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AN ANALYSIS OF THE DYNAMICS OF SEISMICALLY ISOLATED STRUCTURES TAKING INTO ACCOUNT ITS TORSIONAL VIBRATIONS

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ABSTRACT

In this paper are researched torsional fluctuations of structure located on the pendulum system of seismic isolation (SIS). Torsional oscillations are caused by the incongruence of the center of rigidity (CR) with the center of mass (CM) of the structure and influence of rotational ground motions. In the calculations given below is considered only the first reason. The lack of coincidence of CR and CM leads to an asynchronous motion of different pendulum devices and to their different longitudinal deformation. Thereby, the tension and compression forces of the devices are different. This leads to the torsion, rocking and vertical displacement of the seismically isolated structure. This paper investigates the influence of the above-mentioned effects.

Keywords: pendulum system of seismic isolation; mathematical model; torsional oscillations.

1. INTRODUCTION

The analysis of seismic isolation systems (SIS) is generally performed under kinematic excitations defined by horizontal ground motions. The influence of vertical and rotational components of earthquake excitation on the response of SIS (Basu, Whittaker, & Constantinou, 2015) and the influence of the related with these components torsional vibrations of the structure is practically not investigated (Basu et al., 2015).

However, these excitations significantly affect the oscillation behavior of the idealized system: Protected Superstructure (PS)-SIS and may reduce the effectiveness of SI devices.

The influence of the rotational excitations can be clearly seen on the example of the pendulum type SIS. The ground rotation and thus the base rotation of the structure leads to an asynchronous motion of different pendulum devices and to their different longitudinal deformation. Thereby, the tension and compression forces of the devices are different. This leads to the torsion, rocking and vertical displacement of the seismically isolated structure. Similar considerations can be made with regard to seismic isolation based on the application of rubber bearings. The main purpose of this research is to investigate the influence of the above-mentioned effects. This paper presents the analysis of the structure behavior isolated by pendulum type SIS.

A mathematical model that allows investigating the influence of not only horizontal but also vertical and rotational components of earthquake excitation on the response of SIS is analyzed. This mathematical model consists of several groups of equations.

This paper presents the analysis of the degree of influence of the eccentricity between the center of mass and rigidity on the SIS efficiency.

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2. STATEMENT OF THE PROBLEM

If only the horizontal components of the ground motion are considered in the dynamic analysis, then we can simplify this problem to a single degree of freedom system (SDoF), shown in Fig. 1:

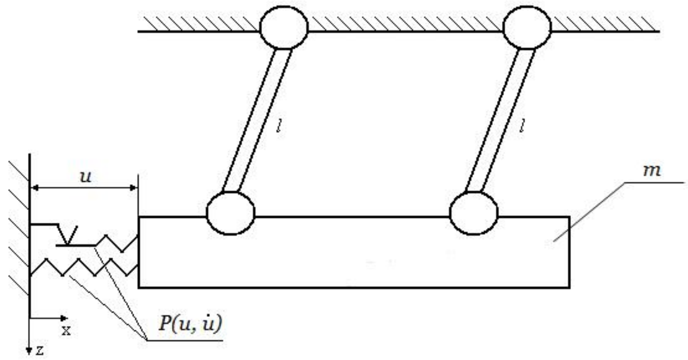

Figure 1. Dynamic model of the pendulum type SIS idealized by a SDoF system

In Cartesian coordinates x, z, the equation describing the model in Fig. 1 has the form (Rutman 2012):

$$m\frac{l^2}{l^2-u^2}\ddot{u}+ml^2\frac{u\cdot\dot{u}^2}{\left(l^2-u^2\right)^2}=-m\left(g-\ddot{z}(t)\right)\frac{u}{\sqrt{l^2-u^2}}-P(u,\dot{u})-\alpha\dot{u}-m\ddot{x}(t) \quad (1)$$

Where m is the mass of the superstructure (protected structure (PS)), l is the length of the pendulum, g is the acceleration due to gravity, x and z are the coordinates, describing the motion of the structure foundation, u is the coordinate that describe the displacements of the protected system related to the foundation, α is the linear damping constant and the $P(u,\dot{u})$ is the bilinear restoring force of the plastic damper of the pendulum type SIS.

When deriving the equation (1) it was assumed that the pendulum rods, constituting the SIS were absolutely rigid. In fact, the rods can deform in their longitudinal direction. As a result of this, vertical and torsional oscillations of the PS will occur. If we take into account the deformation of the rods and consider the PS as a rigid body, the SIS becomes a 6 DoF system. Taking into account the principal properties (its structure and geometry) of the considered pendulum bearings, the model of the Idealized system: protected object – SI device has the following view, Fig. 2:

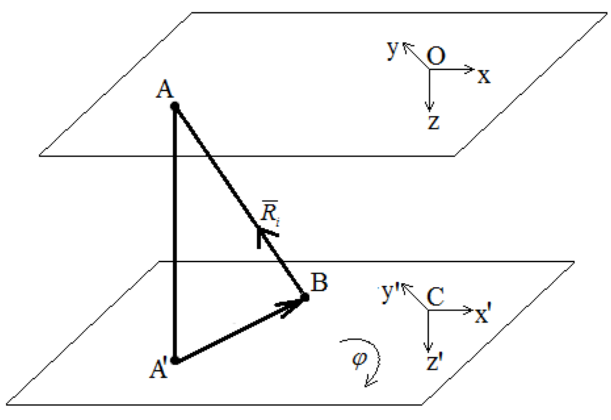

Figure 2. Idealized system: protected object – SI device

2.1. Adopted assumptions

- The upper plane is fixed
- The lower plane has mass and moments of inertia (PS):

$$I_x = I_y = \infty, I_z < \infty$$

- *X, Y, Z* – is the fixed coordinate system centered at the point
- *X', Y', Z'* – is a movable coordinate system centered at the point
- *A'* – projection of *A* when $t=0$
- *B* – current position of *A'* when $z\neq 0$
- The bar *AB* at the node *A* has a spring of rigidity *k*
- Displacements of the center of mass *C* on the Cartesian coordinates *X, Y, Z* are U_c, V_c, W_c
- Angles of rotation of the PS (bottom plane) relatively to the coordinates *X, Y, Z* are φ_x, φ_y, φ_z
- Internal forces and moments relatively to C of each bearing are

$$\bar{R}_i = (R_{x,} R_{y,} R_z)$$

and

$$\vec{M}_{R_i} = (M_{ic_{x,}} M_{ic_{y,}} M_{ic_z})$$

2.2. First group of equations. Equations of motion

Taking into account the motion of center of mass as motion of point *C* relatively mobile system of coordinates of *XYZ* according to formulas of relative motion:

$$\vec{\xi}_a = \vec{\xi}_r + \vec{\xi}_e + \vec{\xi}_c \tag{2}$$

Where $\vec{\xi}_a$ – absolute acceleration of point *C*, $\vec{\xi}_r$ – relative acceleration, $\vec{\xi}_e$ – acceleration in translation, $\vec{\xi}_c$ – Coriolis acceleration.

Relative acceleration is defined by accelerations of relative coordinates of a point *C*:

$$\vec{\xi}_r = (\ddot{u}, \ddot{v}, \ddot{w})$$

Acceleration in translation is defined by

$$\vec{\xi}_e = \vec{\xi}_o + \vec{\varepsilon} \times \vec{r}_{oc} + \vec{\varpi} \times (\vec{\varpi} \times \vec{r}_{oc}) \tag{3}$$

Where $\vec{\varepsilon}$ – vector describing angular acceleration of the rotational movement of ground. (Yin et al., 2016)

$\vec{\varpi}$ – vector describing the angular speed of the rotational movement of ground.

$\vec{\xi}_o = (a_{x,} a_{y,} a_z)$ - the vector describing linear accelerations of mobile system of coordinates (ground). Components of this vector are set by accelerograms.

Coriolis's acceleration is defined by a formula:

$$\vec{\xi}_c = 2\vec{\varpi} \times \vec{V}_r \tag{4}$$

Where $\vec{V} = (\dot{u}, \dot{v}, \dot{w})$ – the relative speed of a point of *C*.

Vector of absolute angular acceleration of PS:

$$\ddot{\vec{\varphi}}_a = \ddot{\vec{\phi}} + \dot{\vec{\varpi}} \tag{5}$$

Where $\ddot{\vec{\phi}}$ – a vector of accelerations of relative angles of rotation of PS concerning axes of *X,Y,Z*.

In a formula (5) angles are considered as small. At this assumption of the equation of the movement PS have an appearance:

$$m\vec{\xi}_a = \vec{R} + \vec{P} + \psi_1(\vec{V}) \tag{6}$$

$$I\ddot{\vec{\varphi}}_a = \vec{M}_R + \vec{M}_p + \psi_2(\phi) \tag{7}$$

Where

$\vec{R}$ - the vector describing summarized force from all rods;

$\vec{M}_R$ - the vector of the summarized moment from forces in rods concerning the center of masses *C*;

$\bar{P}$, $\vec{M}_p$ - summarized force and moment from plastic dampers (hysteresis devices);

$\psi_1(\vec{V})$, $\psi_2(\dot{\varphi})$ - summarized forces and moment arising from viscous damping.

The first group of equations describes the dynamics of the PS under the above-mentioned forces and external kinematic effects. For the pendulum type SIS in the absence of rotational excitations and absolutely rigid devices these equations permanently transform into the famous equations of oscillations of the physical pendulum.

In a formula (3) as a result of small two last composed it is possible to neglect. Then, substituting formulas (2)-(5) in (7), (8) after some transformations we receive:

$$m\ddot{u}_C = R_x + 2m\dot{v}\omega - P_x - \psi_{1x} - ma_x(t) \quad (8)$$

$$m\ddot{v}_C = R_y - 2m\dot{u}\omega_z - P_y - \psi_{1y} - ma_y(t) \quad (9)$$

$$m\ddot{w}_C = R_z + mg - P_z - \psi_{1z} - ma_z(t) \quad (10)$$

$$I_z\ddot{\phi}_z = M_{cz} - M_{pz} - \psi_{2z} - I_z\dot{\omega}(t) \quad (11)$$

$$I_x\ddot{\varphi}_x = M_{cx} - M_{px} - \psi_{2x} \quad (12)$$

$$I_y\ddot{\varphi}_y = M_{cy} - M_{py} - \psi_{2y} \quad (13)$$

Where

$R_x = \sum R_{ix}$; $R_y = \sum R_{iy}$; $R_z = \sum R_{iz}$; $\overline{M}_C = \sum \overline{M}_{iC} = (M_{cx}, M_{cy} M_{cz})$

$\overline{M}_{iC} = \bar{r}_{BC} \times \overline{R}_i$

$\bar{r}_{BC}$ — vector BC

Initial conditions:

$\dot{u}_C(0) = \dot{v}_C(0) = \dot{w}_C(0) = 0$

$u_C(0) = v_C(0) = 0$

$w_C(0) = \dfrac{mg}{nk}$

where n is the number of bearings of the SIS.

2.3. Second group of equations

The first group of equations describes the relationship between the PS generalized coordinates and the displacements of the PS attachment points to pendulum devices or SI bearings. Wherein, PS (superstructure) is assumed to be an absolutely rigid body, i.e. its dynamics is described by six coordinates.

The displacement of center of mass C: U_c, V_c, W_c.

The coordinates of node B on the fixed coordinate system:

when $t = 0$: node B $\longrightarrow$ node A' $\rightarrow (X_{A'}, Y_{A'}, Z_0)$

when $t \neq 0$: node B $\rightarrow$ $(u_C + X_{A'} \cdot \cos\varphi + Y_{A'} \cdot \sin\varphi, v_C - X_{A'} \cdot \sin\varphi + Y_{A'} \cdot \cos\varphi, w_C + Z_0)$;

$X_A = X_{A'} = X_{0i}$; $Y_A = Y_{A'} = Y_{0i}$; $Z_{A'} = Z_0$

Finally, we obtain: $\bar{r}_{AB} = (u_C + X_{0i} \cdot \cos\varphi + Y_{0i} \cdot \sin\varphi - X_{0i}, v_C - X_{0i} \cdot \sin\varphi + Y_{0i} \cdot \cos\varphi - Y_{0i}, w_C + Z_0)$

$\bar{r}_{BC} = (X_{0i} \cdot \cos\varphi + Y_{0i} \cdot \sin\varphi, Y_{0i} \cdot \cos\varphi - X_{0i} \cdot \sin\varphi, 0)$

$\overline{R}_C = \sum \overline{R}_i$; $\overline{M}_C = \sum \overline{M}_{iC}$; $\overline{M}_{iC} = \bar{r}_{BC} \times \overline{R}_i$

$$M_{iC} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ X_{0i} \cdot \cos\varphi + Y_{0i} \cdot \sin\varphi & Y_{0i} \cdot \cos\varphi - X_{0i} \cdot \sin\varphi & 0 \\ R_{ix} & R_{iy} & R_{iz} \end{vmatrix} =$$

$$= \bar{i} \cdot (Y_{0i} \cdot \cos\varphi - X_{0i} \cdot \sin\varphi) \cdot R_{iz} - \bar{j} \cdot (X_{0i} \cdot \cos\varphi + Y_{0i} \cdot \sin\varphi) \cdot R_{iz} +$$
$$+ \bar{k} \cdot \left[(X_{0i} \cdot \cos\varphi + Y_{0i} \cdot \sin\varphi) \cdot R_{iy} - (Y_{0i} \cdot \cos\varphi - X_{0i} \cdot \sin\varphi) \cdot R_{ix} \right]$$

2.4. Third group of equations

The second group of equations describes the relationship between the devices internal forces (or bearings internal forces) and the PS attachment points displacements. The internal force R_i in the bar AB is determined in the following way:

$$\Delta r_i = -\left(\left|\bar{r}_{AB}\right| - \left|\bar{r}_{AA'}\right|\right),\ k \cdot \Delta r_i = R_i\ ,\ \bar{R}_i = R_i \cdot \frac{\bar{r}_{AB}}{\left|\bar{r}_{AB}\right|} = k \cdot \left(-\left|\bar{r}_{AB}\right| + \left|\bar{r}_{AA'}\right|\right) \cdot \frac{\bar{r}_{AB}}{\left|\bar{r}_{AB}\right|}$$

$$\bar{r}_{AB} = \left(X_B - X_A, Y_B - Y_A, Z_B\right),\ \bar{r}_{AA'} = \left(0,0,Z_0\right)$$

The length of the rod of the bearing when $t \neq 0$ is

$$l_i = \sqrt{r_{ix}^2 + r_{iy}^2 + r_{iz}^2}$$

The deformation of the rod is

$$\Delta l_i = -l_i + Z_0$$

The reaction components are

$$R_{ix} = k \cdot \Delta l_i \cdot \frac{r_{ix}}{l_i}\ ,\quad R_{iy} = k \cdot \Delta l_i \cdot \frac{r_{iy}}{l_i}\ ,\quad R_{iz} = k \cdot \Delta l_i \cdot \frac{r_{iz}}{l_i}$$

The total reactions relatively to the center of mass are:

$$\bar{R}_C = \sum R_i\ ;\ R_x = \sum R_{ix}\ ;\ R_y = \sum R_{iy}\ ;\ R_z = \sum R_{iz}$$

The moment of one reaction relatively to *C* is:

$$\bar{M}_{iC} = \bar{r}_{BC} \times \bar{R}_i$$

The total moment of all bearing reactions, constituting the relatively to *C* is:

$$\bar{M}_C = \sum \bar{M}_{iC}$$

3. SOLUTION OF THE PROBLEM

Let us apply the mathematical model obtained above to investigate the effect of the nonalignment of the point of application of the total force *R* and the center of mass *C* on the efficiency of SI. The considered object has characteristics:

- a=b=25.5 m
- l=1,5 m
- m=11766500 kg
- I_z=1275194438 kg*m^2
- T=2,46 s (object with SIS), T=0.6 s (object without SIS)
- n=184

Where *a,b* – dimensions of PS in plan, *m* is the mass of PS, *l* is the length of the pendulum, I_z - moment of inertia, *T* - period of natural oscillation, *n* – number of bearings.

Further on, the distance between these points will be called the eccentricity. The eccentricity arises from the asymmetric arrangement of the bearings with respect to the center of mass.

In the case of eccentricity, the angular velocity and angular acceleration do not become zero and contribute to the absolute acceleration of points. Moreover, depending on the distance of the bearings from the center of mass, this absolute acceleration will be different and increase, according to the distance of the bearing from the center of mass. The absolute acceleration of the point D located at the periphery will be expressed by the formula:

$$\vec{\xi}_D = \vec{\xi}_C + \ddot{\vec{\varphi}} \times \vec{r}_{DC} + \dot{\vec{\varphi}} \times (\dot{\vec{\varphi}} \times \vec{r}_{DC}) \tag{14}$$

Below a test problem of the pendulum SIS with the following initial data is presented:

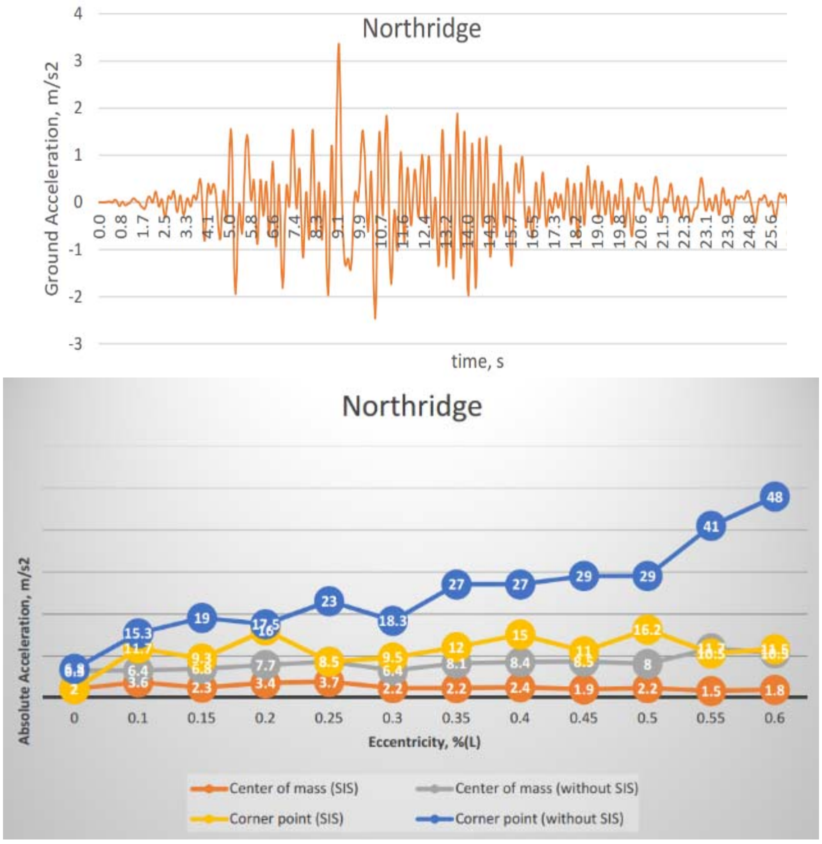

Figure 3. Acceleration time history and the corresponding points of absolute acceleration in protected structure with SIS and without. (Earthquake – Northridge 1994)

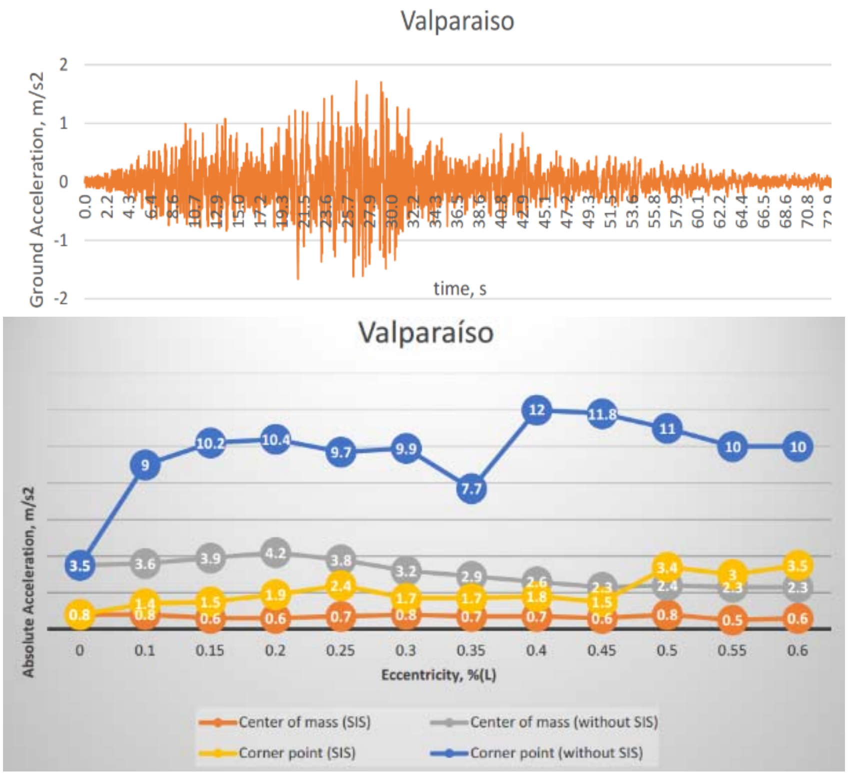

Figure 4. Acceleration time history and the corresponding points of absolute acceleration in protected structure with SIS and without. (Earthquake – Valparaiso 1985)

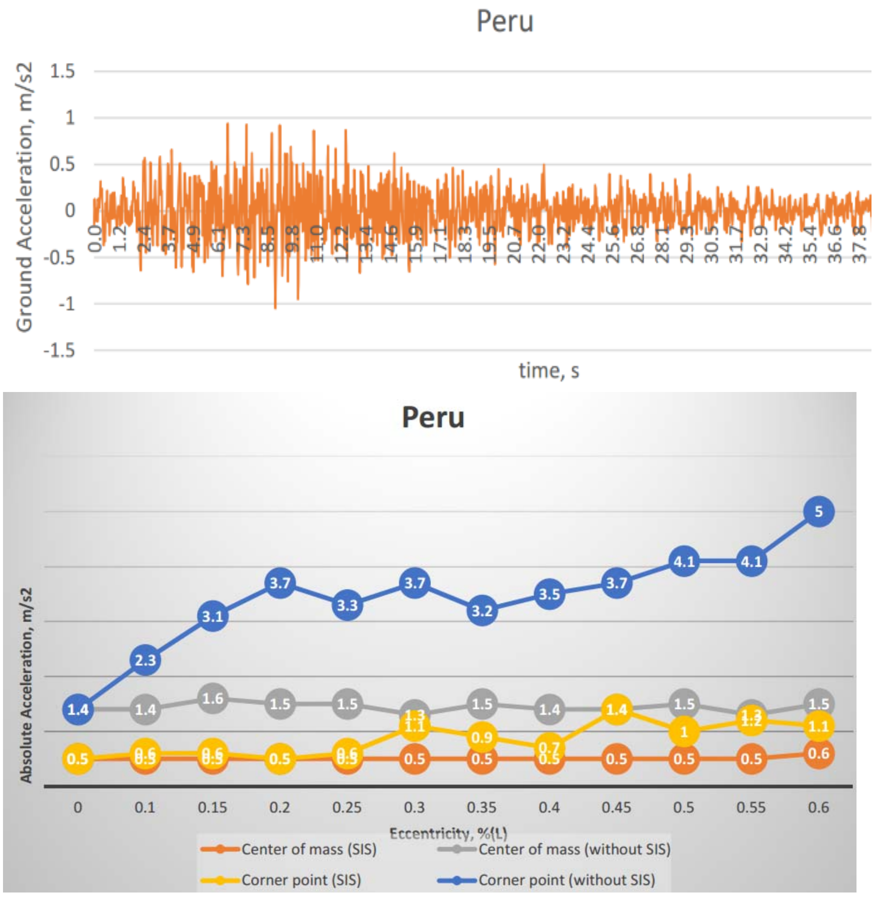

Figure 5. Acceleration time history and the corresponding points of absolute acceleration in protected structure with SIS and without. (Earthquake – Peru)

4. CONCLUSIONS

This research presented a mathematical model that allows to investigate the influence of not only horizontal but also vertical and rotational components of earthquake excitation on the response of SIS is analyzed. This defined mathematical model consists of three groups of equations.

The application of this mathematical model shows that the nonalignment (eccentricity) between the center of mass and center of stiffness of SIS significantly increases the acceleration of the peripheral points of the building, as well as the center of mass of the building.

The presented graphs show the efficiency of pendulum SIS for both translational and rotational components of seismic effects. The SIS decrease the absolute accelerations of the corner building points by two or more times compared to the non-isolated one.

At the same time, when taking into account this eccentricity, the total accelerations of the PS are equal or higher than those of the ground motions (PGA). Nevertheless, a comparison between a non-isolated structure and isolated one shows the effectiveness of the seismic isolation. However, the accelerations on the PS are high.

We must mention that in the presented example neither the summarized force nor moment from plastic dampers (hysteresis devices) were taken into account, since one of the goals of the paper was validate at this stage of research the proposed mathematical model. At the same time, it should be noted that the problem of including both summarized force and moment from plastic dampers into the equations of motion is not simple and is linked to stiff ODES and finding a stable numerical method of stiff ODES integration.

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