

CONSTANT DUCTILITY INELASTIC SPECTRA FOR STRUCTURES EQUIPPED WITH VISCOUS DAMPERS

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ABSTRACT

In the present study, the definition of constant ductility high damping spectra, which are suitable for estimating the response of structural systems with supplemental linear viscous damping, is presented. First, damping reduction factors (B) are introduced to generate high damping elastic demand spectra. Given the elastic spectra, the inelastic performance of the structure can be obtained by relationships that relate the strength demand reduction factor (R) with the ductility demand factor (μ) and the period of the structure (T). As such, expressions that link the above quantities, known as R – μ – T relations, for different damping levels are presented. Moreover, in order to calculate with accuracy the values of the viscous dampers velocities, velocity corrective factors (B_v) are reported for different levels of damping and ductility. Finally, to evaluate the results of the proposed relations, the whole process is applied to two MDOF structural systems equipped with linear viscous dampers.

Keywords: passive energy dissipation systems, high damping spectra, inelastic spectra, linear viscous dampers

1. INTRODUCTION

The control of the displacement demand in the structure in the seismic design process is necessary, with or without the consideration of additional damping systems. Therefore, provisions regarding the analysis methods of such buildings have been added to regulations like FEMA 368 and FEMA 273 for new and existing buildings accordingly. The Nonlinear Dynamic Procedure (NDP) seems to be the most accurate analysis method since all the contemporary regulations accept the nonlinear behavior of the structure, both for the seismic design and assessment. Despite the reliability of the NDP even though the uncertainties related with those (Whittaker et al. 2001), simplified static procedures like the Linear Static Procedure (LSP) (FEMA 273, FEMA 368) and Nonlinear Static Procedure (NSP) (FEMA 273) were introduced by the regulations.

NSP introduced by FEMA 274 (Method 2) is based on the Capacity Spectrum Method (CSM) which was initially developed by Freeman (Freeman et al. 1975, Freeman 1978). In nearly most alternatives of NSP which were investigated throughout the years, the performance point of the structure is calculated by an repetitive process so as to equilibrate the demand with the capacity given initially the pushover curve (capacity) and the elastic demand spectra with damping ratio $\xi = 5\%$. An equivalent SDOF is considered throughout this operation with period $T = T_{eq}$ and viscous damping $\xi = \xi_{eq}$. The equivalent damping ξ_{eq} is specified as the sum of the inherent damping (ξ_o), the additional damping due to the damping devices (ξ_D) and the hysteretic damping expressed in terms of equivalent viscous damping (ξ_{hyst}). Damping Reduction Factors (B) for certain values of the effective damping, are used to decrease the elastic spectra. The values of the B factor are provided in the literature (Sadek et al.

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2000, Ramirez et al. 2002a, Palermo et al. 2013) and in the seismic codes.

An alternative CSM, for structures without additional damping, by using inelastic spectra, have been examined by Fajfar and Gašperšič (1996) and Fajfar (1999), as well as Chopra and Goel (1999a, b). This method differs from the classical CSM as the reduction of the capacity spectra is performed through relationships between the strength demand reduction factor of the structure R and the ductility μ . These types of relationships are referred to as $R - \mu - T$ and there are many proposed in the literature (Miranda and Bertero 1994, Vidic et al. 1994, Hidalgo and Arias 1990, Riddell and Newmark 1979). Except of Riddell and Newmark (1979) and Palermo et al. (2013) who proposed $R - \mu - T$ relationships for systems with higher values of viscous damping, the rest of the studies proposed $R - \mu - T$ relationships for damping ratio 5%, while Ramirez et al. (2002a) associates the elastic displacement with the expected inelastic, taking into account the additional damping not by using the $R - \mu - T$ relationships but through a coefficient C_1 .

Considering that for stiff structural systems and in the case of high values of ductility, the inelastic spectra are more accurate than the elastic one (Fajfar and Gašperšič 1996, Fajfar 1999, Chopra and Goel 1999a, b), a CSM based on constant ductility inelastic spectra has been used, in order to evaluate the seismic response of structures equipped with elastic viscous dampers. To perform this method, damping reduction factors (B) are presented, as well as $R - \mu - T$ relationships for systems with damping ratio up to 50%. Furthermore, pseudo-velocity correlation factors are introduced for the estimation of damper forces. Finally, the proposed high damping-inelastic spectra is performed into a four and six - storey RC and steel frame respectively using the CSM.

2. HIGH DAMPING SPECTRA

2.1 Elastic high damping spectra

The most accurate way to define an elastic spectra is to integrate the differential equation of motion throughout the time. Nevertheless, reduction factors are usually used to reduce the elastic spectra that correspond to 5% damping ratio in order to take into consideration either the additional damping influence or the inelastic response. In the case of high damping elastic spectra the reduction has been performed by using damping reduction factors, defined as follows (Ramirez et al. 2002a):

$$B = \frac{S_d(5\%, T)}{S_d(\xi, T)} \quad (1)$$

where $S_d(T, 5\%)$, is the demanded displacement for 5% damping ratio and $S_d(T, \xi)$ is the demanded displacement for damping ratio ξ .

Several expressions of the reduction factor B are already presented in the literature and regulations (FEMA274, FEMA 368, EC-8, Sadek et al. 2000, Ramirez et al. 2002a, Palermo et al. 2013). Most of them are specified by bilinear or trilinear models. The common feature of all the proposed models is that the reduction factor remains constant beyond the period values that correspond to the constant acceleration area of the response spectra.

Analyses were performed using a set of 20 ground motions (Table 1) which have been scaled to the EC-8 response spectra for soil type C. It can be observed that the reduction did not remain constant above the area of the constant spectral acceleration but it tends to descend (Figure 1). Thus, in the present study the construction of a single continuum expression for the damping reduction factors that could take into account their reducing tendency at the higher values of periods is attempted.

The maximum acceleration of an oscillation with damping ratio ξ , larger than 5%, is smaller, due to the larger damping of the system. To specify the reduced seismic demand, it can be assumed that the variation of the maximum potential energy between the systems with damping ratio 5% ($E_{P,0.05}$) and ξ ($E_{P,\xi}$) would be equal with the energy that is dissipated due to an increase of the damping by $\Delta\xi$ ($E_{D,\Delta\xi}$) (Equation 2).

$$E_{P,0.05} = E_{P,\xi} + E_{D,\Delta\xi} \quad (2)$$

Table 1. Earthquake events used in this study.

Date	Earthquake	M _s	Station	Component
1941	Northern Calif-01	6.4	FerndaleCityHall	315
1951	Imperial Valley-03	5.6	ElCentroArray #9	000
1952	KernCounty	7.36	TaftLincolnSchool	021
1961	Hollister-01	5.6	HollisterCityHall	271
1966	Parkfield	6.19	Cholame - ShandonArray #12	050
1967	Northern Calif-05	5.6	FerndaleCityHall	314
1968	BorregoMtn	6.63	ElCentroArray #9	180
1971	SanFernando	6.61	Castaic - OldRidgeRoute	291
1973	PointMugu	5.65	PortHueneme	270
1976	Friuli Italy-01	6.50	Barcis	000
1978	SantaBarbara	5.92	CachumaDamToe	250
1978	TabasIran	7.35	Dayhook	L1
1979	Imperial Valley-06	6.53	BrawleyAirport	225
1980	Livermore-01	5.80	APEEL 3E Hayward CSUH	146
1980	Mammoth Lakes-01	6.06	Long Valley Dam (Upr L Abut)	090
1980	VictoriaMexico	6.33	CerroPrieto	315
1981	Taiwan SMART1(5)	5.90	SMART1 007	EW
1981	Westmorland	5.90	ParachuteTestSite	225
1984	MorganHill	6.19	San Juan Bautista_ 24 Polk St	213
1986	Mt. Lewis	5.60	HallsValley	090

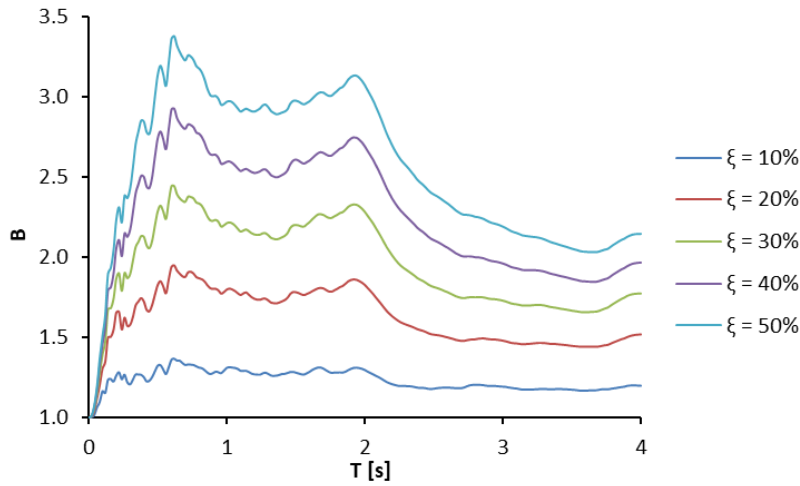


Figure 1. Mean reduction of the elastic spectra due to damping

Therefore, the value of the dissipated energy due to the increase of the damping by $\Delta\xi$ must be defined. Considering a forced vibration by an harmonic external force $P(t) = P_0 \cdot \sin(\omega t)$, the dissipated energy under one cycle of loading due to viscous damping $\xi^* = \Delta\xi$ is (Chopra 2001):

$$E_D = \int f_D du = \int_0^{2\pi/\omega} c^* \dot{u}^2 dt = 2\pi\xi^* \frac{\omega}{\omega_n} k u_0^2 \quad (3)$$

Combining the Equations 1, 2 and 3 results to:

$$B = \frac{Sa_{0.05}}{Sa_{\xi}} = \sqrt{1 + 4\pi\Delta\xi \frac{T_n}{T}} \quad (4)$$

where T is the period of the harmonic external force $P(t)$ and T_n the natural period of the oscillator.

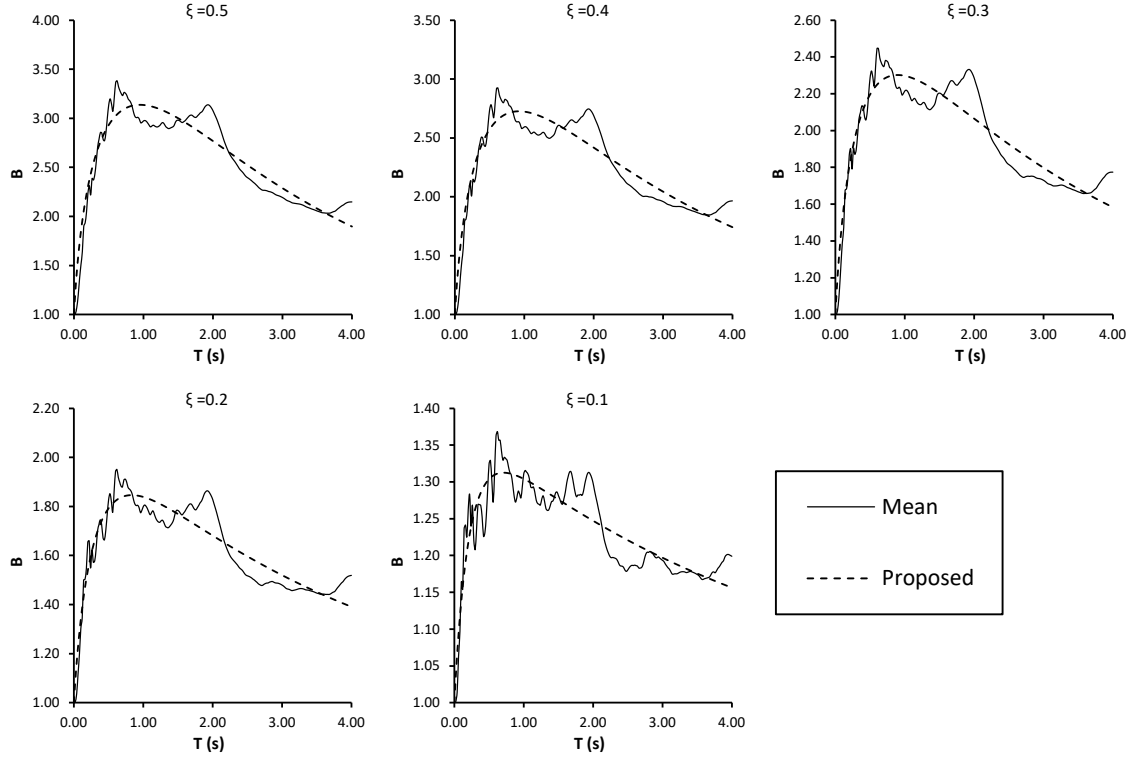


Figure 2. Reduction factor B diagrams

As the term T is difficult to specify mainly due to the uncertainties of the ground motions and the non-harmonic shape of a natural ground motion, a set of 20 ground motions were used to conclude to a function of the general form $f(T, \xi)$ in order to describe the ratio T_n / T . By calibrating the Equation 3 to the analytical results, the reduction factor of the spectra are given by the following relationships:

$$B = \frac{Sa_{0.05}}{Sa_{\xi}} = \sqrt{1 + 4\pi(\xi - 0.05)f(T, \xi)} \quad (5)$$

$$f(T, \xi) = a \left(e^{b \frac{T}{T_0}} - e^{c \frac{T}{T_0}} \right) \quad (6)$$

where a , b and c are based on the damping levels and they are listed in the Table 2 and T_0 is the period that corresponds to the beginning of the constant spectral velocities area.

Table 2. Values of Equation's 5 factors a , b and c .

	ξ				
	0.1	0.2	0.3	0.4	0.5
a	1.46	1.92	2.34	2.82	3.40
b	-0.15	-0.20	-0.24	-0.27	-0.30
c	-2.56	-1.75	-1.45	-1.28	-1.15

Figure 2 displayed comparatively the damping reducing factors from Equation 3 and the mean reduction calculated from the 20 ground motions by time history analysis. A special characteristic of the proposed continuum nonlinear expression is that it can be applied to the whole range of the spectra periods, while it describes the descending reducing factor beyond the constant spectral acceleration branch.

2.2 Constant ductility inelastic high damping spectra

As in the case of elastic response spectra, the most accurate way to define the inelastic spectra is to integrate the differential equation of motion throughout the time. Nevertheless in practice, strength reduction factors (R) are usually used to reduce the elastic spectra. The strength reduction factor defined as the ratio of elastic demand strength (F_{el}) to inelastic demand strength (F_y) as follows (Miranda and Bertero 1994, Chopra and Goel 1999 a, b):

$$R = \frac{F_{el}}{F_y} = \frac{S_{a,el}}{S_{a,y}} \quad (7)$$

Regarding the inelastic spectra, a number of different $R - \mu - T$ relationships have been presented in the literature (Miranda and Bertero 1994, Vidic et al. 1994, Hidalgo and Arias 1990, Riddell and Newmark 1979, Chopra 2001). However, as noticed above, the viscous damping ratio of all these models was assumed to be between 1-10%. By the implementation of passive dissipation control systems the damping of the equivalent SDOF system can reach 30% of the critical damping. Thus, in order to examine the effect of the high damping on the inelastic constant ductility spectra, analyses with the same set of ground motions were performed for damping ratio with the range $\xi = 5-50\%$ and ductility values $\mu = 1.5, 2, 2.5, 3$ and 4 . Subsequently, the main reduction factor was determined for each level of damping and ductility. The main reduction factors for the inelastic spectra for damping values $\xi = 0.05-0.5$ are presented in Figure 3.

The single expression of the reduction factor spectra for different levels of damping and ductility are based on the Hidalgo and Arias (1990) relationship and is given by the following equation:

$$R_{\mu} = 1 + \frac{T}{aT_0 e^{b\mu T} + \frac{T}{c\mu - 1}} \quad (8)$$

where T_0 is the period that corresponds to the beginning of the constant spectral velocities area and a , b and c rely on the damping ratio and are listed in Table 3.

Table 3. Values of Equation's 7 factors a , b and c .

	ξ		
	0.05	0.10	≥ 0.20
a	0.31	0.25	0.24
b	-0.97	-0.47	-0.13
c	1.00	0.95	0.94

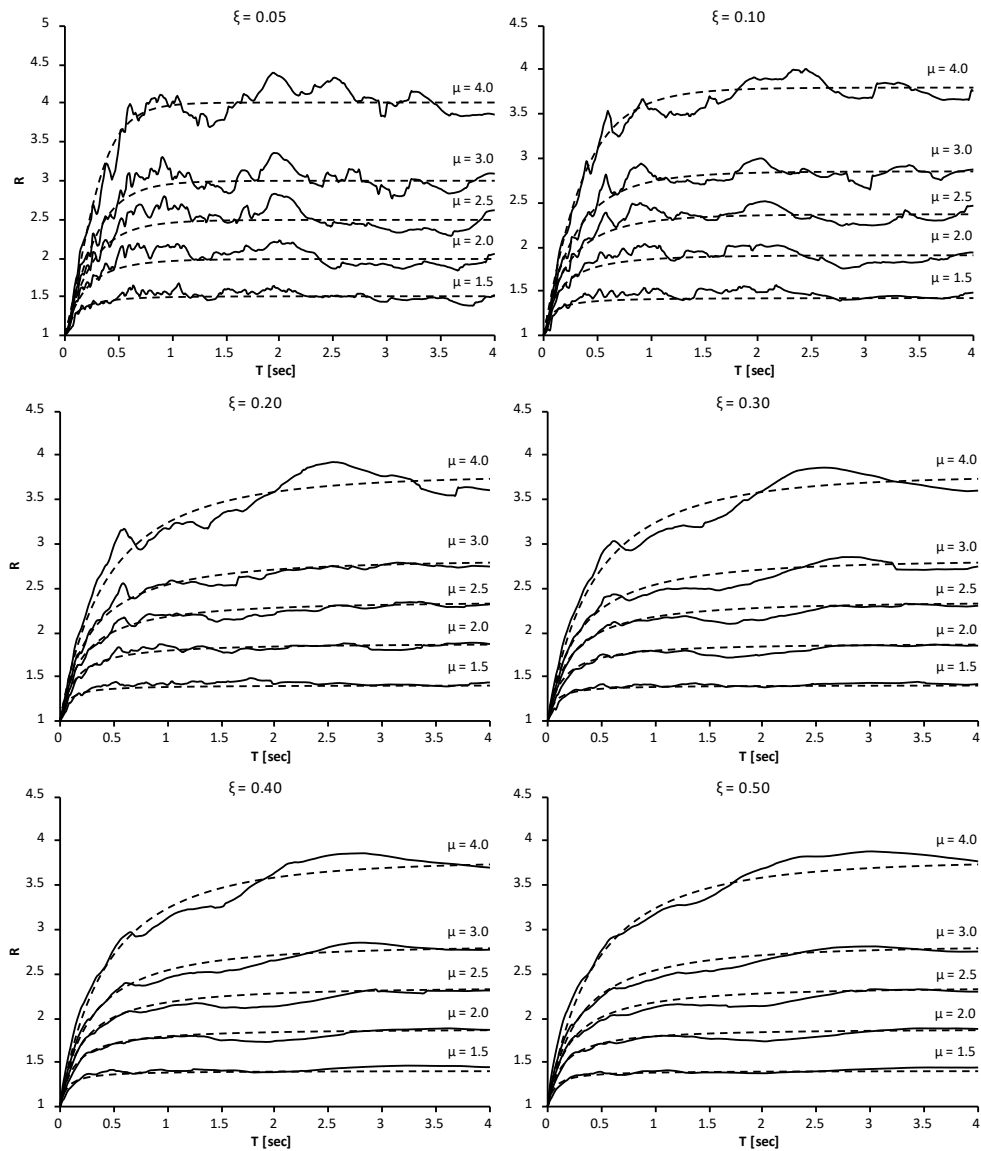


Figure 3. Strength reduction factors R , by the analysis (solid line) and by the proposed relationships (dash line)

The damping level did not affect significantly the form of the reduction spectra for the construction of the constant ductility inelastic spectra. It is obvious that for damping ratios higher than 20% the reduction remains constant.

It can be seen by Figure 3, that for low damping ratios ($\xi=0.05$), the assumption of equal displacements, comparing the elastic and the inelastic response, of long period structures is verified. On the other hand, assuming high damping ratios ($\xi > 0.10$), that assumption leads to non-conservative results ($R < \mu$). The above remark is taken into account by the c factor of Equation 7, as for long period structures the strength reduction factor take values $R = c \cdot \mu$.

2.3 Velocity corrective factors

In order to design the energy dissipation systems the estimation of the devices forces are of great importance. The prediction of the dampers ends' relative velocity is necessary to calculate the damping force as viscous dampers are velocity-dependend devices. The most simplified method to calculate it is by using the pseudo-velocity spectra (PS_v) of the equivalent SDOF oscillator.

After that, the velocity values are distributed to the structure storeys based on the 1st mode. The PS_v can be computed by the displacement spectra given the relationship:

$$PS_v = \omega S_d \quad (9)$$

while for the inelastic systems,

$$PS_v = (\omega_{el} / \sqrt{\mu}) S_d \quad (10)$$

where ω_{el} is the structures' natural circular frequency of vibration and $\mu = 1$ for elastic seismic response.

However, even for elastic systems, the assumption of the equivalence between the velocity spectra (S_v) and the pseudo velocity spectra (PS_v) is valid for oscillators with period values near to $T = 0.5$ s (Sadek et al. 2000). For period values larger than $T = 0.5$ s and as the damping ratio increases, this assumption leads to underestimation of the developed velocities, whereas for lower periods it leads to overestimation. Owing to this, a corrective factor (B_v) is introduced by using the ground motions of Table 1. B_v is equal to the pseudo velocity spectra divided by the velocity spectra (Equation 10).

$$B_v = \frac{PS_v}{S_v} = \frac{\omega_{el} / \sqrt{\mu} S_d}{S_v} \quad (11)$$

The expression of the B_v calculated by a regression analysis is presented in Equation 11, and is demonstrated for different values of damping ratios and ductility in Figure 4.

$$B_v = (a_1 \mu^2 + a_2 \mu + a_3) T^{a_4 \mu^2 + a_5 \mu + a_6} \quad (12)$$

where a_1 - a_6 , are coefficients listed in Table 4 for different damping ratio levels.

Table 4. Values of Equation's 11 factors.

ξ	a_1	a_2	a_3	a_4	a_5	a_6
0.05	0.014	-0.089	1.058	0.008	-0.095	-0.043
0.10	0.015	-0.105	1.056	0.006	-0.083	-0.098
0.20	0.020	-0.169	1.080	0.014	-0.140	-0.131
0.30	0.013	-0.106	1.002	0.000	-0.038	-0.272
0.40	0.012	-0.104	0.984	-0.004	-0.014	-0.338
0.50	0.006	-0.072	0.946	0.000	-0.031	-0.375

According to the results depicted in Figure 4, it can be observed that as the ductility demand and the effective damping ratio increase, the corrective factor B_v is increased for stiff structures and decreased for long-period ones. Particularly, for structures with damping ratio $\xi = 0.05$ and ductility $\mu = 1$ the B_v ranges from 1.3 to 0.9, while for ductility demand $\mu = 4$, ranges from 1.7 to 0.7. In addition, for structures with damping ratio $\xi = 0.20$, B_v take values from 2.25 to 0.6 and from 2.4 to 0.5, for ductility demand $\mu = 1$ and $\mu = 4$ respectively.

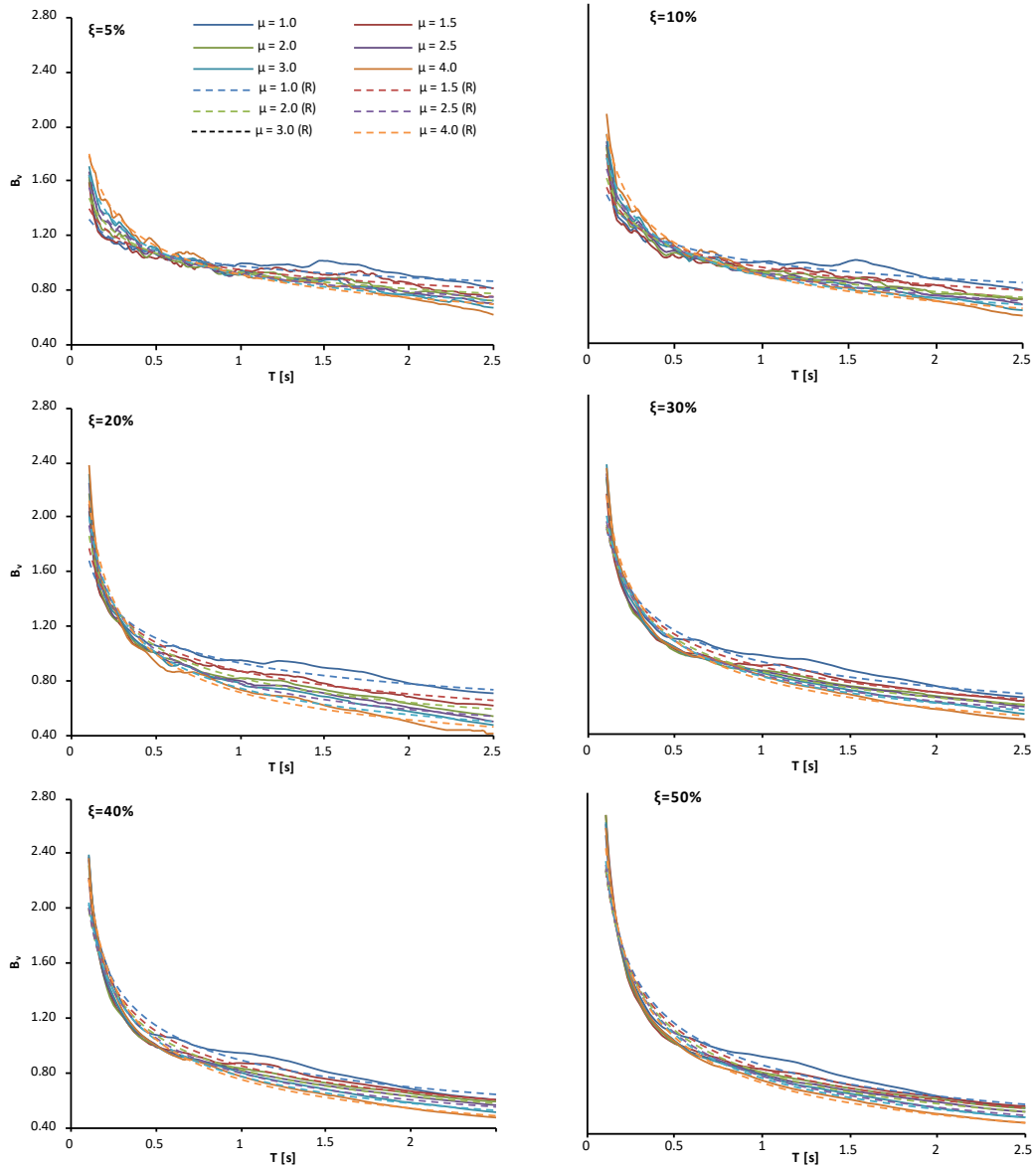


Figure 4. Corrective factor B_v for different ductility and damping levels

3. VERIFYING THE PROPOSED SPECTRA

To examine the performance of inelastic constant ductility - high damping spectra to MDOF systems with linear viscous dampers, two frame buildings were considered. A four - storey reinforced concrete frame building, designed according to EC-2 and EC-8 provisions, and a six - storey steel frame building, designed according to EC-3 and EC-8 provisions, were analyzed (Figure 5). Regarding the damping systems, elastic viscous dampers were assumed with a damping constant of $C = 2000 \text{ kN s/m}$, placed at 26.56° from the horizontal for the RC building, and $C = 3000 \text{ kN s/m}$, placed at 18.56° from the horizontal for the steel one. The dampers are implemented at the central opening of each floor for both cases. The effective damping of the examined frames are equal to $\xi_{\text{eff}} = 20.8\%$ and $\xi_{\text{eff}} = 16\%$, for the RC frame and the steel frame respectively.

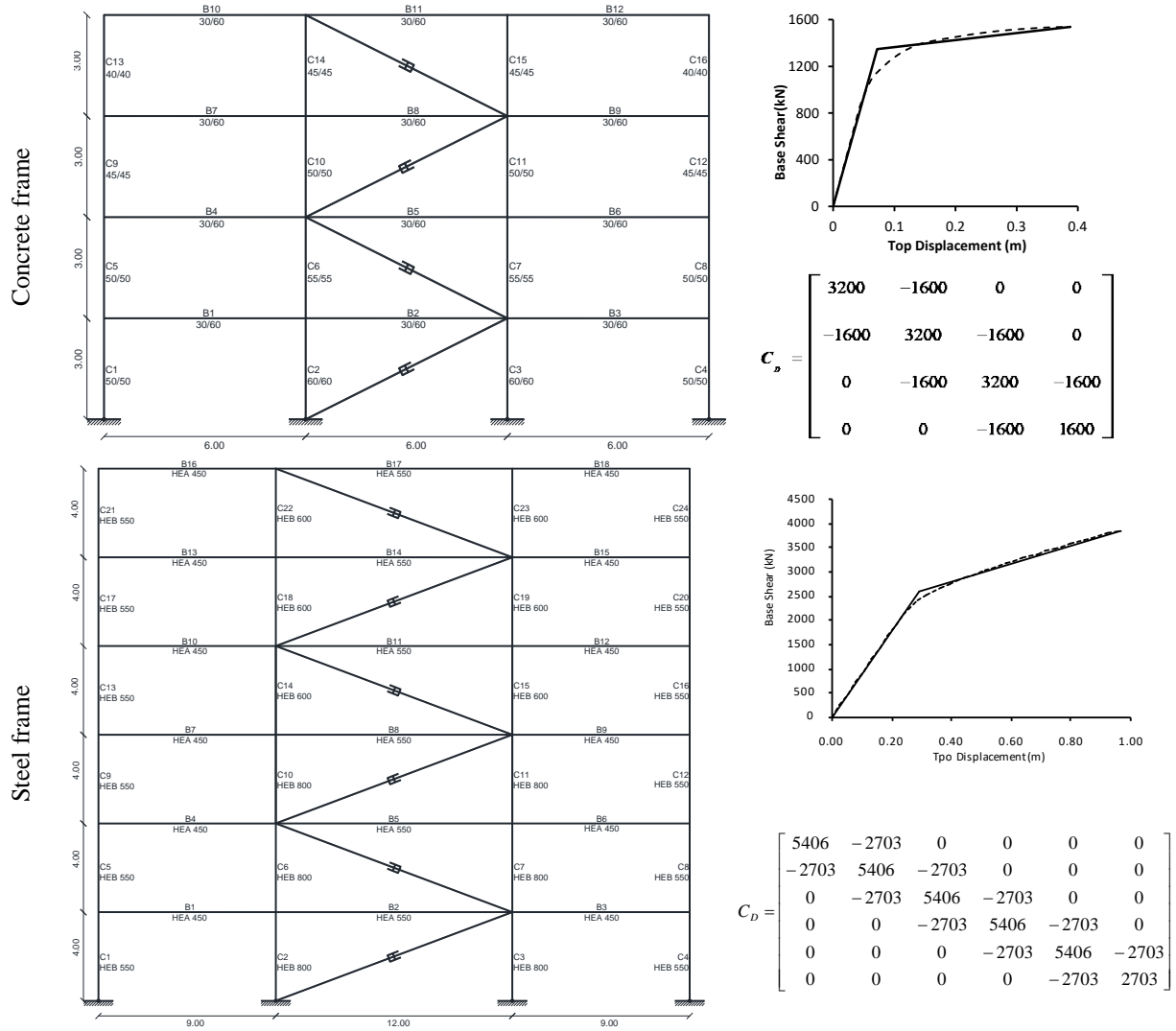


Figure 5. Examined frames

A total of 28 time history analyses (TH) were performed for each frame. Table 5 presents the 5 ground motions along with their scale factors used to verify the proposed methodology. Incremental factors were used in order to investigate the method performance at different levels of ductility. In order to evaluate also the approximate relations of reducing the elastic spectra due to high damping (Equations 3, 4 and 5), as well as the $R - \mu - T$ relations (Equation 7) for the construction of the high damping constant ductility spectra, the capacity spectrum method combined with the proposed relations, was applied for the mean spectrum for each scale factor. A graphic method was applied, in which the ductility is defined by the cross of the demand spectra with the bilinear capacity spectra at the yield point (Aschheim and Black 2000). The results are displayed in Table 6. Using the proposed relations leads to notable estimation of the top displacement compared with the average and the maximum results obtained by the TH analyses. The results of both methods estimate with a satisfactory accuracy the TH results.

Moreover, the viscous dampers forces are calculated using the Equation 11, to correct the PS_v values. The B_v factors for each ductility levels are listed to the Table 7. Moreover, Table 7 displays the ratio of the viscous dampers' median forces calculated by the corrected PS_v , by the average ($F_{d,Ave}$) and maximum ($F_{d,Max}$) forces computed by the TH analyses. Finally, the total average of the ratios $F_d / F_{d,Ave}$ and $F_d / F_{d,Max}$ are shown.

Table 5. Earthquakes and scale factors applied for the analyses.

Earthquake	Component	Scale Factors
Northern Calif-01	315	x1 x1.25 x1.50 x1.75 x2 x3
Kern County	21	x1 x1.25 x1.50 x1.75 x2 x3
Northern Calif-05	314	x1 x1.25 x1.50 x1.75 x2
San Fernando	291	x1 x1.25 x1.50 x1.75 x2x3
Santa Barbara	250	x1 x1.25 x1.50 x1.75 x2

Table 6. Spectral peak displacement using the proposed R- μ -T relationships.

	Scale Factor	x1	x1.25	x1.5	x1.75	x2.00	x3.00	Average
4-Storey	$S_d / S_{d,Ave}$	0.91	0.95	0.99	1.00	1.03	1.41	1.05
	$S_d / S_{d,Max}$	0.80	0.83	0.87	0.84	0.82	1.29	0.91
6-Storey	$S_d / S_{d,Ave}$	0.87	0.88	0.88	0.91	0.94	0.92	0.90
	$S_d / S_{d,Max}$	0.80	0.81	0.80	0.80	0.80	0.80	0.80

It can be observed that the PS_v values are similar to S_v in the case of 4 - storey frame. Mentioned above, for structures period near to 0.5s the PS_v are nearly equal to S_v . Thus, as the period of the examined structure is $T = 0.61s$, the result was expected to be the present. On the other hand the in the case of 6 - storey frame, due the longer period compared to the RC frame ($T = 1.69 s$) the PS_v are less than S_v , thus the application of the velocity corrective factors is necessary.

Table 7. Values of the corrective factor B_v , S_v and the ratios $F_d, Push / F_{d,TH}$.

Scale Factor		x1	x1.25	x1.5	x1.75	x2.00	x3.00	Average
μ	4-st	1.00	1.00	1.18	1.39	1.63	2.64	-
	6-st	1.00	1.00	1.00	1.08	1.25	1.80	-
B_v	4-st	1.06	1.06	1.04	1.02	1.01	0.94	-
	6-st	0.84	0.84	0.84	0.83	0.81	0.75	-
S_v	4-st	0.40	0.51	0.60	0.67	0.73	1.00	-
	6-st	0.59	0.75	0.89	1.03	1.13	1.47	-
$F_d / F_{d,Ave}$	4-st	0.90	0.95	1.00	0.98	0.96	0.95	0.96
	6-st	0.85	0.86	0.87	0.89	0.88	0.85	0.87
$F_d / F_{d,Max}$	4-st	0.82	0.84	0.90	0.87	0.86	0.89	0.86
	6-st	0.73	0.75	0.77	0.79	0.79	0.77	0.77

4. CONCLUSIONS

In the present study high damping constant ductility spectra to assess the response of RC frame buildings with viscous dampers was investigated. Moreover, applications of the proposed methodology on a RC 4-storey and 6-storey steel frame with viscous dampers were presented.

Regarding the reducing of the elastic spectra to construct high damping spectra, a continuum nonlinear expression was indicated, which can be applied to the whole range of the spectra periods. A particular feature of the proposed relationships is that it could describe the descending reducing factor beyond the constant spectral acceleration area.

As done for the elastic spectra a continuous $R - \mu - T$ relationship was proposed for the inelastic spectra taking into consideration the effect of the damping level. The fact that has to be mentioned is that the damping level does not notably affect the reduction spectra. In fact the reduction remains

constant for damping ratio values higher than 20%.

Once the viscous dampers are velocity-dependent devices, the accurate estimation of the dampers ends' relative velocity are of great importance in order to calculate the damping forces. Owing to this, an expression that relates the S_v with the PS_v is presented by introducing a corrective factor (B_v) which is affected by the damping ratio and the ductility demand of the structure.

Throughout the analysis of two frame building equipped with viscous dampers, the high damping spectra seems to estimate with great accuracy the top displacement and the dampers forces.

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