SUPERSTRUCTURE MODE IDENTIFICATION IN A BASE ISOLATED BUILDING FROM PUSH AND SUDDEN RELEASE TESTS

Anastasia ATHANASIOU¹, Giuseppe OLIVETO²

ABSTRACT

A three-story base isolated building in Augusta, Italy, was subjected to a series of push and sudden release tests in March 2013. During the tests, the building was displaced slowly to the desired amplitude and then it was left free to oscillate. The records include the displacements at the isolation level and the floor accelerations. The latter were processed for the removal of the low frequency noise and the estimation of reliable floor velocities and displacements. The adjusted records were used in the identification of the non-linear isolation system and of the superstructure. This paper focuses on the identification of the modal properties of the linear, in the range of interest, superstructure. The identification is accomplished in the time domain in a twofold manner. First, the superstructure is modelled as a three degree-of-freedom system, fixed at the base and excited by the acceleration recorded at the isolation level during the tests. The three degree-of-freedom physical model is identified using the Covariance Matrix Adaptation Evolution Strategy, a stochastic algorithm for non-linear, black-box optimization. At a second stage, the superstructure is modelled as a first-order, single-input multiple-output linear time-invariant system, and the identification is repeated using the Eigensystem Realization Algorithm together with the Observer Kalman filter IDentification algorithm. The single input refers to the base acceleration, while the multiple output refers to the superstructure floor accelerations. The system order is one of the unknowns of the inverse problem. The capabilities and limitations of the two identification procedures are discussed.

Keywords: seismic isolation; structural identification; in-situ test; evolution strategies; ERA/OKID

1. INTRODUCTION

Dynamic identification is a highly effective tool for the detection of damage in Structural Health Monitoring, where damage is defined as changes in the geometric and/or material properties of the system. These changes could be the result of extreme loading, aging, temperature, fatigue, manufacture and/or installation errors etc. Structural identification is often synonymous to the time or frequency domain identification of the modal parameters of equivalent linear models used for the representation of the actual systems. The data used in the identification are usually ambient noise records instead of strong motion records, for the obvious reason that the probability of a monitored building to be excited by an earthquake during its nominal life is small. Nevertheless, identification studies using earthquake data can be found in the literature and date back to the late ’70 - early ’80s, Beck and Jennings (1980). In (Stewart et al. 1999) the authors performed identification analyses using the strong motion records from four base isolated buildings in California applying cumulative and recursive prediction error identification methods, CEM and RPEM respectively. The RPEM allows for the time-variation of the model used in the identification, while the CEM does not. The authors used an equivalent linear model to describe the isolation system; the identified parameters included the equivalent modal parameters of the base isolated building, i.e. fundamental mode frequencies, damping ratios and mode shapes. The authors observed frequency reduction (stiffness softening) with increasing ground motion amplitude and highlighted that the hysteretic action is strongly dependent on

¹ Postdoctoral fellow, DICAR, University of Catania, Italy, thanasiou@dica.unict.it
² Professor (retired), DICAR, University of Catania, Italy, goliveto@dica.unict.it
the shaking amplitude. Nagarajaiah and Xiaohong (2000) published the results they obtained from the modal identification of the base isolated University of Southern California hospital, using the building's recordings from the 1994 Northridge earthquake. They implemented parametric and non-parametric methods for the identification of an equivalent linear dynamic system. Oliveto et al. performed the dynamic identification of a hybrid base isolated building in Solarino, Italy, from full scale free vibration tests (Oliveto et al. 2008). The identification was done in the frequency domain and provided the optimal equivalent viscous models for the isolation system under the considered excitation. The identified linear models were unable to account for the period shortening with the decay of amplitude. The identification analyses of the Solarino isolation system were repeated in (Oliveto et al. 2010) with more detailed models for the description of the isolators. In both (Oliveto et al. 2008) and (Oliveto et al. 2010) the least squares method was used for the identification. The least squares method introduced several complications to the inverse problem, since it required numerical approximation of the gradient and moreover interaction with the user during the iterative procedure. In the years that followed the Covariance Matrix Adaptation Evolution Strategy (CMA-ES), a state of the art algorithm for difficult non linear optimization (Hansen 2016), was applied to the Solarino identification study. The CMA-ES outperformed the previously used methods and improved the obtained solutions (Athanasiou et al. 2013). The implementation of the CMA-ES allowed for the consideration of more sophisticated models regarding the description of the isolation system. However, in (Athanasiou et al. 2013) the use of CMA-ES was limited to the identification of the properties of the isolation system.

In this study, the dynamic response of a three-story base isolated building in Augusta, subjected to a series of low amplitude, push and sudden release tests, is used for the identification of the properties of the superstructure. The identification of the nonlinear properties of the isolation system was done at an earlier stage (Athanasiou 2016). Herein, the identification is performed assuming second and first order models of the superstructure. In the first case, the superstructure is modelled as a three degree-of-freedom system, fixed at the base and excited by the acceleration recorded at the isolation level during the tests. The three degree-of-freedom physical model is identified using the CMA-ES. In the second case, the superstructure is modelled as a first-order, single-input multiple-output linear time-invariant system, and the identification is repeated using the ERA-DC/OKID algorithm (Juang et al. 1993, Vicario 2014). The single input refers to the base acceleration, while the multiple output refers to the superstructure floor accelerations. The two methods point to similar vibrational properties for the building. The use of the ERA-DC/OKID allows the identification of a fourth frequency that is present in the structural response of the building. The clear advantage of ERA-DC is that it identifies the first-order, rather than the second-order, model of the structure without making assumptions on the damping type. Moreover, ERA-DC provides sufficiently accurate results even when the data are noisy and limited.

2. CASE STUDY

The Augusta base isolated building was designed according to the provisions of the latest Italian seismic regulations (CS.LL.PP. 2008, NTC hereafter). Upon completion, the building was subjected to a series of push and sudden release tests. The tests were performed at low amplitudes to ensure that no damage would occur in the finished structure. The building consists of a basement, two storeys above the ground level and a penthouse. The structure is 35.70m long and 16.00m wide, the maximum height above the ground level is 10.50m and the basement story height is 3.60m, see Figure 1. The foundation lies predominantly on deposits of stiff clay, i.e. site class B following NTC. The building is isolated at the base using 16 HDRB and 20 LFSB, see Figure 2. All isolators were produced by FIP Industriale S.P.A.(SI-N 500/150 elastomeric bearings and VM 25-150-200/600/600 sliding bearings). The isolation plane runs along the top of the pillars of the basement story, slightly above the ground level. A set of ten release tests were performed on the Augusta building on March 20-22, 2013. During the tests, the building was displaced statically, in the longitudinal direction, up to the desired initial displacement, and then suddenly released. The imposed displacements varied from 58 to 117mm. The testing apparatus consisted of the loading device, the measurement equipment and the data acquisition system. The loading device consisted of a reaction wall, a hydraulic jack, a sudden release device and
Figure 1. East and south views of the Augusta building (37.2508 N, 15.2133 E), (a) and (b).

Figure 2. Peak reconstructed response of the Augusta building under tests 1, 4, 5, 6, 9 and 10, in terms of floor accelerations, velocities and displacements relative to the ground (sub-plots (a) to (c)), and relative to the base displacements and inter-story drifts (sub-plots (d) and (e)).

A load cell, and could apply a maximum force of approximately 2000 kN. The histories of the loading force, displacements and accelerations were recorded throughout the experiments. The horizontal and vertical accelerations were measured at the various floor levels using 16 sensors. The horizontal displacements were measured at various positions, below and above the isolation level. At the end of each test the building was brought to the initial zero displacement position by means of a simple re-centering mechanism (Athanasiou et al. 2017). The observed residual displacements varied between 6 mm and 19 mm. A simple baseline fitting scheme was developed for the removal of the low frequency noise from the Augusta records (Athanasiou et al. 2017). The method does not require significant computational effort, and accounts for initial and end conditions, provided that those are known. The baseline fitting scheme was applied for the processing of the absolute and relative acceleration signals obtained from the full Augusta data-set and provided the adjusted response in terms of absolute and relative floor accelerations, velocities, displacements and inter-story drifts, Figure 2. The predicted ground floor displacements matched very satisfactorily the observed ones.

3. SYSTEM IDENTIFICATION USING THE CMA-ES

3.1 Two step identification

The dynamic identification of the system was performed in the time domain using the CMA-ES, a stochastic algorithm used in difficult, non-convex optimization (Hansen 2016). The identification was performed at two stages, for the separate identifications of the isolation system and the superstructure. The objective function was defined as the distance between simulated and experimental response.
In the identification of the isolation system, the rubber bearings were modelled using a bilinear system characterized by three parameters: the elastic and post-elastic stiffnesses, $k_0$, $k_1$ and the characteristic strength $Q$ (Naem and Kelly 1999). The friction sliders were modelled using the Coulomb model, defined only by the coefficient of friction $\mu$. The response of the isolated system was evaluated using a single degree-of-freedom analytical model, described in detail in (Oliveto et al. 2012). The results obtained from the identification of the isolation system are described in detail in (Athanasiou 2016). The Augusta superstructure was designed to respond elastically at excitations that do not exceed the design level following the prescriptions of the Italian regulations, NTC. In terms of design spectrum, for reinforced concrete structures this implies a strength reduction factor $q = 1.5$. It is reasonable to assume that the superstructure responded within the linear elastic range under the Augusta free vibration tests, since those were performed at very low amplitudes to ensure no damage to the finished building. For the purposes of the identification, the superstructure is modeled as a viscously damped, linear structural system fixed at the base, see Figure 3. The non-linear contribution of the isolation system is considered indirectly, by exciting the superstructure by the acceleration developed at the isolation system, i.e. the acceleration recorded at the ground floor, just above the isolation system, during the experiments.

The equation of motion for the linear superstructure model takes the following form:

$$[M_s][\ddot{u}_s(t)] + [C_s][\dot{u}_s(t)] + [K_s][u_s(t)] = -[M_s][t]\ddot{\iota}(t)$$

where $\{u_s\} = \{u_1, u_2, u_3\}^T$ is the vector of the relative to the base longitudinal floor displacements. $[M_s]$, $[C_s]$, $[K_s]$ are the mass, damping and stiffness matrices of the superstructure and $\{t\} = \{1, 1, 1\}^T$ the influence vector. $[M_s]$ is a diagonal matrix, with the lumped floor masses $m_i$ ($i = 1, 2, 3$) as diagonal entries, while $[C_s]$ and $[K_s]$ are full $[3\times3]$ matrices. Equation 1 represents a coupled system of second order differential equations in which the independent variable is the time, $t$, and the dependent variables are the horizontal displacements, $\{u_s\}$. If the damping is classical, i.e. if the modes are orthogonal with respect to the system's damping matrix, the equations of motion can be transformed to an uncoupled set of modal equations, Equation 2.

$$\ddot{q}_n(t) + 2\zeta_n\omega_n\dot{q}_n(t) + \omega_n^2q_n(t) = -\Gamma_n\ddot{\iota}(t), \text{ where } \Gamma_n = \frac{\{\varphi_n\}^T [M_s][t]}{\{\varphi_n\}^T [M_s][\varphi_n]}$$

$$\{u_s(t)\} = \sum_{n=1}^{3} \{u_{sn}(t)\} = \sum_{n=1}^{3} \{\varphi_n\}q_n(t)$$

Each modal equation 2(a) is solved for the modal coordinate $q_n$ applying any known analytical or numerical method to determine the modal response contributions $\{u_{sn}\}$ (Chopra 2012).
responses \{u_{na}\} are then combined to obtain the total response \{u_t\}, Equation 2(b), where \{\phi_n\} is the \nt mode of the undamped system and \omega_n, \zeta_n the corresponding modal frequency and damping ratio.

### 3.2 System parameter vector and fitness function

Equation 2 implies that the model parameters required for the description of the superstructure response are 18: 3 modal participation factors \Gamma, 3 modal periods \T, 3 damping ratios \zeta and 9 mode components \phi_{ij} (i, j = 1, 2, 3). However, the essential model parameters required for the simulation of the system response can be reduced to the following 12: 3 floor masses \m, 3 modal periods \T, 3 modal damping ratios \zeta and the 3 first mode components \phi_13. In the reduced system parameter vector, the modal participation factors have been replaced by the floor masses, which physically are more meaningful and easily quantifiable. Given the mass distribution and the mode shapes the modal participation factors can be evaluated from Equation 2(a). The reduced system parameter vector includes only the first mode shape \{\phi_1\} = \{\phi_{11}, \phi_{21}, \phi_{31}\}^T excluding the higher modes \{\phi_2\} = \{\phi_{12}, \phi_{22}, \phi_{32}\}^T and \{\phi_3\} = \{\phi_{13}, \phi_{23}, \phi_{33}\}^T. In fact, if \{\phi_1\} is known, \{\phi_2\} and \{\phi_3\} can be found solving the following constrained non-linear minimization problem:

\[
\text{Minimize : } f = \begin{bmatrix}
\phi_1^T & \{\phi_1\}^T & \{\phi_2\}^T & \{\phi_3\}^T
\end{bmatrix} \begin{bmatrix}
\m_1 & \{\phi_1\}^T \m_1 & \{\phi_2\}^T \m_1 & \{\phi_3\}^T \m_1
\end{bmatrix} \begin{bmatrix}
\phi_1 \{\phi_1\} \{\phi_2\} \{\phi_3\}
\end{bmatrix}
\]

subject to:

\[
\begin{align*}
\text{c}_s &= \phi_{12} \phi_{32} < 0, \quad \text{c}_s = \phi_{33} \phi_{13} < 0, \quad \text{c}_s = \phi_{23} \phi_{12} < 0, \\
\text{c}_{eq,1} &= \{\phi_2\}^T \m_1 \{\phi_2\} = 1, \quad \text{c}_{eq,2} = \{\phi_3\}^T \m_1 \{\phi_3\} = 1
\end{align*}
\]

The solution to the above minimization problem provides a set of mode vectors which are orthonormal with respect to the mass matrix, Equation 3. The inequality constraints 3(b) control the mode shape; for the typical three degree-of-freedom, bi-dimensional system considered herein, the first mode has no nodes (no sign change between adjacent components), the second mode has one node (one change of sign between adjacent components) and the third mode has two nodes (two sign changes between adjacent components). The constrained optimization problem is solved in MATLAB using the function \textit{fmincon} (MATLAB, 2015). The algorithm does not suffer accuracy and stability issues. The optimization problem for the identification of the Augusta superstructure is formulated as the minimization of the following fitness function:

\[
f = \sum_{i=1}^{3} \left( \frac{1}{\sqrt{\sum_{i=1}^{3} (\tilde{u}_{i,exp}(t) - \tilde{u}_i(t))^2}} \sum_{i=1}^{3} (\tilde{u}_{i,exp}(t))^2 dt + \sum_{i=1}^{3} (\tilde{u}_{i,exp}(t) - \tilde{u}_i(t))^2 dt \right) / \sum_{i=1}^{3} (\tilde{u}_{i,exp}(t))^2 dt
\]

where \tilde{u}_{i,exp}(t), \tilde{u}_i(t) are the relative to the base i-floor acceleration and velocity histories, obtained from the processing of the Augusta free vibration records and \tilde{u}_i(t), \tilde{u}_i(t) the corresponding simulated acceleration and velocity histories. The relative superstructure response is generated for each candidate system parameter vector \{S\} = \{m_1, m_2, m_3, T_1, T_2, T_3, \zeta_1, \zeta_2, \zeta_3, \phi_{11}, \phi_{21}, \phi_{31}\} implementing classical modal superposition, Equation 2. The optimal solution is the solution providing floor velocities and accelerations which match the observed ones. Displacements are not included in the fitness function, Equation 4, since they are suspected for very low signal to noise ratio SNR (Athanasiou et al. 2017). For the identification of the second-order system, 2sec long response histories were used (2000 data points, sampled at \Delta t=0.001sec).

The CMA-ES was ran 10 times on the free vibration data of tests 4, 5, 6, 9 and 10. The population size was doubled in every restart of the algorithm, while all remaining CMA-ES parameters were set to their default value. The identification was performed in a bounded search space to ensure the occurrence of feasible solutions. The start point and the search space considered in the identification are given in Table 1. Please note that an extra parameter was added to the identification problem: the superstructure mass, \m_0. The algorithm was restrained to look for solutions for which \m_0=m_1+m_2+m_3. This was done by adding the penalty term \(m_0^2/m_0^2\) to the fitness function \(f\), Equation 4.

The initial guess for \m_0 was 1390tons, i.e. the superstructure mass for the finished building as estimated by the structural designer. This mass was distributed to the floors according to their areas,
The lower bound \( \phi_s \) primarily the experimental response; - \( \phi \phi \) \( \phi \phi \); 1s optimal solution, i.e. the one which corresponds to a of di

The superstructure model parameters obtained 

3.3 Identification results

The superstructure model parameters obtained from the identification of tests 4, 5, 6, 9 and 10 are given in Table 2. The dominant mode \( \{ \phi_1 \} \) was obtained from the identification, while higher modes \( \{ \phi_2 \} \), \( \{ \phi_3 \} \) were evaluated a posteriori from the solution of the minimization problem shown in Equation 3. The identified system vectors given herein correspond to the optimal solutions provided by the CMA-ES for each test. The solution provided by the CMA-ES is not unique; the identification of different tests results to somewhat different systems. Independent identification runs on the same set of data provide often different system parameter vectors. The solution shown in Table 2 is the optimal solution, i.e. the one which corresponds to a local minimum of the fitness function shown in Equation 4 and shows increased repeatability. The obtained results show that:

- The identified systems reproduce satisfactorily the experimental response; the error for the optimal solutions was less than 10%. Figures 4 and 5 show the matching between experimental and identified relative response under test 6. The matching is very satisfactory in terms of relative velocities and accelerations, but it is less satisfactory in terms of displacements. The identified displacements resemble very well the experimental ones in the initiation of motion, when peak response occurs, but the traces become somewhat different ever after. This is due to the fact that relative displacement is of very small amplitude and probably suffers from low SNR (Athanasiou et al. 2017).
- The superstructure mass \( m_s \), the two first modal periods \( T_1, T_2 \), the first mode \( \{ \phi_1 \} \) and the second modal damping ratio \( \zeta_2 \) are the parameters identified with higher level of certainty. The first and third damping ratios \( \zeta_1 \) and \( \zeta_3 \) are the parameters showing the highest scatter.
- The identified superstructure mass seems to be somewhat higher than the one estimated by the structural designer under testing conditions, approximately 7 - 15% higher.
- Tests 5, 6, 9 and 10 are tests of similar amplitude and lead to the same modal periods, with one exception however; \( T_3=0.035sec (f_3=33Hz) \) for test 10 and \( T_3=0.06sec (f_3=18Hz) \) for the remaining tests. This is a very interesting output implying that four structural frequencies can be found in the system response. However, the model considered herein is a three degree-of-freedom model unable to capture more than three modal frequencies.
- Test 4, the test of lowest amplitude, resulted to somewhat smaller modal periods with respect to tests 5, 6, and 9.
- The first mode shapes, identified from tests 4, 5, 6 and 9, are very similar, see Figure 6(a).
Table 2. Identification results for the Augusta superstructure obtained by ten independent runs of the CMA-ES on the available test data. The number of times the optimal solution was repeated in the runs is shown in brackets next to the error value ($f_{\text{min}}$). The tests are ordered in terms of increasing Peak Base floor Acceleration (PBA).

<table>
<thead>
<tr>
<th>Test no.</th>
<th>4</th>
<th>10</th>
<th>9</th>
<th>5</th>
<th>6</th>
<th>avg</th>
<th>cov [%]</th>
</tr>
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<tr>
<td>PBA [g]</td>
<td>0.33</td>
<td>0.36</td>
<td>0.37</td>
<td>0.42</td>
<td>0.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m₁ [tons]</td>
<td>710</td>
<td>400</td>
<td>667</td>
<td>690</td>
<td>784</td>
<td>650</td>
<td>23</td>
</tr>
<tr>
<td>m₂ [tons]</td>
<td>544</td>
<td>840</td>
<td>536</td>
<td>466</td>
<td>446</td>
<td>567</td>
<td>28</td>
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<tr>
<td>m₃ [tons]</td>
<td>282</td>
<td>248</td>
<td>254</td>
<td>291</td>
<td>363</td>
<td>288</td>
<td>16</td>
</tr>
<tr>
<td>m₄ [tons]</td>
<td>1544</td>
<td>1488</td>
<td>1459</td>
<td>1453</td>
<td>1594</td>
<td>1508</td>
<td>4</td>
</tr>
<tr>
<td>T₁ [sec]</td>
<td>0.26 (3.9)</td>
<td>0.27 (3.7)</td>
<td>0.27 (3.7)</td>
<td>0.27 (3.8)</td>
<td>0.27 (3.7)</td>
<td>0.27 (3.7)</td>
<td>1</td>
</tr>
<tr>
<td>T₂ [sec]</td>
<td>0.08 (12.1)</td>
<td>0.09 (11.7)</td>
<td>0.09 (11.6)</td>
<td>0.09 (11.6)</td>
<td>0.09 (11.4)</td>
<td>0.09 (11.7)</td>
<td>2</td>
</tr>
<tr>
<td>T₃ [sec]</td>
<td>0.05 (18.8)</td>
<td>0.03 (33.2)</td>
<td>0.06 (18.1)</td>
<td>0.06 (18.1)</td>
<td>0.06 (18.0)</td>
<td>0.05 (21.2)</td>
<td>22</td>
</tr>
<tr>
<td>ζ₁ [%]</td>
<td>1.1</td>
<td>1.8</td>
<td>3.2</td>
<td>4.2</td>
<td>2.5</td>
<td>2.6</td>
<td>46</td>
</tr>
<tr>
<td>ζ₂ [%]</td>
<td>6.0</td>
<td>5.4</td>
<td>6.3</td>
<td>6.4</td>
<td>5.4</td>
<td>5.9</td>
<td>8</td>
</tr>
<tr>
<td>ζ₃ [%]</td>
<td>1.8</td>
<td>4.0</td>
<td>2.5</td>
<td>2.7</td>
<td>2.4</td>
<td>2.7</td>
<td>30</td>
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<td></td>
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<td></td>
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<td>θ₂₁,</td>
<td>0.805, 0.724, 0.812, 0.817, 0.821, 0.798, 5,</td>
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<td></td>
<td></td>
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<tr>
<td>θ₃₁,</td>
<td>1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 0,</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>$f_{\text{min}}$ [%]</td>
<td>7.7(2/10)</td>
<td>8.1(3/10)</td>
<td>6.7(4/10)</td>
<td>6.3(6/10)</td>
<td>6.3(8/10)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4. Matching between first floor and roof simulated and experimental accelerations, sub-plots (a) and (b). The simulated response is evaluated using the superstructure model identified by CMA-ES for test 6.

Figure 5. Matching between second floor simulated and experimental velocity and displacement, sub-plots (a) and (b). The simulated response is evaluated using the superstructure model identified by CMA-ES for test 6.

Higher modes are somewhat different. Figures 7 show the maximum superstructure response under test 6, constructed using n=1, 2 and 3 modes. Mode 1 dominates the system response, however higher modes are necessary for reproducing successfully the first floor accelerations.

- All tests, but test 10, indicate a decrease of the mass with the superstructure height; for tests 4, 5, 6 and 9 $m₁≈0.45m$, $m₂≈0.35m$, $m₃≈0.20m$. These results are reasonable; the second floor
Figure 6. Identified superstructure modes for the Augusta building. Sub-plot (a) shows the modes identified by CMA-ES using the data of tests 4, 5, 6, 9 and 10. Sub-plot (b) shows the comparison of the modes obtained by application of the CMA-ES and ERA-DC/OKID on the data of test 6.

Figure 7. Peak relative acceleration, velocity and displacement for test 6; sub-plots (a), (b) and (c). The peak response is approximated using $n=1$, 2 or 3 modes. When $n=3$ modes are considered the response is the exact one.

has the same area as the first one but includes a terrace, and there dead and live load is reduced. The roof lies at the top of the building and occupies a significantly smaller area than any other floor, hence it is expected to be identified with the lowest mass. For test 10, $m_1=0.27m$, $m_2=0.51m$, $m_3=0.17m$.

- The first and third modal damping ratios are identified with an average value of 3%. This is a common value used in the design of RC structures at the working stress levels (Chopra 2012). The identification of Test 4, a test of smaller energy input, provides first and third modal damping ratios of the order of 1%, implying that energy dissipation capacity decreases with decreasing amplitude. All tests point to high second mode damping ratios, of the order of 6%.

4. SUPERSTRUCTURE IDENTIFICATION USING ERA-DC/OKID

The CMA-ES provided the optimal parameters for the considered Augusta superstructure model, i.e. the ones that reproduce satisfactorily the observed response. However, the CMA-ES does not allow identification of the physical model - the system order is among the known parameters of the problem. Moreover, the solution provided to the identification problem is not unique; it represents a local minimum of the nonlinear error function. The solution provided by the CMA-ES is the solution that shows increased repeatability and is physically meaningful. To assess the reliability of the CMA-ES, the identification of the superstructure is repeated using the Eigensystem Realization Algorithm with
data correlation (ERA-DC) together with the Observer/Kalman filter Identification algorithm (OKID). ERA/OKID was originally developed at NASA for the identification of lightly-damped structures; the method is applicable to any linear system and has found numerous applications (Juang et al. 1993, Vicario 2014, Hong et al. 2009). The superstructure is modelled as a first-order single input (\( \ddot{u}_b \)) and multiple output (\( y_1=\ddot{u}_1, y_2=\ddot{u}_2, y_3=\ddot{u}_3 \)) system, Figure 3. Additional model refinement is performed using the non-linear optimization technique proposed by Luş et al. (2002). ERA-DC/OKID performs well when the input and output data are sufficiently long, the SNR is large, the noise is white and of zero mean, and the data are sufficiently rich so that all modes are sufficiently excited (Hong et al. 2009). The main advantages of the ERA methodology is that no assumptions are required regarding the system order and the nature of damping, i.e. classical or non-classical damping, (Luş et al. 2002).

4.1 Basic formulations and minimal order realization

The dynamics of the second-order system, Equation 1, can be expressed in the following first-order matrix differential equations:

\[
\begin{align*}
\dot{x}(t) &= [A]x(t) + [B]\ddot{u}_b(t), \\
\dot{y}(t) &= [C]x(t) + [D]\ddot{u}_b(t)
\end{align*}
\] (5a)

\[
[A] = \begin{bmatrix}
0 & H[1] \\
-M[1] & -M[1][C]_1
\end{bmatrix}, \\
[B] = \begin{bmatrix}
0 \\
1
\end{bmatrix}, \\
[C] = \begin{bmatrix}
0 \\
-M[1] & -M[1][C]_1
\end{bmatrix}, \\
[D] = \begin{bmatrix}
0
\end{bmatrix}
\] (5b)

where \( \{x\} = \{u_1, \dot{u}_1\}^T \) is the six-dimensional state vector and \( \{y\} = \{\ddot{u}_1\} \) the three-dimensional output vector. \([A], [B], [C]\) and \([D]\) are the time invariant, input, output and direct transition matrices. Discretization of Equations 5 in the time domain lead to the following formulation:

\[
\begin{align*}
\dot{x}(k+1) &= [\Phi]\{x(k)\} + [\Gamma]\ddot{u}_b(k), \\
\dot{y}(k) &= [C]\{x(k)\} + [D]\ddot{u}_b(k) \quad (k \text{ stands for } k\Delta t)
\end{align*}
\] (6a)

\[
[\Phi] = e^{[A]\Delta t}, \quad [\Gamma] = \left( \int_0^{\Delta t} e^{[A]\sigma}d\sigma \right) [B] \quad \text{for zero-order hold sampling assumption}
\] (6b)

The matrices \([\Phi], [\Gamma]\) contain the information regarding the vibrational characteristics of the system. ERA/OKID (or ERA-DC/OKID) is used to determine the system's Markov parameters, \([Y(0)]=\[D]\) and \([Y(k)]=[C][\Phi^{k-1}][\Gamma]\) \((k=1,2,3,..)\), that are needed to derive a minimum order realization of the matrices \([\Phi], [\Gamma], [C]\) and \([D]\). The Markov parameters are used for the construction of the Hankel matrix \([H(k)]\), Equation 7. In ERA, singular value decomposition of the Hankel matrix at step 0, \([H(0)]\), provides the matrices that are required for the definition of \([\Phi], [\Gamma]\) and \([C]\). In ERA-DC, the matrices \([\ddot{H}(0)]=[H(0)][H(0)]^T\) and \([\ddot{H}(1)]=[H(1)][H(0)]^T\) are defined. If noise in the Markov parameters is not correlated, then \([\ddot{H}(0)]\) and \([\ddot{H}(1)]\) will contain less noise than \([H(0)]\) and \([H(1)]\) (Vicario 2014). Singular value decomposition of \([\ddot{H}(0)]\), Equation 8, provides the matrices that are required for the definition of \([\Phi], [\Gamma]\) and \([C]\).

\[
[H(k)] =
\begin{bmatrix}
[C][\Phi^0][\Gamma] & [C][\Phi^1][\Gamma] & \ldots & [C][\Phi^{i-1}][\Gamma] \\
[C][\Phi^0][\Gamma] & [C][\Phi^1][\Gamma] & \ldots & [C][\Phi^2][\Gamma] \\
& \ddots & \ddots & \ddots \\
[C][\Phi^0][\Gamma] & [C][\Phi^1][\Gamma] & \ldots & [C][\Phi^{i+j-2}][\Gamma]
\end{bmatrix}
\] (7)

where \(i, j \in Z^+\) determine the size of \([H(k)]\)

\[
[\ddot{H}(0)]=[U][\Sigma][V]^T = [\begin{bmatrix} U_1 \end{bmatrix}, \begin{bmatrix} U_2 \end{bmatrix}], [\begin{bmatrix} S \end{bmatrix}, \begin{bmatrix} 0 \end{bmatrix}] [\begin{bmatrix} V_1 \end{bmatrix}, \begin{bmatrix} V_2 \end{bmatrix}] = [U_1][S_1][V_1]^T
\] (8)
Table 3. Identified modal properties of the Augusta superstructure as obtained by application of ERA-DC/OKID on the data of tests 4, 5, 6, 9 and 10.

<table>
<thead>
<tr>
<th>Test no.</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>9</th>
<th>10</th>
<th>avg</th>
<th>cov [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PBA [g]</td>
<td>0.33</td>
<td>0.36</td>
<td>0.37</td>
<td>0.42</td>
<td>0.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₁ [sec]</td>
<td>0.26</td>
<td>0.27</td>
<td>0.27</td>
<td>0.26</td>
<td>0.27</td>
<td>0.26</td>
<td>1</td>
</tr>
<tr>
<td>T₂ [sec]</td>
<td>0.08</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>2</td>
</tr>
<tr>
<td>T₃ [sec]</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>2</td>
</tr>
<tr>
<td>T₄ [sec]</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>1</td>
</tr>
<tr>
<td>ζ₁ [%]</td>
<td>2.0</td>
<td>1.6</td>
<td>2.8</td>
<td>3.9</td>
<td>2.2</td>
<td>2.5</td>
<td>35</td>
</tr>
<tr>
<td>ζ₂ [%]</td>
<td>6.0</td>
<td>5.9</td>
<td>6.6</td>
<td>6.6</td>
<td>5.6</td>
<td>6.1</td>
<td>7</td>
</tr>
<tr>
<td>ζ₃ [%]</td>
<td>1.8</td>
<td>3.4</td>
<td>3.7</td>
<td>3.3</td>
<td>4.1</td>
<td>3.2</td>
<td>27</td>
</tr>
<tr>
<td>ζ₄ [%]</td>
<td>5.8</td>
<td>5.8</td>
<td>5.5</td>
<td>5.8</td>
<td>4.6</td>
<td>5.8</td>
<td>9</td>
</tr>
</tbody>
</table>

Once $S₁$, $U₁$, $V₁$ are evaluated, $[C]$ can be read as the first three rows of $[U₁][S₁]^{1/2}$ and $[Γ]$ as the first column of $[S₁]^{1/2}[U₁]^T[H(0)]$. $[Φ]$ equals $[S₁]^{1/2}[U₁]^T[H(1)]/[V₁][S₁]^{-1/2}$. If the system was a three degree-of-freedom discrete system, noise free, the singular value decomposition of the Hankel matrix at step 0 would provide only six non-zero singular values. However, due to the presence of measurement noise, additional modes, with small singular values, are identified. In such case the order of the system is $n>6$ and $\{x\} \in \mathbb{R}^n$, $[A] \in \mathbb{R}^{nxn}$, $[B] \in \mathbb{R}^n$, $[C] \in \mathbb{R}^{nxn}$. The truncation of higher ‘noisy’ modes may affect the accuracy of the reconstructed response; this is why Luş et al. (2002) recommended further refinement of the identified model.

### 4.2 Identified superstructure properties by ERA-DC/OKID

For the identification of the first-order system 0.90sec long acceleration histories (300 data points) were used. The input and output signals were down-sampled by a factor of 3, i.e. $Δt=0.003$sec. Twelve Markov parameters were considered ($p=12$, 4% of the data length). The considered $p$ value is rather small, however larger values of $p$ led to unstable results, probably due to bias introduced in the data by noise. The singular values $s_1$, i.e. the diagonal entries of $[Σ]$, were considered non-zero when greater than 0.001$s_1$, where $s_1$ is the first singular value of $[Σ]$. ERA-DC/OKID provided the complex eigenvectors and eigenvalues of the identified first-order system $λ_j$, and $ψ_j$, i.e. the eigenvalues and eigenvectors of the identified system matrix $[A]$. These, for classically damped systems appear in complex conjugate pairs ($λ_j$, $λ_j^*$) and ($ψ_j$, $ψ_j^*$) where $j = 1, 2, \ldots n/2$. The modal frequencies $ω_j$, damping ratios $ζ_j$ and modes $ψ_j$ of the second-order system can be derived from the first-order properties using Equation 9. The identified modes refer to the three translational degrees of freedom, where structural response was recorded.

$$ω_j = \sqrt{\text{Re}(λ_j)^2 + \text{Im}(λ_j)^2}, \ ζ_j = -\left(λ_j + λ_j^*\right)/2ω_j, \ \{φ_j\} = [C]\{ψ_j\}[Λ]^{-2}, \ [Λ] = \text{diag}(λ_j, λ_j^*)$$

(9)

Comparison of Tables 2 and 3 shows how ERA-DC/OKID and CMA-ES provide similar periods and damping ratios. However, the coefficient of variation for the identified modal properties tends to be smaller when using ERA-DC. The error between simulated and experimental response is not provided in Table 3, since ERA-DC is looking for models of any dimension that match exactly the observed response. Hence, compared to CMA-ES, the model identified by ERA-DC reproduces more satisfactorily the system response, compare Figures 4 and 8. ERA-DC identifies a fourth frequency that seems to be present in the Augusta superstructure response. Once more, the identification reveals that damping ratios are identified with less certainty and take values that are as high as 6% for the second mode. Figure 9 shows the power spectral density functions of the experimental and simulated first floor accelerations evaluated using the superstructure models identified for test 6 by CMA-ES and...
Figure 8. Matching between first floor and roof simulated and experimental accelerations, sub-plots (a) and (b). The simulated response is evaluated using the superstructure model identified by ERA-DC/OKID for test 6.

Figure 9. Power spectral density functions of the experimental and simulated first floor accelerations, evaluated using the superstructure models identified by CMA-ES and ERA-DC/OKID for test 6, (a) and (b).

ERA-DC. Again, ERA-DC provides a better matching between experimental and identified curves in the frequency range from 0 to 35 Hz, i.e. the range of frequencies identified by the method.

5. CONCLUSIONS

Two state-of-the-art methodologies, the CMA-ES and ERA-DC/OKID are implemented herein for the identification of the properties of the superstructure of a base isolated building in Augusta subjected to push and sudden release tests in March 2013. Both methods point to similar vibrational properties for the superstructure. The CMA-ES requires the a priori definition of a second-order model for the superstructure. Given the low amplitude system response, the superstructure was modeled herein as a three degree-of-freedom viscously damped linear system, excited at the base by the reaction of the isolation system. The CMA-ES was ran several times on the different test data and provided the optimal floor mass distributions and modal properties for the considered model (modal frequencies, damping ratios, modes of vibration). All tests, but one, provided very similar results, showing that the inherent properties of the superstructure do not depend on the input excitation (in the linear range). There was evidence that the second mode provided significant contribution to the flexible superstructure acceleration response. The excitation of higher frequencies is related to the type of induced excitation; when the building was released, an impulsive force acted at the base. This force was transmitted to the superstructure and attenuated with distance and time. The identification was repeated with ERA-DC/OKID. ERA-DC allowed the identification of a first-order state model of the superstructure from the histories of the input ground motion and the recorded response, without making assumptions on the system order and the type of structural damping. ERA-DC revealed that there is a fourth frequency present in the system response that could not be captured by the three degree-of-freedom model of the superstructure used in CMA-ES. The identification of the system using ERA-DC was significantly faster, since it did not require neither the evaluation of the candidate system response at every step nor restarts. However, the disadvantages of CMA-ES (computationally expensive, a priori knowledge of the system order) are counterbalanced by its two main advantages,
i.e. the identification of physical models of the superstructure that can be easily interpreted and the evaluation of the floor mass distributions, that may be used in the evaluation of the system stiffness and damping matrices.

6. ACKNOWLEDGMENTS

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7. REFERENCES


