

## A QUICK METHOD FOR ESTIMATING THE LATERAL AND TORSIONAL STIFFNESS OF MRF IN 3D MODEL

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### ABSTRACT

This study presents an approximate and quick method for estimating the lateral and torsional stiffness of 3D asymmetric plan shape with steel moment resistant frame. This research further more prepares a method for evaluating lateral displacement stories, torsional stories and natural period with regarding different lateral load patterns. This paper proposes a new method for computing lateral and torsional stiffness for each frame in two directions, and then changing the stiffness of all frames to one frame and to calculate the deformation and natural period for two directions. The main idea of the suggested innovative method was expanded through the force method to assess the lateral displacement and stiffness of 2D building structures and then, the mentioned procedure was developed into 3D building structures. An example has been modeled to compare the new method with linear analysis. The results indicate that maximum error of lateral displacement, torsional values for each stories and first mode period less than 3.5%, 7.3% and 10.2%, respectively. Also, it is shown that the suggested method able to assess 3D irregular structure with appropriate accuracy compared with linear analysis.

*Keywords: Lateral stiffness; Torsional Stiffness; Lateral Displacement; Torsional Story; Natural Period*

### 1. INTRODUCTION

Structural analysis and design software need initial values of cross-section, material properties and masses for computing of lateral displacement, rotation of stories and period time of the first mode. Cross-section and material properties values are used for evaluating stiffness and strength in analysis and design procedures. These initial values are obtained by some preliminarily calculations. Approximate method for structural analysis include cantilever and portal method are limited to only regular geometric 2D moment frames. Also, Spurr, Bowman and Witmer [1] methods are usable for regular 2D moment frames. Above mentioned methods cannot calculate lateral and torsional stiffness for the 3D building systems. Moreover the calculation of lateral load pattern distribution to determine the lateral displacement will be more necessary and important when there are eccentricity between mass center and rigid center of the 3D frame system. The kan and PCA methods have been presented for regular moment 2D frame with shear walls, in addition, The results kan and PCA methods for 2D frame system reached up more than 50% in magnitude of lateral displacements. However, these methods are not appropriate for estimating lateral displacement of 3D frames. Grigorian [2] presented a simple approach for assessment of the lateral response of regular high-rise frames. Using of this method is not suitable for evaluation the lateral response of 3D irregular frame under lateral load. Moreover, in 3D irregular structure, the eccentricity between the mass center and the rigid center is important. Miranda and Taghavi [3] suggested an approximate method to evaluation floor acceleration demand and dynamic characteristics of the building. Subsequently, Miranda and Akkar [4] extended the method of Heidebrecht and Stafford [5] to computed generalized drift spectrum with higher mode

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effects. They showed that the effect of lateral stiffness ratio was important for buildings with period of the first mode longer than about 1.2 second. Gengshu et al. [6] studied effects of second order and buckling on buildings through the closed form solutions of continuous systems. Eroglu and Akkar [7] proposed estimating the lateral stiffness in frames for approximate analysis of tall structures consists of linear and nonlinear static analyses. Shahrouzi [8] suggested a simplified method evaluating the eigenvalues of regular and irregular chain structures. Hosseini and Imagh-e-naiini [9] studied a quick method for estimating the lateral stiffness of building structures, including regular and irregular moments and braced 2D frames. They modeled a regular moment frame, an irregular moment frame and two regular frames with braced and shear wall. Numerical results of kan method, ETABS analysis and proposed method illustrated that the proposed method is more efficient than existing methods. More recently, Shadman and Golara [10] presented new method for estimation dynamic characteristics of MRF using 3D model. They developed formulation of estimating lateral and torsional stiffness for 3D irregular structure. Their results demonstrated that the proposed method can provide the lateral displacement and torsional story values with maximum error of less than 12%.

## 2. IMPLIMENTATION OF APPROXIMATE APPROCH

Hosseini and Imagh-e-naiini suggested the main idea of method that based on the simplification of 2D modeling. In this idea, the simplified equivalent 2D system with definite mechanical specifications and multi-bays is changed to many one-bay 2D frames combined by hinges. These 2D frames can be summarized by a one-bay, one-story 2D frame, hereinafter referred to as a module of the simplified system. Figure 1,2,3 show a schematic view of the proposed method to simplify a 2D frame system.

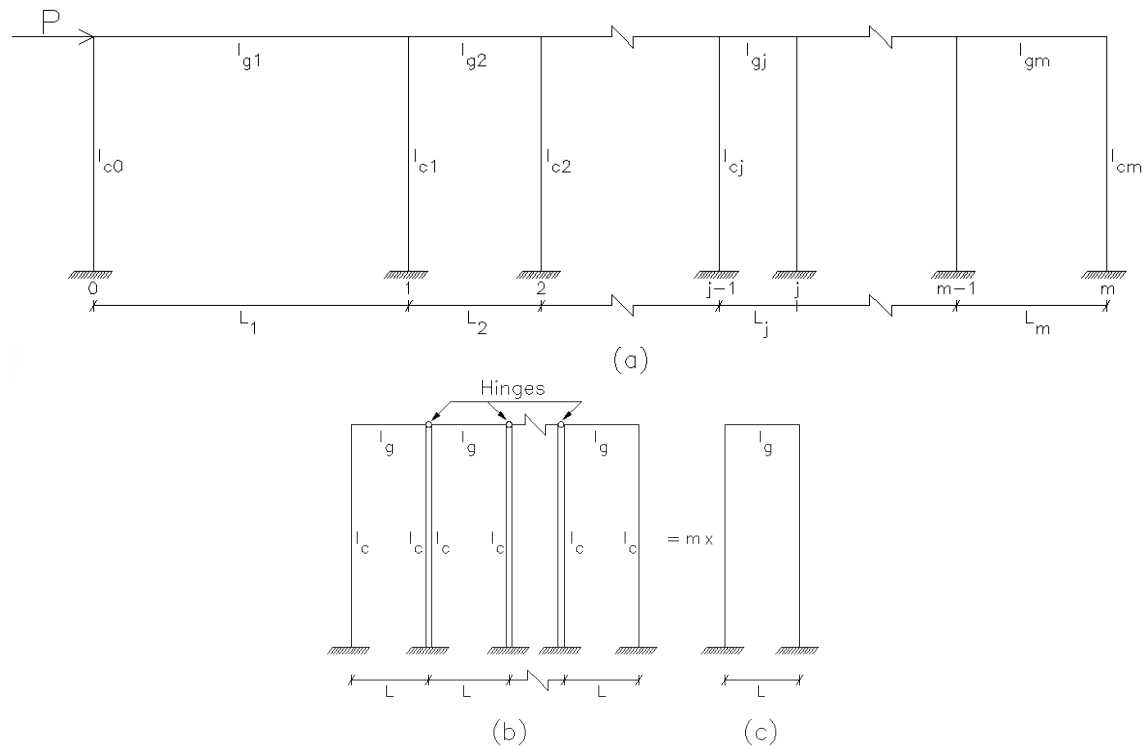


Figure 1. The simple multi-bay frame under lateral load, and its simplified equivalent systems: (a) the main 2D system, (b) the simplified equivalent 2D system, (c) the basic module of simplified

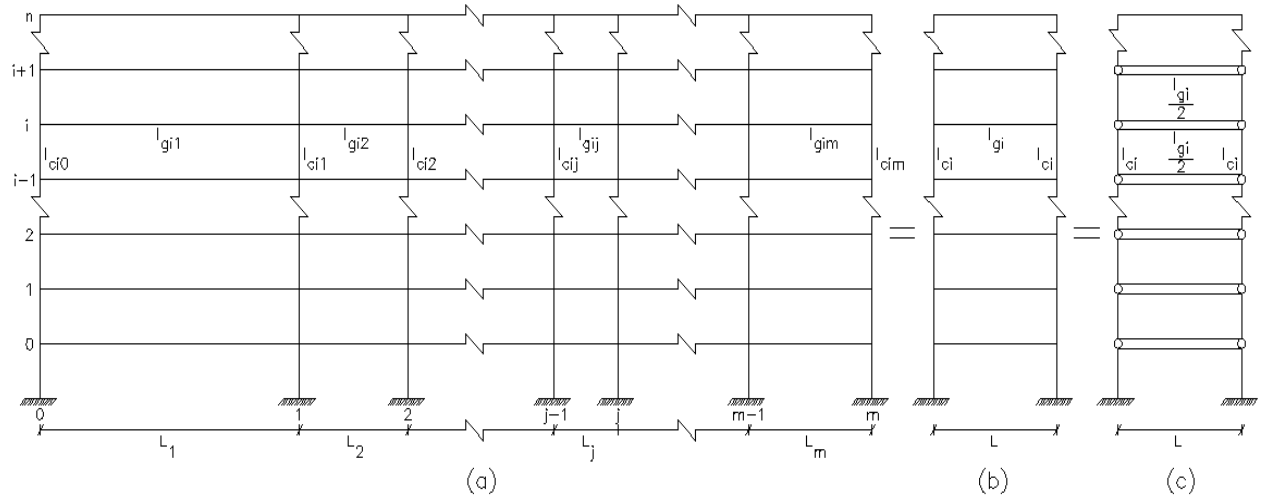


Figure 2. Model of m-bay, n-story 2D moment frame, and its simplified equivalent systems: (a) the main 2D system, (b) the one-step-simplified equivalent 2D system, (c) the final simplified equivalent system

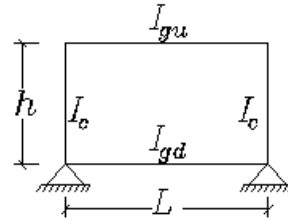


Figure 3. The main frame module of the simplified 2D system for regular moment frames

### Equations of proposed method Hosseini and Imagh-e-naiini

Equations	Number of Equations
$I_c = \frac{\left( \sum_{j=1}^m I_{cj} \right)}{2m}$	(1)
$\frac{I_g}{L} = \frac{\left[ \sum_{j=1}^m \left( \frac{I_{gi}}{I_j} \right) \right]}{m}$	(2)
$I_{cj} = \frac{1}{2} \left( \sum_{j=1}^m I_{cij} \right)$	(3)
$\frac{I_{gi}}{L} = \sum_{j=1}^m \left( \frac{I_{gij}}{I_j} \right)$	(4)
$k_{jm} = \left( \frac{12k_c}{h^2} \right) \left( \frac{k_c(k_d + k_u) + 6k_d k_u}{k_c^2 + 2k_c(k_d + k_u) + 3k_d k_u} \right)$	(5)
$k_c = \frac{EI_c}{h}$	(6)
$k_d = \frac{EI_{gd}}{h}$	(7)
$k_u = \frac{EI_{gu}}{h}$	(8)

Where  $I_c$ ,  $I_g$ ,  $L$ ,  $m$ ,  $h$ ,  $I_{gd}$ ,  $I_{gu}$ ,  $E$  and  $k$  are moment of inertia for column, moment of inertia for beam, span length, number of spans, height of the frame module, cross-sectional properties of the frame module, modulus of elasticity, lateral stiffness, respectively.

### 2.1 Base concept of estimation lateral and torsional stiffness for 3D irregular structures

The lateral and torsional stiffness in the asymmetric plan building with 3D moment frame for X and Y directions were calculated with the approximating method for the 2D moment frame. Then, the simplified stiffness in the X and Y directions were summed in each direction, and the 3D system was exchanged for two 2D moment frames in each direction of X and Y.

Equations of suggested method Shadman and Golara	
Equations	Number of Equations
$k_{fm_x} = \sum_{i=1}^n k_{fm_i}$	(9)
$k_{fm_y} = \sum_{j=1}^m k_{fm_j}$	(10)
$X_{CR} = \frac{k_{fmy} \times \bar{x}}{\sum k_{fmy}}$	(11)
$Y_{CR} = \frac{k_{fmx} \times \bar{y}}{\sum k_{fmx}}$	(12)
$k_{\theta_{x,i}} = \sum_{i=1}^n k_{fmy_i} \times x_i^2$	(13)
$k_{\theta_{y,i}} = \sum_{i=1}^n k_{fmx_i} \times y_i^2$	(14)
$k_{\theta_{c,i}} = \sum_{i=1}^n \frac{GJ}{h}$	(15)
$J = I_{xc} + I_{yc}$	(16)
$k_{\theta_i} = \sum_{i=1}^n (k_{fmy_i} x_i^2 + k_{fmx_i} y_i^2 + \frac{GJ}{h})$	(17)
$k_{\theta_i} = k_{\theta_{x_i}} + k_{\theta_{y_i}} + \sum \frac{GJ}{h}$	(18)
$G = \frac{E}{2(1+\nu)}$	(19)
$\Delta_{xi} = \frac{V_{xi}}{k_{fmx}}$	(20)
$\Delta_{yi} = \frac{V_{yi}}{k_{fmy}}$	(21)
$\theta_i = \frac{T_i}{k_{\theta_i}}$	(22)
$m_{\theta_i} = \frac{1}{2} m(a^2 + b^2) + md^2$	(23)

Where  $i$ ,  $n$ ,  $x_i$ ,  $y_i$ ,  $X_{CR}$ ,  $Y_{CR}$ ,  $k_{\theta_{x_i}}$ ,  $k_{\theta_{y_i}}$ ,  $k_{\theta_{c_i}}$ ,  $G$ ,  $J$ ,  $\nu$ ,  $E$ ,  $\Delta_{x_i}$ ,  $\Delta_{y_i}$ ,  $V_{x_i}$ ,  $V_{y_i}$ ,  $k_{fmx}$ ,  $k_{fmy}$ ,  $k_{\theta_i}$ ,  $T_i$ ,  $\theta_i$ ,  $a$ ,  $b$ ,  $m$ ,  $m_{\theta_i}$  and  $d$  are counter of each story, number of moment frames in the each direction, distance from each frame to stiffness center of each story in two directions, coordinate of the center of stiffness, torsional stiffness of each story, torsional stiffness of columns, shear modulus, polar inertia of column, poisson ratio, modulus of elasticity, estimated lateral displacement in the each direction, story shear force in the each direction, estimated lateral stiffness, estimated torsional stiffness, torsional moment of rigidity center, total in-plan rotation of each story, length and width of panel, distance between mass center of panel and total mass center of each story, total mass of each story,

### 3. ASSESSMENT OF PROPOSED APPROACH FOR 3D IRREGULAR BUILDING

A numerical example with asymmetric plan is conducted to show the appropriate efficiency of the proposed method with calculating the lateral displacement, torsion of each story, and main period of structures. Irregular building is consisting of six-story steel frame. In model, the plan view is the same (Figure 4) and the height of the stories and the length of bays are 3m and 5m, respectively. The modulus of elasticity is supposed to be  $2100 \text{ tonfcm}^{-2}$ . The shear modulus of steel material was considered  $784 \text{ tonfcm}^{-2}$ . For calculating natural period and base shear force, a mass of 448 tons has been considered for all floors. For calculation of lateral loading have been used the regulations of Iranian seismic design code (IS 2800-05) [11]. Lateral loading of frames is defined as static loads. Basis acceleration and soil period for soil type II is considered 0.35 and 0.5 second, respectively. The IS 2800-05 is derived from UBC 1994 and BOCA 1978 and have undergone major changes over the years. For sections of beams were used from IPE-450, IPE-400 and IPE-330. Also, sections of columns were introduced HE-340B, HE-320B and HE-300B. The beams and columns are modeled as elastic elements with steel02 material. All of connections between beams and columns within the structure are assumed to be fixed. Also, the P-delta effects are considered in calculations. The masses are lumped at floor levels; moreover, the degrees of freedom have been defined horizontal. The Rayleigh damping coefficient for steel has been assumed 0.05 at the first-vibration-mode, and the effect of nonstructural elements was not considered. The specification of 3D modeled six-story frame with irregular plan is shown in Figure 4. The model is modeled by employing ETABS software. The exact values of lateral and torsional deformation were calculated ETABS software. For this structure is computed the lateral displacement, torsional stories and the period time of first mode using the proposed method. Finally, the values calculated by the proposed method will be compared with the exact analysis values.

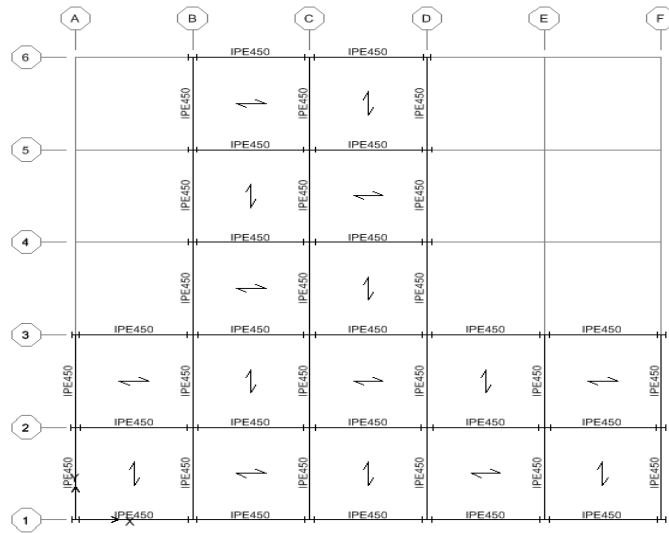


Figure 4. Schematic 2D Plan view of studied structure with asymmetric distribution stiffness

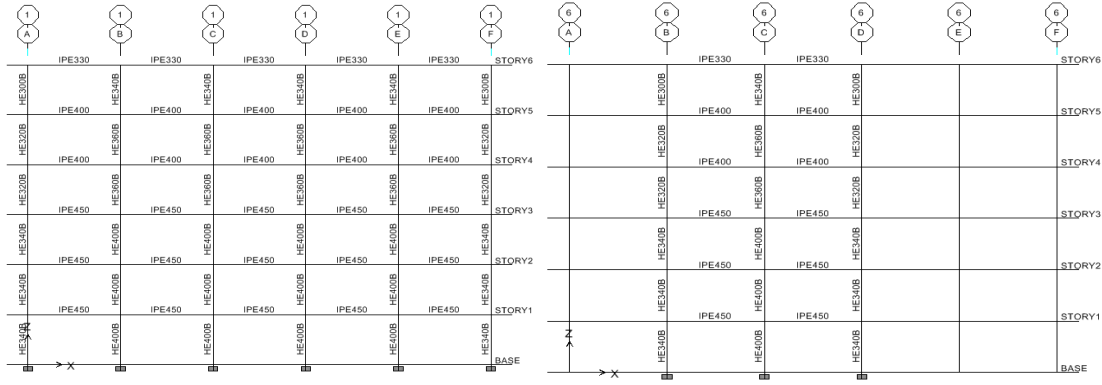


Figure 5. Schematic 2D view of AF-1 and AF-6 frames of studied structure

The stiffness center in X and Y direction and the values of stiffness based on the proposed method are showed in Table 1 and Table 2 to 4, respectively. Torsional stiffness of columns in each story should be computed, and its value should be sum separately with the torsional stiffness of each frame that the results are demonstrated in Table 5. Based on lateral stiffness values, the story shear force, and Hooke's law, can be computed story lateral displacement by Equations 20 and 21 that are presented in Table 6. The total in-plan rotation of each story can be computed by using Equation 22 and Table 7 presents estimated rotation values calculated by stiffness center and in-plan torsional moment provided by determined lateral load distributions in the X and Y direction for considered structure. In order to calculate the main period of structure, lateral and torsional stiffness values should be computed based on the above-mentioned equations 15 to 18. Thus, the rotation and transition component of mass moment inertia for each story is calculated by using equation 23 which the results are shown in Table 8.

Table 1. Specifications of selected structure

Story	Story height (m)	Center of Rigidity (m)	
		X	Y
story 6	3	11.656	10.061
story 5	3	11.654	10.057
story 4	3	11.652	10.066
story 3	3	11.650	10.054
story 2	3	11.650	10.054
story 1	3	11.644	10.072

Table 2. The obtained values of stiffness based on simplified model in proposed method  
Unit is (kg.cm)

Frame 1					
Story	h(cm)	$k_c = \frac{EI_c}{h}$	$k_u = \frac{EI_{gu}}{h}$	$k_d = \frac{EI_{gd}}{h}$	$k_{fm_x} = \left(\frac{12k_c}{h^2}\right) \left(\frac{k_c(k_d + k_u) + 6k_d k_u}{k_c^2 + 2k_c(k_d + k_u) + 3k_d k_u}\right)$
story 6	300	544684866.7	191992240	188648280	37609.36
story 5	300	649795316.7	188648280	188648280	38987.40

story 4	300	649795316.7	188648280	275183440		44969.98
story 3	300	837213400	275183440	275183440		55229.03
story 2	300	837213400	275183440	275183440		55229.03
story 1	300	837213400	275183440	1.00E+20		111107.68

Table 3. Calculation of the stiffness center based on simplified model in proposed method for the structure in Y direction

Story	$k_{fmy}$						Calculate of $X_{CR}$		
	Frame A	Frame B	Frame C	Frame D	Frame E	Frame F	$\Sigma k_{fmy}$	$k_{fmy} * \bar{x}$	$X_{CR}$
story 6	12413.2	26094.9	26798.6	26094.9	13166.8	12413.2	116981.5	1363548.3	11.656
story 5	12768.9	26762.0	27541.5	26762.0	13469.8	12768.9	120073	1399271.9	11.654
story 4	13927.1	29325.7	30270.5	29325.7	14770.2	13927.1	131546.2	1532799.3	11.652
story 3	15675.5	33331.3	34568.7	33331.3	16817.0	15675.5	149399.3	1740541.2	11.650
story 2	15675.5	33331.3	34568.7	33331.3	16817.0	15675.5	149399.3	1740541.2	11.650
story 1	20550.9	44726.2	47095.2	44726.2	22649.0	20550.9	200298.5	2332229.7	11.644

Table 4. Calculation of the stiffness center based on simplified model in proposed method for the structure in X direction

Story	$k_{fmx}$						Calculate of $Y_{CR}$		
	Frame 1	Frame 2	Frame 3	Frame 4	Frame 5	Frame 6	$\Sigma k_{fmx}$	$k_{fmx} * \bar{y}$	$Y_{CR}$
story 6	37609.4	42856.7	40848.3	20906.5	20906.5	19194.5	182321.9	1834356.6	10.061
story 5	38987.4	44254.4	39342.9	21554.9	21554.9	19887.5	185582.0	1866308.5	10.057
story 4	44970.0	52413.9	45455.9	25373.8	25373.8	23024.7	216612.2	2180330.9	10.066
story 3	55229.0	63876.8	55976.1	31043.7	31043.7	28028.4	265197.7	2666384.5	10.054
story 2	55229.0	63876.8	55976.1	31043.7	31043.7	28028.4	265197.7	2666384.5	10.054
story 1	111107.7	190772.7	115293.8	82850.7	82850.7	57881.4	640757.0	6453611.3	10.072

Table 5. Computed torsional stiffness value for each story based on proposed method

Story	$k_{\theta,xi} = \sum_{i=1}^n k_{fmy} \times x_i^2$	$k_{\theta,yi} = \sum_{i=1}^n k_{fmx} \times y_i^2$	$\sum \frac{GJ}{h}$	$k_{\theta} \left( \frac{kg \cdot cm}{rad} \right)$
story 6	63348546307	117637479576	3951202955	184937228838
story 5	65066654862	121735486419	4849042622	191651183902
story 4	71105217175	141587051341	4849042622	217541311139
story 3	80359453709	173058943471	6216126769	259634523948
story 2	80359453709	173058943471	6216126769	259634523948
story 1	106390577103	392564373740	6216126769	505171077612

Table 6. Estimation of lateral displacement values of rigidity center for each story

Story	$k_{fmx}(kg \cdot cm)$	$k_{fmy}(kg \cdot cm)$	$V_{xi}(kg)$	$V_{yi}(kg)$	$\Delta_{xi}(cm)$	$\Sigma \Delta_{xi}(cm)$	$\Delta_{yi}(cm)$	$\Sigma \Delta_{yi}(cm)$
story 6	182321.9	116981.5	74281.5	74281.5	0.407	4.29	0.635	7.84
story 5	185582.0	120073.0	138833.7	138833.7	0.748	3.88	1.156	7.20
story 4	216612.2	131546.2	190571.3	190571.3	0.880	3.13	1.449	6.04
story 3	265197.7	149399.3	229830.5	229830.5	0.867	2.25	1.538	4.60
story 2	265197.7	149399.3	256064.5	256064.5	0.966	1.39	1.714	3.06
story 1	640757.0	200298.5	269181.4	269181.4	0.420	0.42	1.344	1.34

Table 7. Estimated in-plan rotation values of each story provided by torsional moment for X and Y direction

Story	$k_{\theta_{x,y}} \left( \frac{kg \cdot cm}{rad} \right)$	$T_{i,x,y}(kg \cdot cm)$	$\theta_{x,y}(rad)$	$\Sigma \theta_{x,y}(rad)$
story 6	180986025883	85913515.9	0.00047	0.0052
story 5	186802141280	160589646	0.00085	0.0047
story 4	212692268517	220444229	0.00104	0.0039
story 3	253418397179	265863493	0.00105	0.0028
story 2	253418397179	296213481	0.00117	0.0018
story 1	498954950843	311388475	0.00062	0.00062

Table 8. Calculated lateral and torsional stiffness values, transformed mass moment inertial and torsional mass moment inertial

Story	$k_{fmx}(kg \cdot cm)$	$k_{fmy}(kg \cdot cm)$	$k_{\theta} \left( \frac{kg \cdot cm}{rad} \right)$	Mass X $\left( \frac{kg}{\frac{cm}{sec^2}} \right)$	Mass Y $\left( \frac{kg}{\frac{cm}{sec^2}} \right)$	Mass $\theta$ $(kg \cdot cm \cdot sec^2)$
story 6	182321.9	116981.5	180986025883	438.2	438.2	1230858142
story 5	185582.0	120073.0	186802141280	457.0	457.0	1283569354
story 4	216612.2	131546.2	212692268517	457.8	457.8	1285953968
story 3	265197.7	149399.3	253418397179	463.2	463.2	1301064671
story 2	265197.7	149399.3	253418397179	464.3	464.3	1304104001
story 1	640757.0	200298.5	498954950843	464.3	464.3	1304104001

### 3.1 Calculation of exact values using software

The exact values of lateral and torsional stories can be to compute by ETABS software. These results include lateral displacement values for each direction of the x-axis and y-axis and the torsion of each story around z-axis. Table 9 shows the exact values of lateral displacement in two directions and the torsions of each story. Figure 6 illustrates schematic views of the lateral and torsional deformations of



the sixth stories.

Table 9. Exact values of displacements in two directions and torsional stories

Story	Diaphragm	Load	U <sub>x</sub> (cm)	U <sub>y</sub> (cm)	R <sub>zx</sub> (rad)	Load	U <sub>x</sub> (cm)	U <sub>y</sub> (cm)	R <sub>zy</sub> (rad)
story 6	D6	EX	4.225	0.495	0.005118	EY	0.735	7.665	0.005009
story 5	D5	EX	3.865	0.462	0.004529	EY	0.643	6.959	0.004434
story 4	D4	EX	3.128	0.407	0.003711	EY	0.519	5.86	0.003635
story 3	D3	EX	2.261	0.300	0.002726	EY	0.375	4.476	0.002671
story 2	D2	EX	1.39	0.189	0.001713	EY	0.223	2.989	0.001685
story 1	D1	EX	0.428	0.004	0.00061	EY	0.078	1.355	0.00063

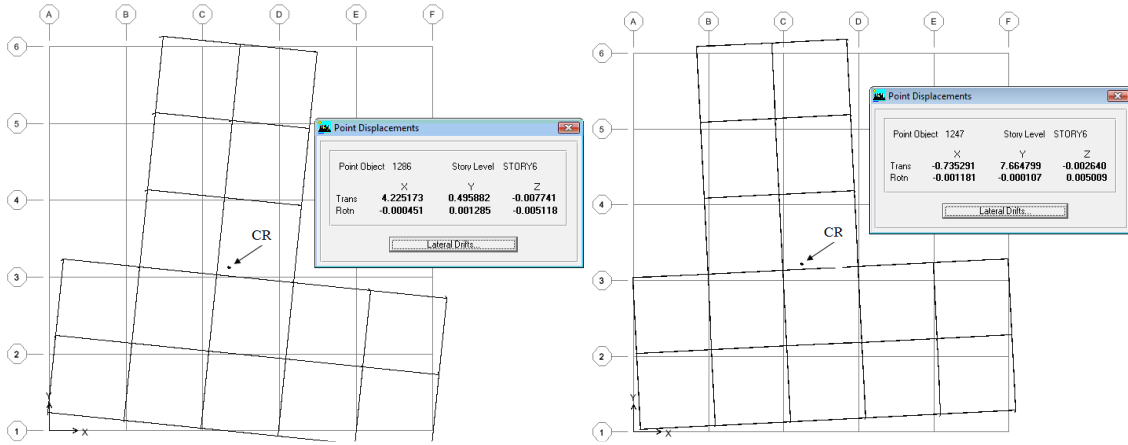


Figure 6. Schematic views of lateral displacement and torsional deformations for sixth stories of studied building under loading for X and Y direction

### 3.2 Error percentage of proposed approach

Given the exact values of ETABS software and approximate values of proposed method can be calculate the percentage of available errors. The calculated errors can also be computed as follows:

$$\%e_{displacement} = \frac{\Delta_{estimate} - \Delta_{exact}}{\Delta_{exact}} \times 100 \quad (24)$$

$$\%e_{rotation} = \frac{\theta_{estimate} - \theta_{exact}}{\theta_{exact}} \times 100 \quad (25)$$

$$\%e_{period} = \frac{T_{estimate} - T_{exact}}{T_{exact}} \times 100 \quad (26)$$

Where  $\Delta$ ,  $\theta$  and  $T$  are lateral displacement of each story, torsion of each story, and period time of first mode, respectively. Table 10 to 12 demonstrate the calculated errors of lateral displacement, torsion and the period time of first mode values of six-story structure for each stories computed by the exact solution and the proposed method.

Table 10. Calculated errors of lateral displacement values for each stories computed by the exact solution and proposed method

Story	$\Sigma\Delta_{xi}$ (cm)	$U_x$ (cm)	% $e_x$	$\Sigma\Delta_{yi}$ (cm)	$U_y$ (cm)	% $e_y$
story 6	4.288	4.225	1.5	7.84	7.665	2.3
story 5	3.880	3.865	0.4	7.20	6.959	3.5
story 4	3.132	3.128	0.1	6.04	5.86	3.1
story 3	2.252	2.261	-0.4	4.60	4.476	2.8
story 2	1.386	1.39	-0.3	3.06	2.989	2.4
story 1	0.420	0.4275	-1.8	1.34	1.355	-1.1

Table 11. Calculated errors of torsional stories values computed by the exact solution and proposed method

Story	$\Sigma\theta_x$ (rad)	$R_{zx}$ (rad)	% $e_{\theta x}$	$\Sigma\theta_y$ (rad)	$R_{zy}$ (rad)	% $e_{\theta y}$
story 6	0.0052	0.005118	1.6	0.0052	0.005009	3.8
story 5	0.0047	0.004529	3.8	0.0047	0.004434	6.0
story 4	0.0039	0.003711	5.1	0.0039	0.003635	7.3
story 3	0.0028	0.002726	2.7	0.0028	0.002671	4.8
story 2	0.0018	0.001713	5.1	0.0018	0.001685	6.8
story 1	0.00062	0.00061	1.6	0.00062	0.00063	-1.6

Table 12. Obtained first mode period of the structure based on the exact solution and the proposed method

Mode	$T_{exact}$ (sec)	$T_{estimate}$ (sec)	% $e_{Period}$
1	1.3774	1.5335	10.2

#### 4. CONCLUSIONS

Evaluation of the proposed method requires accurate structural model. For predict the response of structural system under seismic loads; the building structure should be accurately described. Using reliable analytical software and accuracy definition of material strength, elements stiffness and masses are necessary. This paper, an approximate and quick method suggest for estimating lateral stiffness, torsional stiffness, lateral displacement, torsional stories and first mode period of structure. This paper aims to obtain the error between result proposed method and exact analysis by ETABS software for the asymmetric plan with MRF structure. The results showed that maximum error for calculation of lateral displacement, torsional stories and first mode period of structure 3.5%, 7.3% and 10.2%, respectively. The results illustrated that there is good agreement and insignificant error between values of proposed method and exact solution based on analytical modeling. Thus, the proposed method can provide a proper alternate with acceptable accuracy for estimating initial lateral displacement, torsional stories and also period of the first mode of 3D irregular structure. The proposed method provides the conservative estimation of the displacement, rotation and main period of the structure.

#### 5. ACKNOWLEDGMENTS

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