beta_2^2 \end{cases} \tag{1}$$

We introduce a Cartesian coordinate system x-y-z and consider a harmonic Love wave which propagates along the x-axis, polarized along the y-axis. Denoting its angular frequency as ω, the wave number as k and wave phase velocity as c, we have:

$$k = \frac{\omega}{c} \tag{2}$$

We assume the only non-zero displacements in the two layers as:

$$\upsilon_1 = \left(Ae^{iks_1z} + Be^{-iks_1z}\right)e^{i(\omega t - kx)}, \quad \upsilon_2 = \left(Ce^{iks_2z} + De^{-iks_2z}\right)e^{i(\omega t - kx)}, \tag{3}$$

where s_1 and s_2 are defined as:

$$s_1 = \sqrt{\left(\frac{c^2}{\beta_1^2} - 1\right)}, \quad s_2 = \sqrt{\left(\frac{c^2}{\beta_2^2} - 1\right)}. \tag{4}$$

The assumed displacements in Equation 3 satisfy the equation of motion for transverse wave in the two layers:

$$\nabla^2 \upsilon_i = \frac{1}{\beta_i^2} \frac{\partial^2 \upsilon_i}{\partial t^2}, \quad i = 1,2 \ . \tag{5}$$

Standard Love wave derivations adopts the following set of boundary conditions to determine the arbitrary constants A, B, C and D in Equation 3 and the related Love wave dispersion curve (Novotny 1999):

Condition I: Free surface of the medium (z = 0). Since $\tau_{zx} = \tau_{zz} = 0$, only one component is left:

$$\left(\tau_{zy}\right)_1 = \mu_1 \frac{\partial \upsilon_1}{\partial z} = 0 \qquad \text{For z = 0,} \tag{6}$$

where, $\left(\tau_{zy}\right)_1$ is the tangential stress component.

Condition II: Continuity of displacement at the layer- half space interface (z = h_1)

$$\upsilon_1 = \upsilon_2 \qquad \text{For z = } h_1, \tag{7}$$

Condition III: Continuity of tangential stress at the layer- half space interface (z = h_1)

$$\left(\tau_{zy}\right)_1 = \left(\tau_{zy}\right)_2 \Rightarrow \mu_1 \frac{\partial \upsilon_1}{\partial z} = \mu_2 \frac{\partial \upsilon_2}{\partial z} \qquad \text{For z = } h_1, \tag{8}$$

Condition IV: Zero displacement at infinite depth ($z \to \infty$)

$$\upsilon_2(z) \to 0 \qquad z \to \infty \tag{9}$$

The boundary condition in Equation 9 is satisfied for a displacement of υ_2 the form:

$$\upsilon_2 = Ce^{-ks_2^* z} e^{i(\omega t - kx)}, \quad D = 0 . \tag{10}$$

for any $c < \beta_2$ where

$$s_2^* = \sqrt{\left(1 - \frac{c^2}{\beta_2^2}\right)} \tag{11}$$

The metabarrier is modelled by means of a metasurface of single-degree of freedom mechanical oscillators, oriented along the polarization direction of the Love waves (Palermo et al. 2017) (see Figure 1b). Here each horizontal resonator is placed on the outer surface of the top layer (z = 0) with a spacing A_r on the horizontal plane.

The equation of motion of each resonator reads

$$m\ddot{\upsilon}_r = k_r\left(\upsilon_1(0) - \upsilon_r\right) \tag{12}$$

In the Equation 12, *m* is the mass of the resonator, k_r its stiffness and υ_r the displacement of the resonator mass, $\upsilon_1(0)$ is the displacement at the base of the resonator.

From Equation 12 we evaluate the amplitude of the resonator motion υ_r as:

$$\upsilon_r = \frac{\omega_r^2}{\omega_r^2 - \omega^2}\upsilon_1(0) \tag{13}$$

where, $\omega_r = \sqrt{k_r/m}$ is angular frequency of the resonator.

Thus, we recover the stress transferred by the resonator over the soft layer surface as:

$$\tau_{zy,r} = \frac{F}{A_r} = \frac{k_r}{A_r}\cdot\left(\frac{\omega^2}{\omega_r^2 - \omega^2}\right)\cdot\upsilon_1\cdot\left(A+B\right)\cdot e^{i(\omega t - kx)} \tag{14}$$

We can now derive the dispersion relation for the Love waves coupled with surface resonators by substituting the non-zero tangential stress at the free surface with the stress derived in Equation 14 as:

$$\left(\tau_{zy}\right)_1 = -\tau_{zy,r} \tag{15}$$

By substituting the assumed displacements in Equation 3 in the set of boundary conditions of Equations 7, 8, 10 and 14, we obtain a system of equations whose non-trivial solution provides the Love-metasurface dispersion relation:

$$\tan\left(k\cdot s_1\cdot h_1\right) = \frac{\mu_2\cdot s_2^* \cdot -\frac{1}{k}\cdot\frac{k_r}{A_r}\cdot\left(\frac{\omega^2}{\omega_r^2-\omega^2}\right)}{\mu_1\cdot s_1\cdot\frac{\mu_2}{\mu_1}\cdot\frac{s_2^*}{s_1}\cdot\frac{1}{k}\cdot\frac{k_r}{A_r}\cdot\left(\frac{\omega^2}{\omega_r^2-\omega^2}\right)} \tag{16}$$

We remark that for a vanishing mass (*m=0*), Equation 17 recovers the classical Love wave dispersion relation:

$$\tan\left(k\cdot s_1\cdot h_1\right) = \frac{\mu_2\cdot s_2^*}{\mu_1\cdot s_1} \tag{17}$$

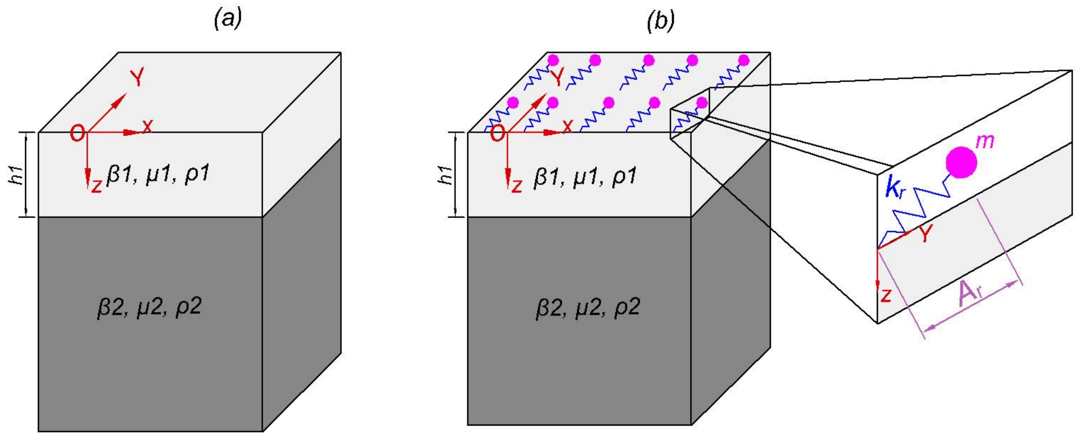

Figure 1. (a) Schematic of layer on half-space require for propagation of the Love wave; (b) interaction with a metabarriers

2.2 Numerical modeling

We utilize the analytically derived dispersion relation to predict the dispersive properties of seismic Love waves interacting with a meter-scale metabarrier. The assumed mechanical parameters for the metabarrier and for the layered soil are given in Table 1.

Table 1. Numerical and geometrical parameters.

Parameters		
Symbol	Definition	Value
β_1	Shear bulk speed in layer	300 $m \cdot s^{-1}$
ρ_1	Density of layer	1600 $kg \cdot m^{-3}$
h_1	Layer height	30 m
μ_1	Shear Modulus in layer	144 MPa
β_2	Shear bulk speed in half-space	1000 $m \cdot s^{-1}$
ρ_2	Density of half-space	1600 $kg \cdot m^{-3}$
μ_2	Shear Modulus in half-space	1600 MPa
A_r	Resonator area	1 m^2
f_r	Resonator frequency	4.5 Hz
m	Resonator mass	Variable
k_r	Resonator stiffness	Variable

In particular, we analyze the interaction between Love waves and a metabarrier of mechanical resonators having a resonance frequency f_r= 4.5 Hz. We consider four different values of resonator mass *m*= [0, 1000 kg, 2000 kg and 3000 kg] whose dispersion properties calculated as per Equation 16 are shown in Figures 2a, 3a, 4a and 5a. In these figures, the red dashed line indicates the classic Love wave dispersion relation (i.e. no resonators) while the black lines mark the dispersion relation of Love waves interacting with the metabarrier. The dispersion curves for a non-zero value of mass (Figures 3a, 4a and 5a) display a classical avoid crossing behavior between two repelling branches (here defined as lower and upper branch) which separate around the resonator resonance frequency (marked by a black dashed-line).

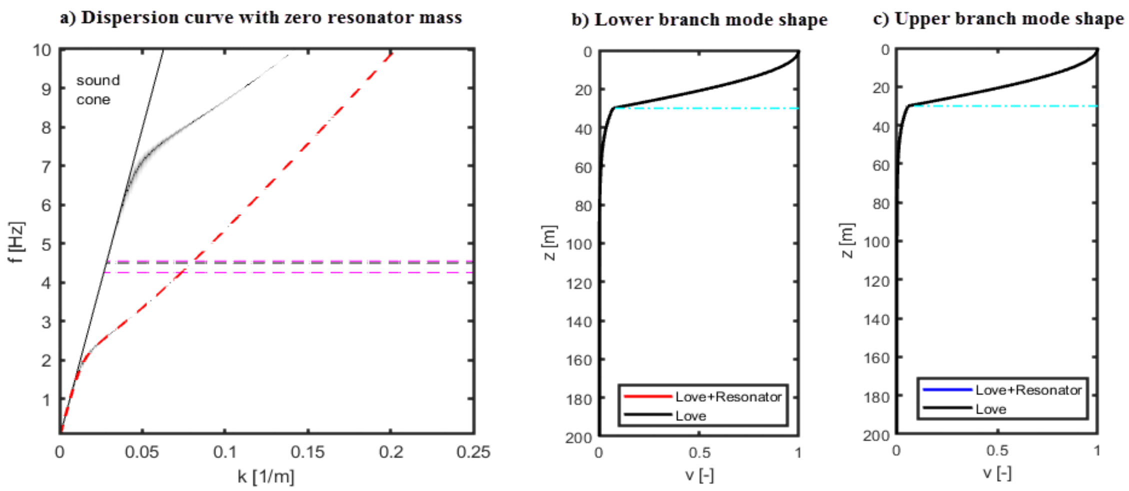

Figure 2. (a) Dispersion curve with zero mass (b) Mode shape of f=4.25 Hz from lower branch (b) Mode shape of f=4.55 Hz from upper branch

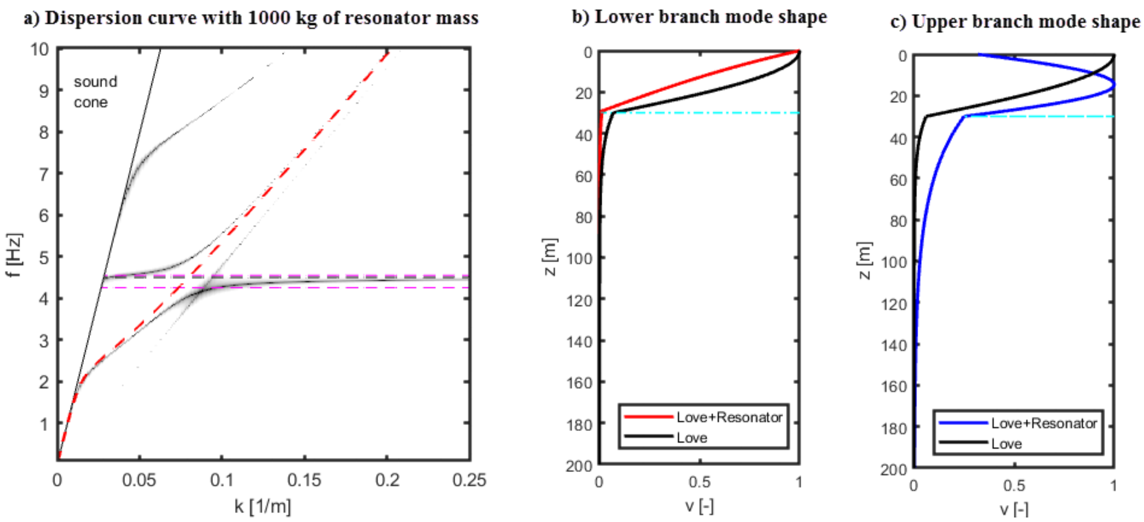

Figure 3. (a) Dispersion curve for resonator mass equals to 1000 kg (b) Mode shape of f=4.25 Hz from lower branch (b) Mode shape of f=4.55 Hz from upper branch

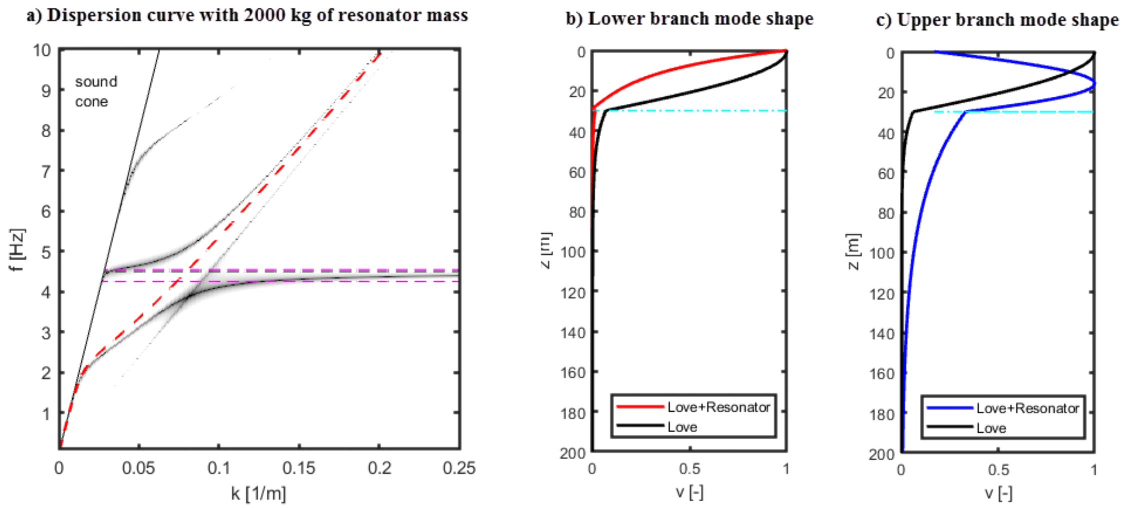

Figure 4. (a) Dispersion curve for resonator mass equals to 2000 kg (b) Mode shape of f=4.25 Hz from lower branch (b) Mode shape of f=4.55 Hz from upper branch

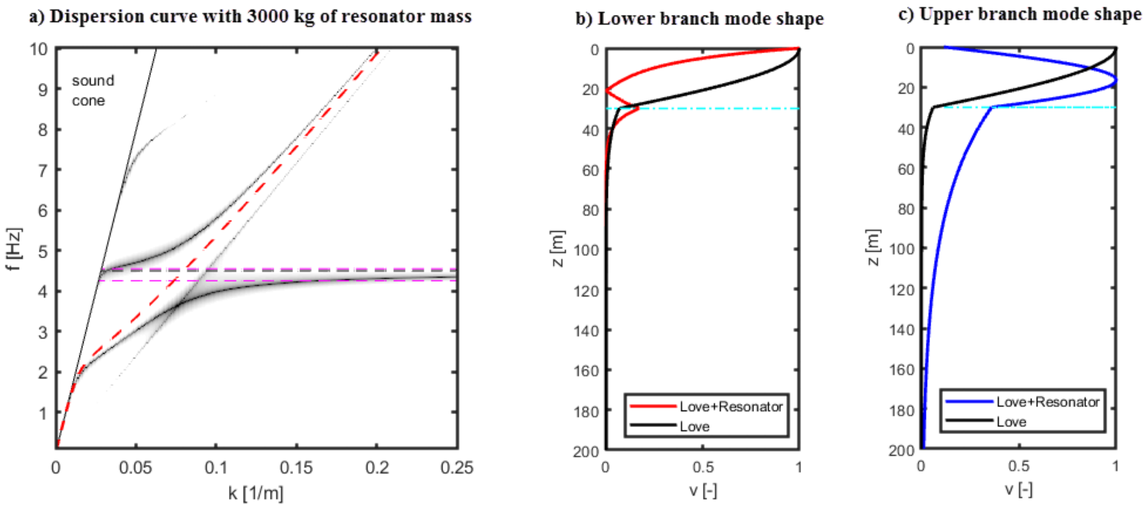

Figure 5. (a) Dispersion curve for resonator mass equals to 3000 kg (b) Mode shape of f=4.25 Hz from lower branch (b) Mode shape of f=4.55 Hz from upper branch

This repulsion increases for an increasing value of mass, resulting in a strong variation of the Love dispersive properties around the resonator frequency. Such variation is accompanied by a significant change in the Love waves shapes around the resonance frequency, as shown in Figures 3b,c, 4b,c, and 5b,c. In particular, the mode shapes of Love waves approaching the resonant frequency (f<=fr, i.e. lower branch) show a significant surface confinement due to the in-phase coupling between the resonators and the propagating waves, with the maximum displacement occurring at the resonator base (Figures 3b, 4b and 5b). Conversely, for the upper branch ($f>f_r$), the surface displacement tends to zero, due to the anti-phase motion between the resonator mass and the resonator base (Figures 3c, 4c and 5c).
Finally, the existence of this avoided crossing behavior leads to a strong variation of the phase velocity of Loves around the resonance. In Figure 6 is shown the Love wave speed variation for two constant frequencies values f=[4.25, 4.75] Hz for different values of the resonator mass. In this figure, blue and red lines mark the phase velocity of the Love wave branches at 4.25 and 4.55 Hz, respectively. An increase in the resonator mass results in an increase of the upper branch phase velocity and in a decrease of the lower branch phase velocity, respectively. This suggests the possibility of manipulating the Love wave speed and consequently the refractive index of the medium. This capability could open to the design of metabarriers and metalenses to control the direction of Love wave propagation.

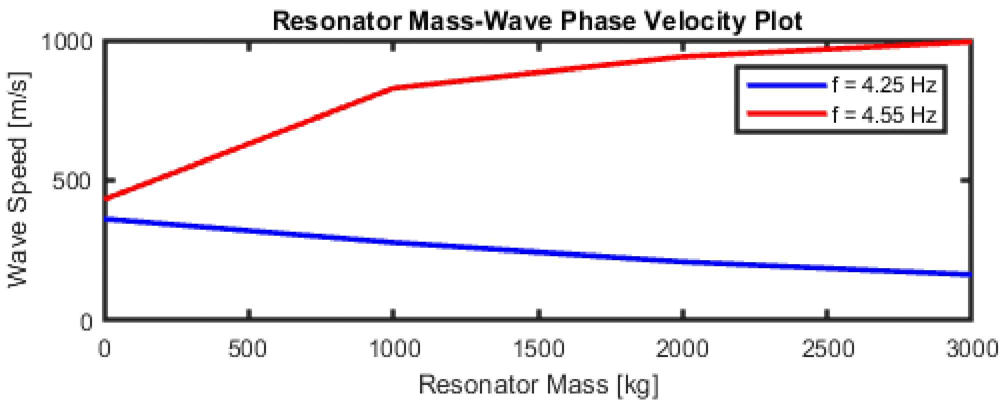

Figure 6. Effect of resonator mass on the Love wave phase speed

3. CONCLUSIONS

The horizontal ground shaking generated by the propagation of seismic Love waves is a primary cause of damages in structures and infrastructures during an earthquake.

In this article, we have investigated the possibility of designing a metabarrier to control the propagation of Love waves in the low frequency range (below 10 Hz), relevant for seismic isolation. The investigation has been guided by an analytical formulation of the dispersive properties of Love waves interacting with a metasurface of horizontal resonators, used as simplified model of the metabarrier. The obtained results show how a metabarrier of meter-scale resonant units can significantly modify the dispersive properties of Love waves in terms of velocity of propagation and mode shapes.
We expect these preliminary findings to be useful for designing a novel class of devices able to attenuate the propagation seismic Love waves.

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