16TH EUROPEAN CONFERENCE ON EARTHQUAKE ENGINEERING THESSALONIKI 18 - 21 JUNE 2018

TWIN BOUNDARY METHOD FOR DYNAMIC SOIL MODELING

Samyar SARRAF[1], Sadyar SARRAF[2], Kiarash DOLATSHAHI[3], Reza RAFIEE[4]

ABSTRACT

With the achievements appeared in FE and return to initial artificial boundary theory of Smith (1974), this paper introduces and evaluates a new innovative approach by improvement and simplification of artificial boundaries. The theory of the proposed boundary is based on introducing a metaphorical twin space boundary at a same place over each edge sides, named by the authors as Twin-Boundary. One of the Twin Boundaries works as the fixed-end boundary and the other plays the role of the free-end. The resultant of these Twin Boundaries is summation of reflected waves at each sides that is null. A well-known fundamental rule of physics is established that two different particles cannot be appeared at a same coordinate in space. Although basic theory of FE is set up to follow physical and mechanical rules governing nature, new advances over virtual space of FE modeling have made it possible to break the mentioned laws of physics with absolute adherence. Twin Boundary is divided into three types based on dimensions, rate of accuracy and specific features to overcome vast user expectations. Application, instruction and feature of each type are explained and formulated. Finally, the ability of Twin Boundary for different incident excitation is clarified and a brief comparison of literature methods and Twin Boundary is presented.

Keywords: Artificial Boundary; Dynamic Analysis; Dynamic Soil Modeling; Twin boundary;

1. INTRODUCTION

Scientific engineering consistently strives to innovate and recuperate numerical or analytical modeling which be able to capture the best compliance with real world phenomena. Science progresses and computer achievements had a great influence on this aspect of engineering recently. Consequently, Finite Element Method (FEM) is one of the famous engineering purposed modeling technique. Although FE method is a young field of computation compared with the other available analytical modeling, it is started in the middle of last century by O.Z. Zienkiewicz, this method has got widely uses for evaluation of solid, fluid or multi-phases behaviors. Not only FE considers different material types but also FE will account different method of analyzing such as static analysis, plastic behavior, fracture mechanic and so on (Khoei 2014). Dynamic analysis is usually placed at the top of FE researching issues because of problem variety consideration and computational volume over vast range of FE analyzing. Therefore, dynamic part always relies on powerful computer and new analyzing methods for improving and facilitating of FE dynamic employment.

The majority of dynamic evaluation is the manner of wave propagation modeling over model body to obtain accurate and reliable outputs compared to available theories and reality of natural phenomena. This majority is one of the main source of principal problem for dynamic analysis. Since nature of soil dynamic modeling usually face up to very large dimensions, engineers are involved to evaluate a semi-infinite media. This issue is occurred while FE can only consider a finite area of the semi-infinite soil media; in fact, FE modeling considers only a limit portion of real behavioral world which leads to creation of wave turbulences over cutting edges. These turbulences are resultant of inability of cutting edges for wave transition function (Towhata 2008).

[1] Correspondent Author, Sharif University of Technology-Kish campus, Tehran, Iran, sarraf@kish.sharif.edu
[2] Civil Engineering Department, Sharif University of Technology-Kish campus, Tehran, Iran, sadyar.sarraf@gmail.com
[3] Civil Engineering Department, Sharif University of Technology, Tehran, Iran, dolatshahi@sharif.edu
[4] School of Civil Engineering, University of Tehran, Tehran, Iran, rezarafiee@ut.ac.ir

Turbulences over edges make error sources in modeling. Thus, engineers attempt to find a suitable solution for this unwanted errors from early stages of the dynamic numerical analysis by imposing a virtual space over cutting edges to transmute waves outside, this imposed space is named artificial boundary. An especial boundary conditions are needed to transit waves through artificial boundary outside of soil modeling. Many researchers have proposed different boundaries to comply this demand (Wolf 1986). Developing of the artificial boundaries are almost started by Lysmer and Kuhlemeyer (1969) which introduced viscous boundary that is one of the popular method because of its simplicity (Towhata 2008). There are other artificial boundary types as consistent boundary by Lysmer and Waas (1972), superposing boundary by Smith (1974), unified boundary by White (1977), paraxial boundary by Engquist (1977), Transmitting boundary by Lio and Wang (1982), dynamic infinite element by Changbin (1989), viscous-spring boundary by Deek (1994) and PML boundary by Berenger (1994), (Jingbo & Yandong 1998 - Kausel 1988). The last achievement in this field over more than of last two decades is known as Boundary Element Method (BEM). Since BEM is developed only for linear homogeneous soil media, coupling method of FEM-BEM is introduced afterward by employment of FEM as near field soil with ability of nonlinear consideration and BEM as waves transmitting part for far field soil. While all these mentioned models try to improve or remove past presented methods' inaccuracy or problems, the field of artificial boundary has not been achieved a widely accepted solution between engineers yet and this report tries to put a step forward along this way.

2. TWIN BOUNDARY

2.1 Artificial Boundaries

By beginning of computer era, the ability of numerical analysis for problems has been extensively improved. Eventually, analysis of soil dynamic behavior has been initiated to be evolving. Differences in boundary over cutting edges of model is the main distinction between static and dynamic modeling of soil. Nature of soil static analysis is based on stresses that vanishes as a results of far field area growth compared to the fixed amount of exerting force over whole model where particles are far from the exerting force position- i.e. 10 times of the model width (Khoei 2014). In contrast, dynamic modeling must be extensively large to show reasonable results by damping out reflected waves' error.

The simplest kind of artificial boundary is cutting edges located far from the area of interest and utilizing damping effect to mitigate reflected waves. This implies that energy dissipation due to nonlinearity (stress–strain loop) leads to both decay of the reflected waves and unaffected response of interested zone consequently. Other simple boundary kinds may state as free of horizontal motion with zero vertical motion or repeated boundary for a symmetric geometry (Towhata 2008).

2.2 Idea which Left Behind

Developing of artificial boundaries has been almost initiated with the idea of superposing solution of two different but conventional boundary conditions (Smith 1974). In fact, considering two types of fixed-end and free-end shows reflected waves from the fix boundary and the free boundary with the same magnitude but different directions that make superposed results null, while the incident wave is remained unchanged as shown in Figure 1. In practice, calculation is conducted twice to find solution for two different boundary conditions and then results are added together and divided by two. This boundary condition works in a time-domain analysis in which an equation of motion is integrated along the time. In practice of two-dimensional analysis framework, the reflected wave hits another boundary and reflects again. These multiple reflections make the superposition very complicated. To avoid this, the superposition is conducted periodically after a short specified time interval. Complication of the reflected wave combination before superposing of the fix and free model and computation cost of twice modeling computation leads to abolishing of Smith (1974) superposing model.

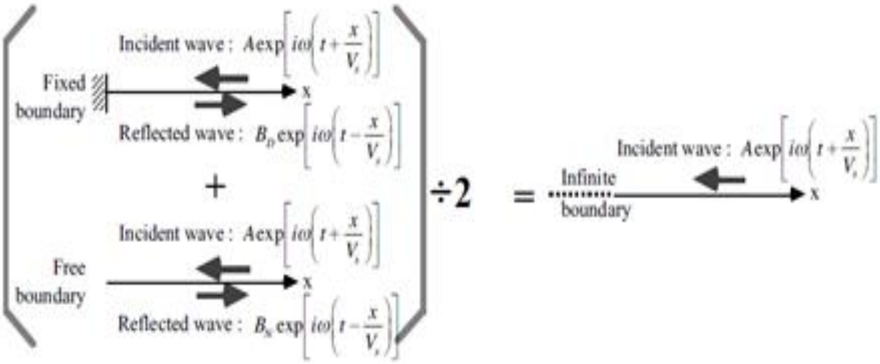


Figure 1. Incident and Reflected Waves Form Fix and Free Ends (Towhata 2008)

2.3 New Method of Artificial Boundary

One of the basic rules of physics expresses that two particles cannot be placed in a same coordinate at the same time. Formation of finite element allows to break this decisive rule; in fact, because FE is based on complete inherent to physics and mechanics laws, this rule breaching happens in such a way that rules are broken meanwhile all fundamentals are followed. Although this claim seems to be contradictory, it works. Point of this idea is based on the lack of separation ability of FE over different particles penetration into each other. This defeat of FE, which is familiar in the field of contact element, is an inspiration for introduction of a new artificial boundaries named Twin Boundary (TB). TB is a step forward through the idea of superposed fixed and free boundaries expressed by of Smith (1972).

Twin Boundary is formed from two overlapped branches which is attached on each cutting edge of model to transit the waves out of each cut. These branches are named coupling side that is formed from two elements with the same specifications located on the same coordinated and are similar to the main model elements. In fact, TB is two separate elements located on each other with the specific side boundary conditions of fixed or free for nodes allocated on the far side of TB elements from the main model and a junction between near side of coupling side nodes and main model cutting edge nodes, Figure 2.

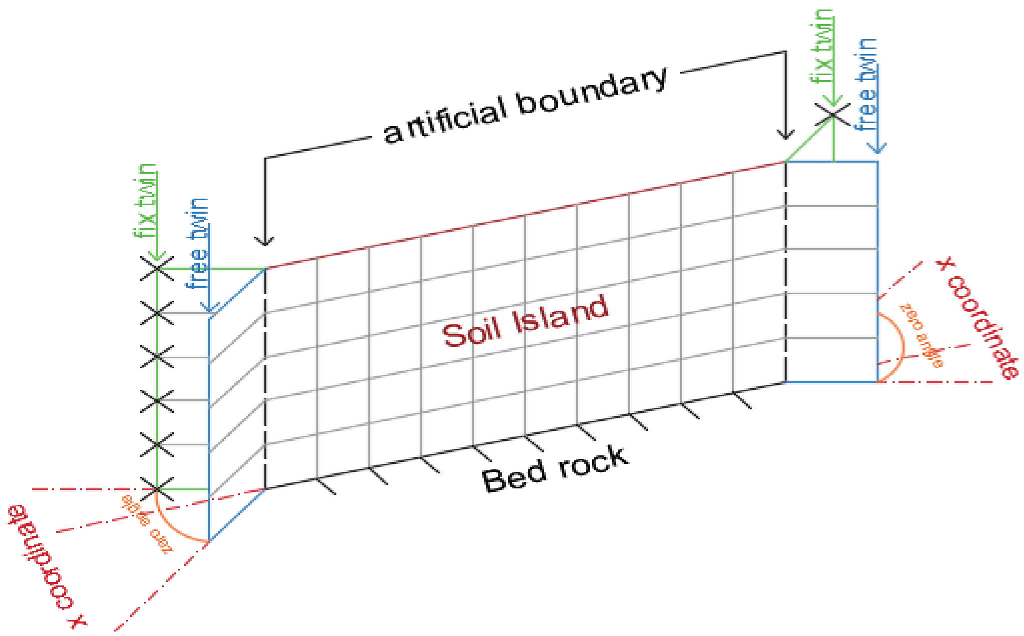


Figure 2. 2D Modeling of Main Model and Twin Boundary – Inclined View

The goal of coupling side is to utilize the ability of the different phase response from each sides to transfer waves outside, while another feature of stress absorption for the fixed branch and displacement absorption for the free branch of coupling side can be considered.

2.4 Advantages of Twin Boundary

Twin Boundary has considerable advantages with respect to conventional artificial boundaries that some of them are as follows:

- TB makes no change in the nature of formulation of FE modeling and no change in the set of wave equations governing dynamic analysis.
- Lack of requirements for parameters and values such as equivalent boundaries, damping and stiffness.
- Other methods completely rely on the damping of the system to mitigate turbulences due to insufficient absorbing system before entering the site area, thus, these boundaries need to model a large length of soil island in FE modeling, while Twin Boundary do not reflect considerable stress waves back to site.

All this makes Twin boundary to be more realistic not only for specific situation, but also for any shape and vast range of imposed excitation motions. This boundary can Apply in the fields of soil dynamic analysis, soil-structure-interaction analysis and earthquake engineering purposes.

3. TWIN BOUNDARY MODELING

3.1 Twin Boundary Types

Range of applicability, application, the desired accuracy compared to the model dimensions and target error are the major issues that divide Twin Boundary into three main groups. Even though all three types show almost equal ability for handling of dynamic problems, each has specific features that distinguishes it from the rest. In the following, each type is examined separately and the superiority and applicability of each is described. These three types are as follows:

1) Analogous TB Element
2) Long TB Element
3) Improved TB Element

3.2 Model Specification

Vast range of parameter considerations is included into this research; however, information that may not be included into an offered evaluation follow these section specifications:

Twin Boundary modeling is performed in 2D plain strain space over a soil-island with elastic modulus of 150 MPa which is mostly considered for dense soil types. Elastic isotropic material constitutive law is applied for stress-strain relation. The model length dimension ranges between 20 to 500 meters by discretizing of (20, 50, 60, 70, 80, 90, 100, 125, 150, 200, 250, 500) and the depth dimension ranges 1 to 30 meters. The model mesh for is made by elements with specification of 1 by 1 square meter 4 noded 2 DOF quad element with 4 Guass integration points. Elements are considered dry and analysis is only on one solid phase.

Soil excitation is usually imposed in the way of uniform displacement approach and force excitation approach. This research assumes bed rock layer to impose uniform harmonic excitation over the base of model. Both vertical and horizontal behavior of Twin Boundary is evaluated by imposing P-wave and Shear-wave excitation respectively. To avoid mixture of vertical and horizontal wave over each case, the value of poison ratio is hold to be equal to 0.05.

3.3 Theoretical Source

Fundamental theory is needed to compute relative error of TB method and its values. Since the framework of excitation is harmonic with a specific frequency, utilizing transfer function of the frequency domain analysis can be a good choice for fundamental theory of error computation. Furthermore, working in one specific frequency helps us to avoid applying Fourier transfer to transmit between frequency spaces (Kramer 1996). The Transfer Function for both vertical and horizontal excitation is shown in Equation 1:

$$TF = \frac{1}{\sqrt{\cos^2(\frac{\omega}{V_i}H)+\sinh^2(\xi\frac{\omega}{V_i}H)}} \tag{1}$$

Where H is depth of soil layer, V is wave velocity, ω is frequency of imposed motion and ξ is damping ratio of soil, as shown on Figure 3 schematically. Subscript 'i' is replaced by 's' for shear-wave and by 'p' for vertical P-wave.

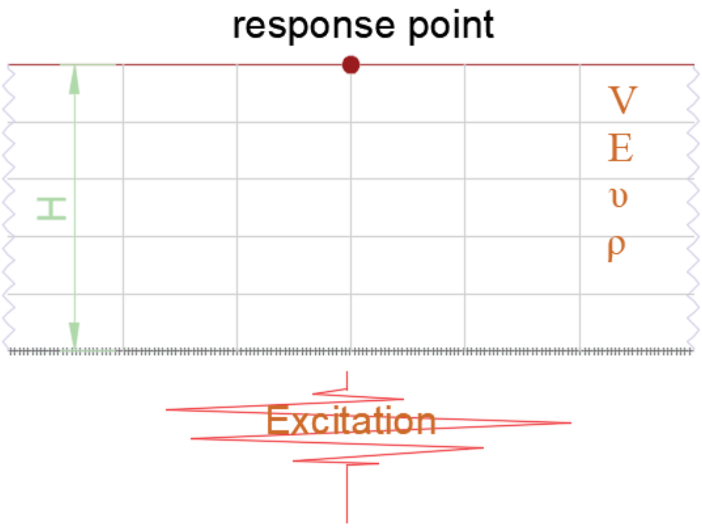


Figure 3. Schematic Side View of Transfer Function Modeling

3.4 Verification

Twin Boundary numerical analysis is performed on the software framework of OpenSees which is one of the most powerful open source softwares in the fields of soil, structure and soil-structure interaction. The model has been passed through error optimization of element size and time step by procedure of error estimation and Adaptivity methods of FEM, while it is literature suggestion to choose element size less than 0.10 of wave length (Semblat & Brioist 2000). The 30,000 application execution, fundamental theory and error estimation adaptivity are all authors credit for verification of results validity.

4. TWIN BOUNDARY TYPES AND APPLICATION

4.1 Analogous TB Element

TB elements are two element columns with equal specification compared to main model element that are allocated on each other and are merged to cutting side elements' nodes to build up coupling side, as shown in Figure 4. While side view does not show the overlapped elements, Figure 2 show better understanding for this issue. It is noticeable to emphasize on the nature of the coupling side which is an ingredient of a fix and free branches. To build these two branches, simply one extra column of nodes is defined beside of the model cutting edges and elements are mapped over these created nodes and main model side nodes. Thus, one side of coupling side is created and do the same work with increasing the nodes and element numbers to build the other side of coupling side. The final stage is to fix the first nodes column of the coupling side.

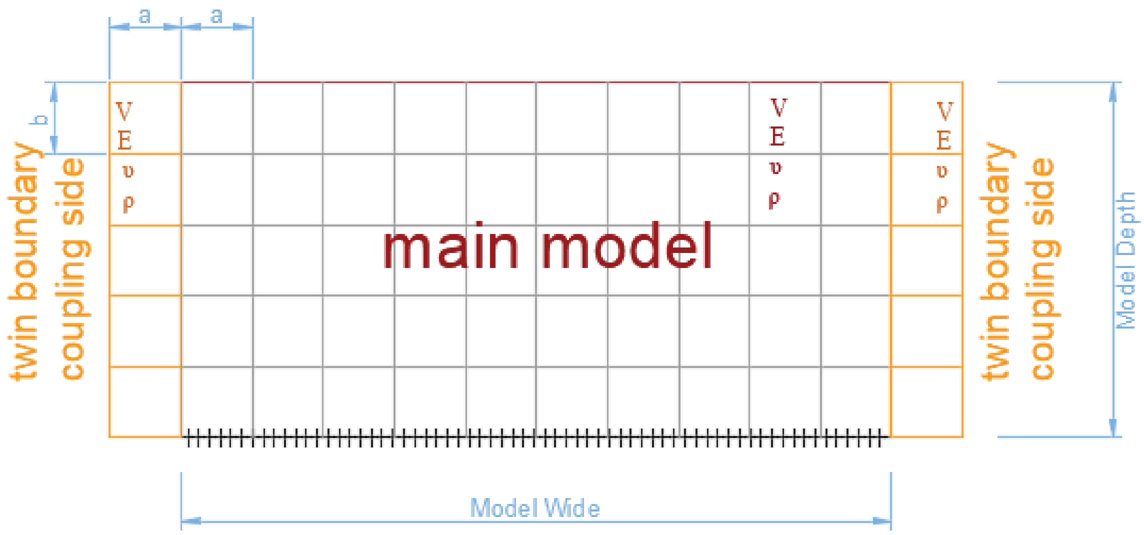


Figure 4. Analogous Twin Boundary Schematic Side View

4.1.1 Analogous TB graph use

Utilizing of analogous Twin Boundary meets a specific instruction; in fact, based on evaluating part of analogous TB, this type of Twin Boundary is exposed by some error source that must be considered to meet allowable results. The user designs the value of model depth and target error of calculation and by entering into Figure 5 the relevant wide range is obtained. It is noticeable that the legend values of Figure 5 shows the model wide value.

4.1.2 Evaluation

Figure 5 reveals some relation between model dimension and source of error:

- Error has direct depth dependency.
- Error has indirect width dependency.
- Perfect result for shallow or wide modeling.
- For accurate deep soil modeling, long width model is needed.

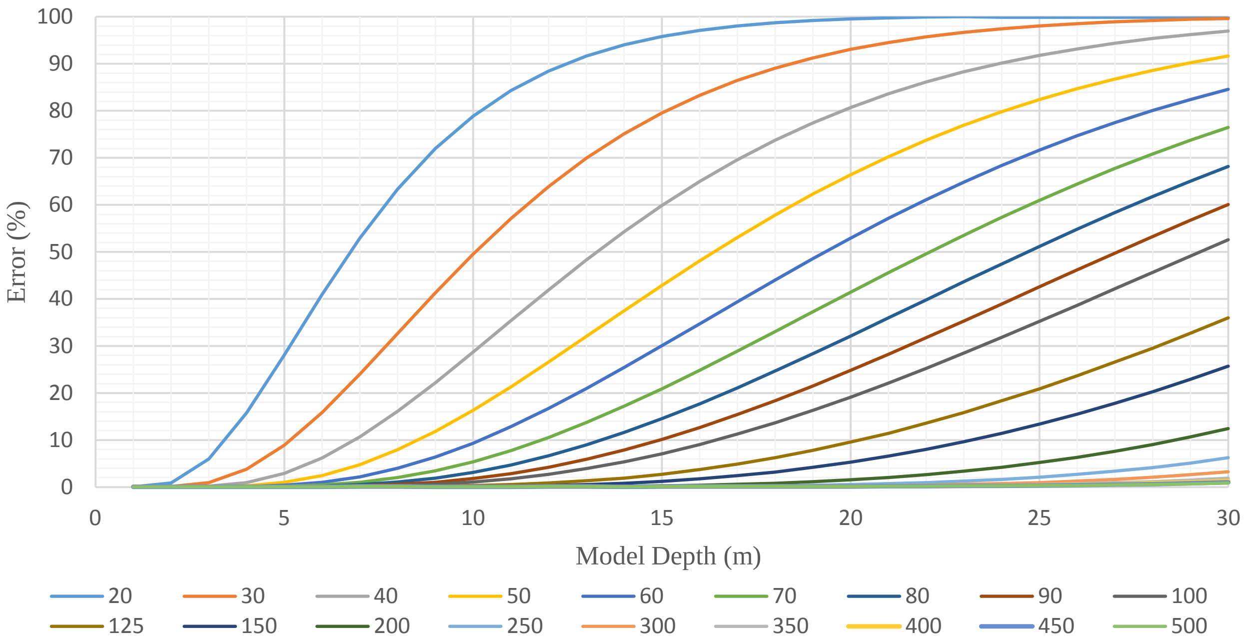


Figure 5. Similar Twin Boundary Element

4.1.3 Error Source

As mentioned, waves reflection summation from the fix and free boundary are null. However, considering both boundary's branches at the same time results to happening of a new phenomenon named *interaction of free-fix* sides. To clarify this phenomenon, suppose that the effects of free side movement works as an imaginary force exerted along the free side branch. Therefore, as shown in Figure 6, this imaginary force not only exposes the main model but also entangles the other side of the fix side branch to collate with this imaginary force. This phenomenon is the main reason why depth of the model has proportional relation to error; actually, by increasing depth, the model experiences more fix-free interaction since the magnitude of the imaginary force is growth as a result of the transfer function growth by increasing depth.

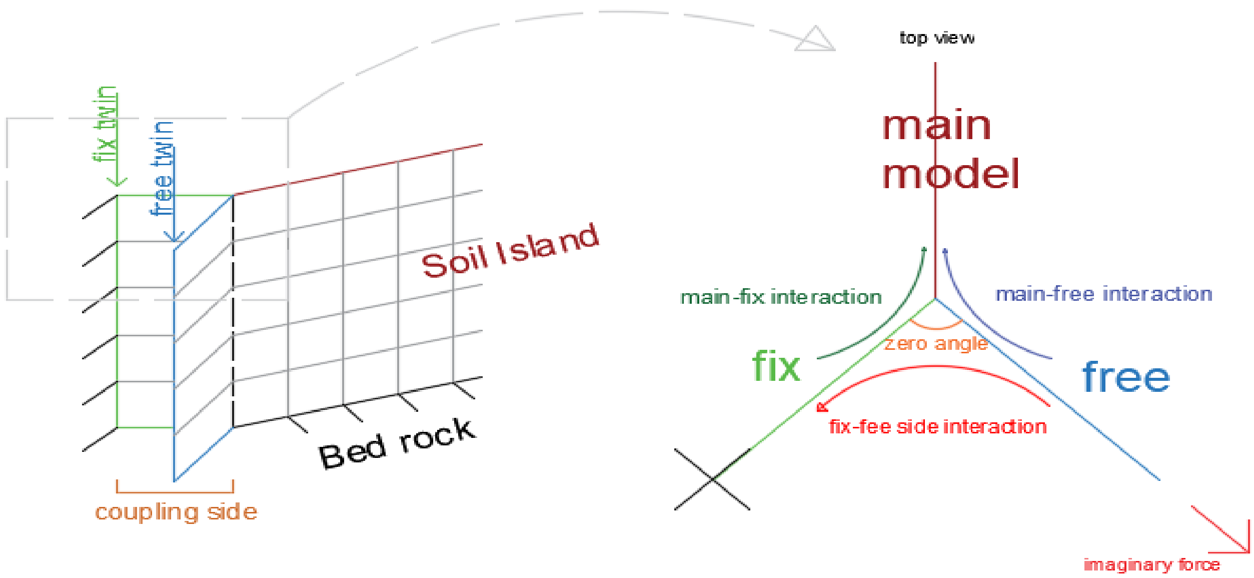


Figure 6. Available Interaction Between Coupling Side Branches and Main Model

4.2 Long TB Element

Although Analogous TB shows very good performances for shallow or wide models, new method is introduced to soothe the free-fix interaction source of error producing. Interaction works as the effect of fix nodes of coupling side on the free nodes of coupling side. Thus, by elongation distance between fix and free nodes, it is desired to mitigate the effect of free-fix interaction. Inspired by this idea, new Twin Boundary of Long TB is introduced as shown in Figure 7. Goal of this method is to minimize the errors in such a way that model with smaller width gains a functional opportunity with an acceptable target error.

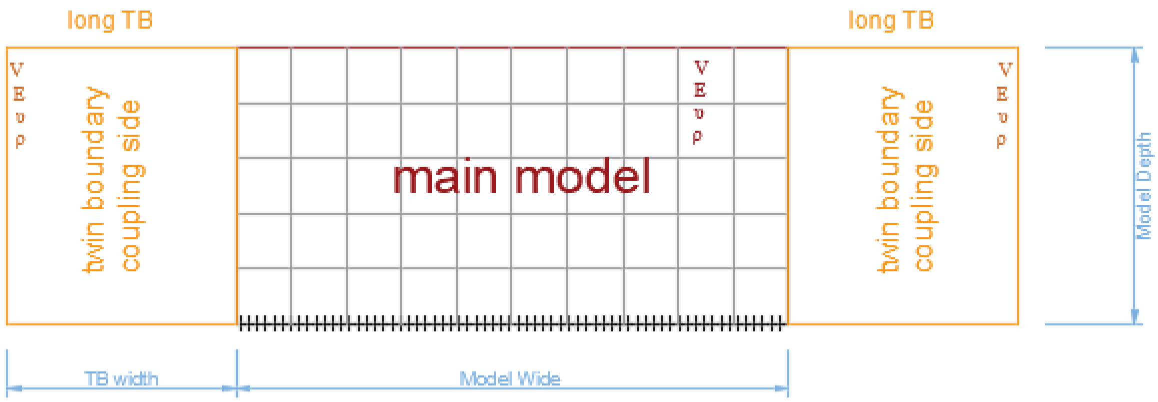


Figure 7. Long Twin Boundary Schematic Side View

4.2.1 Considerable Long TB Types

Figure 8 shows different allowable consideration of the fix and free branches' nodes of coupling side:

1) Side coupling
2) Side-bottom coupling
3) Side-bottom-top coupling

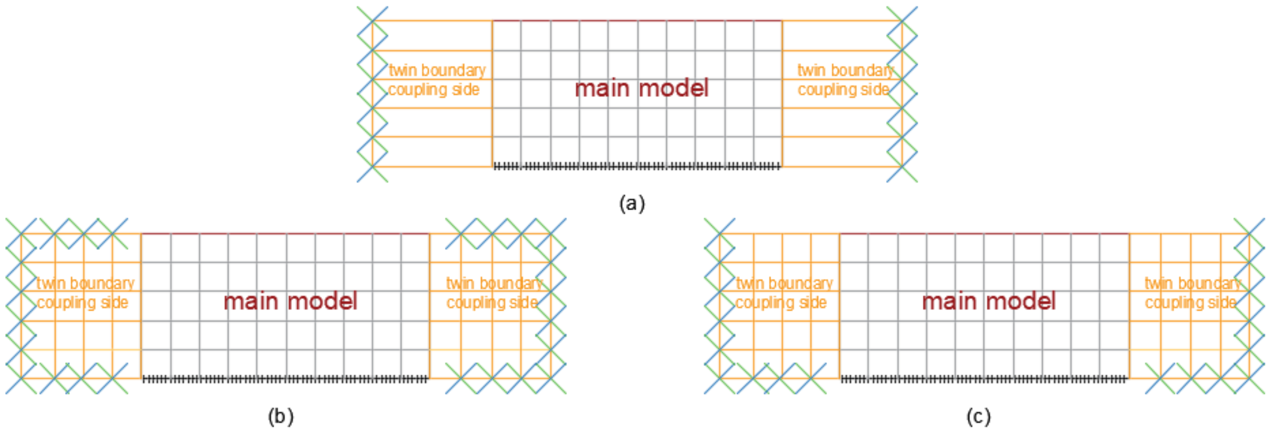


Figure 8. (a) Side Coupling; (b) Side-Bottom-Top Coupling; (c) Side-Bottom Coupling

Computational cost and free-fix interaction effects for both side-bottom coupling type and side-bottom-top coupling type cause elimination of these two types from long TB modeling and only side coupling is evaluated.

4.2.2 Error Sources and Overcoming Solutions

Evaluation of long TB is started with the same material specification between the main model and the Twin Boundary elements. Numerical assessments show two main problem sources of mass for inertia effect and unbalance FE stiffness matrices. Since area of Twin Boundary elements are increased, consequently the proportional mass of these elements are increased; therefore, effect of boundary elements inertia is signified and becomes one of the error sources for long TB. Another source of error is unbalance FEM stiffness matrices that is due to lengthening of element width; in fact, formation of stiffness matrices consists a multiplier named aspect ratio. The aspect ratio is a quantity that represents the fraction of width to length of an element in stiffness matrix (K). To overcome these two error sources, long TB is offered massless and high elastic modulus (E) for Twin Boundary element material specification. Although elimination of mass seems to take boundary analysis outside of dynamic behavior, presence of damping value proves the dynamic nature of Twin Boundary elements.

4.2.3 Special Value for Long TB

Evaluation of long TB indicates some especial parameters for the best optimized accuracy rate responses.

1) No mass for TB elements
2) Width of TB elements is set to be 1000 m
3) Elastic modulus of TB elements is set to be 1.0e9 kPa

4.2.4 Long TB Use

Figure 9's instruction is similar to the analogous TB; by choosing the value of target error and the model depth, relevant wide value of modeling is found. As seen, the proportional error in Long TB decreased considerably compared to analogous TB.

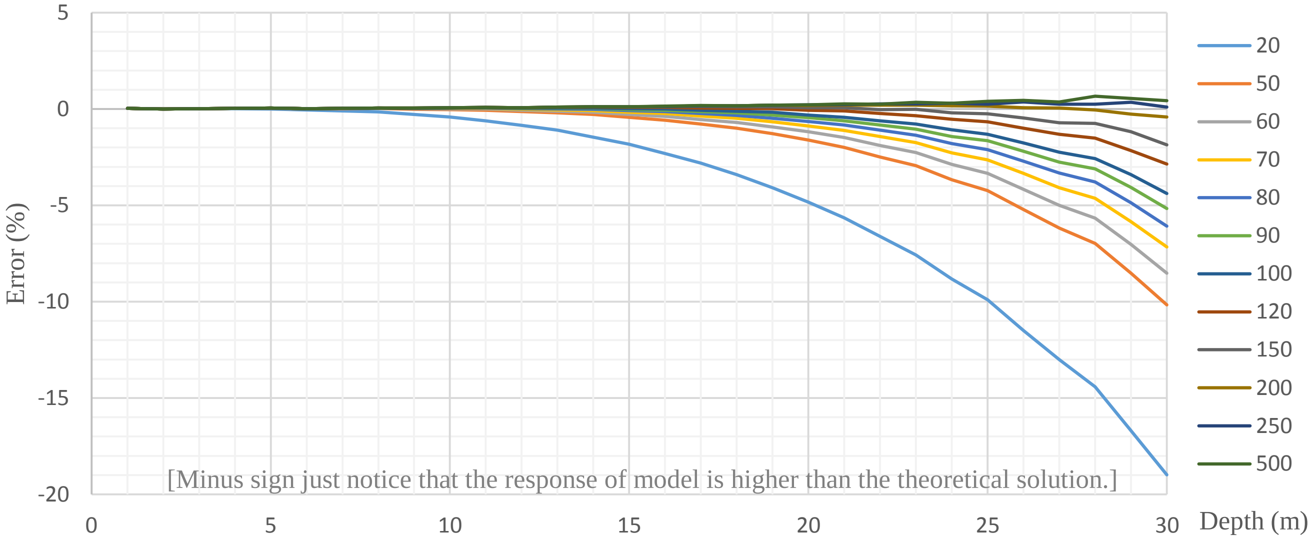


Figure 9. Long Twin Boundary Element

4.2.5 Evaluation

Figure 9 reveals some relation between model dimension and source of error:

- Similar to analogous TB, Long TB shows a higher accuracy level by increasing the width.
- Lengthening of TB elements and changing TB's E values limit approaching to zero error.

4.3 Improved TB Element

Problems and relevant solutions for the last Twin Boundary models are sources of inspiration to introduce a last type as a complementary of Twin Boundary modeling, named Improved TB. The goal of this method is to offer a model with zero error by overcoming the short-comes. Analogous TB's problem is the free-fix interaction effect which is tried to be solved by introducing long TB method; but, other problems as aspect ratio is appeared. The improved TB is aimed to solve both problem types simultaneously. The key role of long TB, to solve the free-fix interaction problem, is lengthening of TB element to reduce the effect of fix side on the rest of the model and changing the stiffness of TB element to reduce unbalance aspect ratio's effects. Improved TB changes the stiffness of boundaries element without elongation of width to prevent unsuitable aspect ratio; in fact, Improved TB utilizes the same element size as of TB elements with reduced modulus of elasticity values on the boundaries.

4.3.1 Formulation

Zero error is the greatest advantages of Twin Boundary that Improved TB can satisfy it. Therefore, this section offers equations for elastic modulus value for boundary's elements, equation 2, 3 and 4. Vast range of soil types and different soil depth is investigated to find proportion elastic modulus value of Twin Boundary for zero error modeling. Investigation divides soil strength formulation into two categories of soft soil and stiff soil. This model specially offers in 50 meters wide.

Stiff Soil: $E_{model} > 100$ MPa

$$E_{TB} = -0.229\ depth^2 + 37.375\ depth - 200.53 \quad (2)$$

Soft Soil: $E_{model} < 100$ MPa

Lower part: depth<20 meters:

$$E_{TB} = -0.1906\ depth^2 + 33.883\ depth - 154.12 \quad (3)$$

Upper part: depth>20 meters:

$$E_{TB} = A.\ depth^2 + B.\ depth + C \quad (4)$$

where,

$$A = 0.0017\ E_{model}^2 - 0.2344\ E_{model} + 6.7765 \quad (5)$$

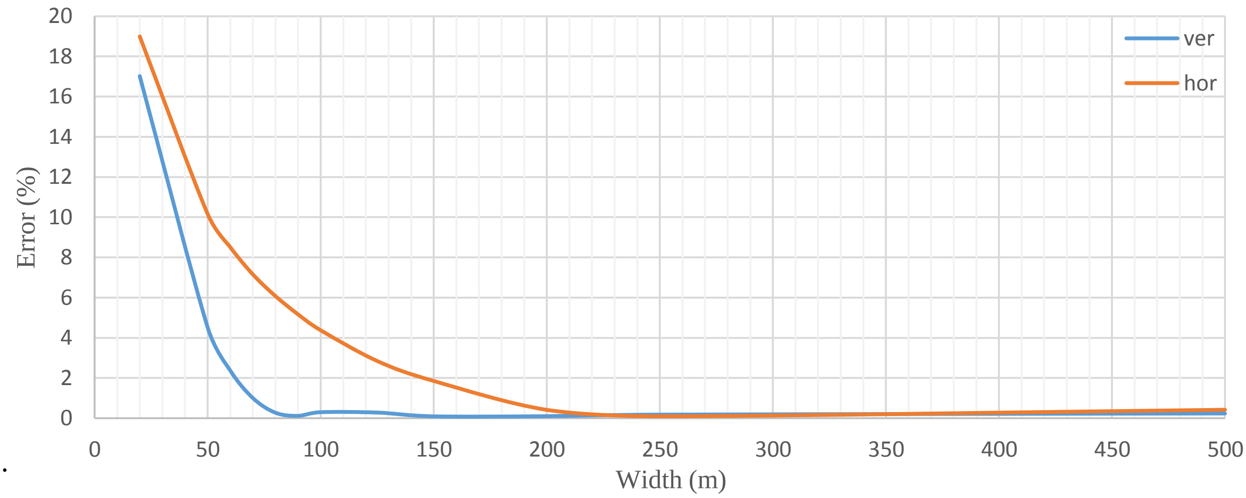
$$B = -0.0467\ E_{model}^2 + 7.5603\ E_{model} - 217.8 \quad (6)$$

$$C = 1.573\ E_{model}^2 - 222.43\ E_{model} + 7087.1 \quad (7)$$

For stiff soil only one 2-ordered polynomial is needed; however, it is desired more statistical regressions for input details of soft soils. For very soft soil, error does not converge to zero, and equations offer the least magnitude of error. Equations 2, 3 and 4 work in the dimension of meter for depth, KPa for TB elastic modulus and MPa for main model elasticity modulus. As previously discussed for Long TB, no mass is considered for Improved TB to prevent inertia effects over coupling side. Therefore, to make sure efficient performances of this method, one branch of coupling side is fixed over x DOF and the other branch is fixed on y DOF.

5. VERTICAL EVALUATION

All evaluated models in the previous sections were investigated for horizontal waves propagation. This section puts a step forward to prove the efficiency of the Twin Boundary models in vertical waves propagation. Figure 10 shows that offered criteria for horizontal waves propagation hold a higher degree of accuracy rather vertical wave effects. Since Twin Boundary shows great performances in both vertical and horizontal directions, by adducing the superposition rule, Twin Boundary will handle any different wave incident combinations.



.

Figure 10. Comparison of Horizontal and Vertical excitation

6. COMPARISON OF TWIN BOUNDARY WITH AVAILABLE LITERATURE METHODS

Lyzmer damper and spring-damper methods are known as a significant achievement in FE artificial boundaries. In consequence, Figure 11 is offered to produce a basic comparison between Twin Boundary method and literature methods which previously mentioned. It is very important to mention that in spite of high accuracy rate of Twin Boundary method, if it is used in instruction form, this evaluation tries to compare all these methods in a same situation; therefore, some inaccuracy of Twin Boundary in Figure 11 is just due to the effects of unfollowing its instruction and to be in consistent with the study of all models in a same manner.

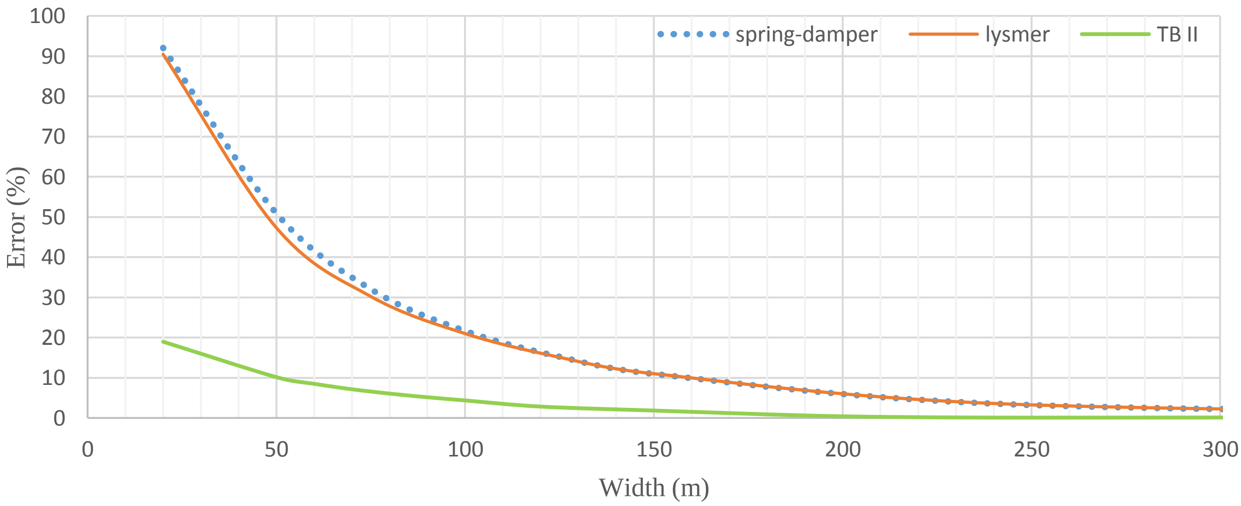


Figure 11. Twin Boundary Vs. Literature Methods

7. CONCLUSION

This research offers a new method of artificial boundary to attenuate the wave at the sides of the main model. This evaluation reveals that Twin Boundary has a higher performance for materials with higher elastic modulus values. Even though this research is focused on linear-elastic evaluation of TB, the behavior of this model under nonlinear-plastic assessments is forecasted to be still operational since the theory behind this idea is independent of any classification. Twin Boundary is divided into three types as follow:

Type I: Analogous TB:

- Great performances for shallow or wide modeling.
- Although increasing the depth value decreases the model error, this model shows better result than available literature modeling.
- Simplicity of modeling, applying coefficients and mathematical changes are not needed such as other methods.

Type II: Long TB:

- More reasonable modeling size and reducing error values for Analogous type.
- Turbulences over edges are dramatically decreased; thus, more usable area of modeling are available.

Type III: Improved TB:

- High rate of accuracy for almost zero error modeling is offered.
- Small modeling with the size of 50 meters, different depth and different soil types is presented.
- Relation formulation offers simple way of modeling.

8. REFERENCES

Jingbo L, Yandong L (1998). A direct method for analysis of dynamic soil-structure interaction based on interface idea. Developments in Geotechnical Engineering, 83(C), 261–276 https://doi.org/10.1016/S0165-1250(98)80018-7.

Kausel E (1988). Local transmitting boundaries, J. Engng. Mech, 114(6), 1011-1027.

Khoei AR (2014). Extended finite element method. extended finite element method: theory and applications. https://doi.org/10.1002/9781118869673.

Kramer SL (1996). Geotechnical earthquake engineering. Prentice-Hall, Inc. http://doi.org/10.1007/ 978-3-540-35783-4, ISBN 0-13-374943-6.

Semblat JF, Brioist JJ (2000). Efficiency of higher order finite elements for the analysis of seismic wave propagation. Journal of Sound and Vibration, 231(2), 460–467. https://doi.org/10.1006/jsvi.1999.2636.

Smith WD (1974). A non-reflecting plane boundary for wave propagation problems. Journal of Computational Physics, 15(4), 492–503. http://doi.org/10.1016/0021-9991(74)90075-8.

Towhata I (2008). Geotechnical earthquake engineering. Springer, ISBN 978-3-540-35782-7.

Wolf JP (1986), A comparison of time domain transmitting boundaries, Earthq. Engng. Struct. DYN., 14, 655-6773.