CURVED SURFACE SLIDERS WITH PASSIVE FRICITION, BOW TIE FRICITION, CONTROLLED FRICITION, LINEAR VISCOUS DAMPING

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ABSTRACT

A curved surface slider (CSS) with different damping mechanisms is computed: passive Coulomb friction as present in conventional CSS, bow tie friction as adaptive but passive damping approach generating position-dependent friction, amplitude proportional friction that can be realized by a CSS with semi-active damper and linear viscous damping as benchmark. First, the damping mechanisms are optimized for minimum peak structural acceleration at the peak ground acceleration (PGA) of the Design Basis Earthquake (DBE) and, subsequently, assessed in terms of peak structural acceleration, peak CSS horizontal force, peak CSS horizontal displacement and re-centring error as function of PGA ranging from very small values and up to values of and beyond the Maximum Credible Earthquake (MCE). The results demonstrate that friction damping can be optimized only at one PGA as optimum friction tuning depends on damper displacement amplitude and therefore PGA, optimized bow tie friction improves the isolation of the structure only at PGAs significantly below that of the DBE while the isolation worsens at all other PGAs, and the optimized semi-active CSS whose actual friction force is controlled in proportion to the actual CSS displacement amplitude performs approx. equally well as the CSS with optimized linear viscous damping. The superior results of the simple approach of linear viscous damping are reasonable by the fact that maximum damping force coincides with maximum velocity of the CSS motion whereby maximum force does not occur at CSS motion reversal as it is the case for bow tie friction.

Keywords: Curved surface slider; Bow tie friction; Friction damping; Friction isolation pendulum; Semi-active control; Viscous damping

1. INTRODUCTION

Curved surface sliders (CSS) represent the most often used anti-seismic devices to isolate the primary structure from the ground acceleration by their low coupling stiffness due to their curvature (Yen and Lee 2000). In addition, the CSS also add damping to the primary structure by energy dissipation due to friction damping. Many studies investigated the impact of the CSS damping on the isolation of the primary structure, i.e. the peak structural acceleration, where the friction damping was linearized (Lai and Soong 1991, Inaudi and Kelly 1993, Kelly 1999, Du and Zhao 2000, Nigdeli et al. 2014). Only few studies considered the nonlinear behaviour of friction damping in their computations to take into account that friction dampers clamp the bearing at very small peak ground accelerations (PGA) and that the friction coefficient, which might be optimally tuned to medium CSS relative motion amplitudes due to the Design Basis Earthquake (DBE), is far from optimum for greater CSS relative motion amplitudes due to the Maximum Credible Earthquake (MCE) (Jangid 2005, Kovaleva et al. 2013, Kamalzare et al. 2015, Ruan and He 2017). This paper aims at understanding the effects of
typical CSS damping mechanisms on peak structural acceleration, peak CSS horizontal force, peak CSS horizontal displacement and re-centring error (Quaglini et al. 2017). Three passive damping mechanisms are considered: passive friction damping as commonly present in CSS, bow tie friction damping as a promising adaptive but passive (position-dependent) damping mechanism (Calvi et al. 2016) and linear viscous damping (Wolff et al. 2015, Zhou and Chen 2017) as benchmark. In addition, a CSS with semi-actively controlled damper is computed that is controlled in real-time to emulate amplitude proportional friction whereby the controlled friction damper generates the same cycle energy as the linear viscous damper (Inaudi 1997, Weber and Boston 2010, Weber et al. 2010). First, all damping mechanisms are optimized at the PGA of the DBE to guarantee optimum CSS tuning for the DBE. Subsequently, the optimized CSS types are assessed in terms of the aforementioned peak values as function of all possible PGAs ranging from very small values up to values related to the MCE and beyond of the MCE (Petrone et al. 2017, Weber et al. 2016b and 2017a). The PGA-dependency of the peak states is then discussed and conclusions are drawn.

2. CURVED SURFACE SLIDERS UNDER CONSIDERATION

2.1 CSS with Friction Damping (FD)

The conventional CSS dissipates energy by friction damping (FD) between slider and sliding surface. The horizontal force $f_b$ of this isolator is given by the following equation (Figure 1)

$$f_b = \text{sgn}(u_b)\mu N_S + \frac{N_S}{R_{eff}} u_b$$

where $\mu$ denotes the friction coefficient, $N_S$ is the vertical load on the CSS, $R_{eff} = g T_{iso}^2 / (2\pi)^2$ is the effective radius given by the selected isolation time period $T_{iso} = 3.5$ s that shifts the isolated structure into the time period range of low seismic energy, $N_S / R_{eff}$ describes the restoring stiffness of the CSS and $u_b$ and $\dot{u}_b$, respectively, denote the horizontal displacement and velocity, respectively, of the CSS relative to ground.

$$f_b = \text{sgn}(u_b)\mu N_S + \frac{N_S}{R_{eff}} u_b$$

Figure 1. CSS with FD: (a) harmonic excitation, (b) excitation due to El Centro NS accelerogram

2.2 CSS with Linear Viscous Damping (VD)

The CSS with linear viscous damping (VD) represents the theoretical benchmark as any friction on the sliding surface is neglected whereby total horizontal isolator force is fully linear (Figure 2).
Figure 2. CSS with VD: (a) harmonic excitation, (b) excitation due to El Centro NS accelerogram

\[ f_b = c \dot{u}_b + \frac{N_S}{R_{eff}} u_b \]  

(2)

where \( c \) denotes the viscous damper coefficient of the oil damper in parallel to the CSS with linear viscous damping (CSS-VD).

### 2.3 CSS with Bow Tie Friction Damping (BTF)

Bow tie friction (BTF) can be realized by increasing the roughness of the sliding surface outwards which results in **position-dependent friction** (Figure 3). The according horizontal CSS force becomes

\[ f_b = \text{sgn}(\dot{u}_b) \left\{ \mu_0 + \left( \frac{\mu_I - \mu_0}{d_I} \right) u_b \right\} N_S + \frac{N_S}{R_{eff}} u_b \]  

(3)

where \( \mu_0 \) is the friction coefficient at centre position of the slider and \( \mu_I \) denotes the friction coefficient generated at slider position \( d_I \). The idea of bow tie friction stems from the fact that its cycle energy increases quadratically with the increase of the damper displacement amplitude \( U_b \) (Equation 4) as for linear viscous damping (Equation 5) in contrast to friction damping whose cycle energy is in proportion to damper displacement amplitude (Equation 6)

\[ (E_{I \times U_b})_{\text{bow tie friction}} = \frac{2}{\pi} \mu_0 \left| i U_b \right| \left( i U_b \right) = i^2 \cdot (E_{I \times U_b})_{\text{bow tie friction}} \quad \left( \mu_0 = 0 \right) \]  

(4)

\[ (E_{I \times U_b})_{\text{linear viscous}} = \frac{\mu_0}{\pi} \left| i U_b \right| \left( i U_b \right) = i^2 \cdot (E_{I \times U_b})_{\text{linear viscous}} \]  

(5)

\[ (E_{I \times U_b})_{\text{Coulomb friction}} = 4 \mu_N \left| i U_b \right| \left( i U_b \right) = i \cdot (E_{I \times U_b})_{\text{Coulomb friction}} \]  

(6)

where harmonic excitation is assumed, Equation 4 is valid for the special case of \( \mu_0 = 0 \) generating triangle-like bow tie friction with zero force at centre position and \( \omega \) is the radial frequency of the bearing relative motion. Bow tie friction generates its maximum force at displacement reversal, i.e. at zero velocity, which is fully the opposite for linear viscous damping where maximum force and maximum velocity coincide and therefore maximize the dissipated power.
2.4 Semi-active CSS with Amplitude Proportional Friction Damping (APF)

The semi-active control approach of amplitude proportional friction damping is derived by balancing the cycle energies of linear viscous damping and friction damping

\[ \pi c \omega U_b^2 = 4 \mu N_s U_b \] (7)

which demonstrates that friction \(\mu\) must be controlled in proportion to damper motion amplitude \(U_b\) in order to dissipate the same cycle energy as linear viscous damping

\[ \mu \sim U_b \] (8)

which linearizes the controlled friction damper in terms of cycle energy while the controlled force still shows Coulomb friction behaviour. The adaptive behaviour described by Equation 8 cannot be realized by a passive damper but requires the adoption of a real-time controlled damper, e.g. magnetorheological or oil dampers with controllable bypass valve (Weber et al. 2016a, Weber et al. 2017b). The sliding surface of the CSS is assumed to be lubricated to minimize the passive, i.e. uncontrollable, friction which yields the horizontal force of the semi-active CSS as follows (Figure 4)
\[
f_b = sgn(\ddot{u}_b) \mu_0 N_S + sgn(\ddot{u}_b) \frac{\mu_l - \mu_0 U_b}{U_l} N_S + \frac{N_S}{R_{eff}} u_b
\]

where \( \mu_0 = 1\% \) denotes the uncontrollable friction coefficient of the lubricated curved surface (observed in Figure 4(a) by the offset \( f_b/N_S = 1\% \)), the formulation of the controlled friction force compensates for the uncontrollable lubricated friction, \( \mu_l \) is the friction coefficient that is emulated by the controlled damper when the actual damper amplitude \( U_b \) is equal to the damper amplitude \( U_l \) and zero force tracking error in the semi-active damper is assumed (Weber and Maślanka 2014, Weber 2015). Figure 4(a) depicts the resulting force displacement trajectories under harmonic excitation and different damper motion amplitudes showing that when \( \mu \) is controlled in proportion to \( U_b \) the friction characteristics are maintained. Under real-time condition the actual value of \( U_b \) is unknown. Common method in real-time control is to use the latest (previous) amplitude value whereby a time delay of half a period and consequently a small tuning error in the actual friction is inherently present for APF (Figure 4(b)).

3. SIMULATION PROCEDURE

3.1 Equations of Motion of Primary Structure with CSS

Structures requiring base isolation typically show a first eigenfrequency \( f_l \) in the vicinity of 1 Hz which is assumed here. Considering that the isolated structure is shifted to the far lower natural frequency of \( 1/3.5 \) s=0.286 Hz \( (T_{iso} = 3.5 \) s, section 2.1) the primary structure may be modelled as a single degree-of-freedom system. The coupled equations of motion of the primary structure mass \( m_s \) and the mass of the fundament plate and top concave plate of the CSS \( m_b \) therefore become

\[
m_s \ddot{u}_s + c_s (u_s - \ddot{u}_b) + k_s (u_s - u_b) = -m_s \ddot{u}_g \tag{10}
\]

\[
m_b \ddot{u}_b + f_b = c_s (u_s - \ddot{u}_b) + k_s (u_s - u_b) - m_b \ddot{u}_g \tag{11}
\]

where \( c_s = 2 \zeta_1 m_s 2\pi f_1 \) denotes the structural viscous damping coefficient where the damping ratio \( \zeta_1 \) of the first mode is assumed as \( 1\% \), \( k_s = m_s (2\pi f_1)^2 \) is the structural stiffness coefficient, \( \ddot{u}_g \) is the ground acceleration, and \( u_s, \dot{u}_s \) and \( \ddot{u}_s \) denote the structural displacement, velocity and acceleration relative to ground. Taking the d’Alembert excitation force \(-m_s \ddot{u}_g \) to the left side of Equation 10 yields the absolute structural acceleration \( \ddot{u}_s + \ddot{u}_g \) that occupants feel. The Coulomb friction force in Equation 11 is approximated by the tangens-hyperbolicus approach (Andersson et al. 2007) to be able to solve Equations 10 and 11 in the time domain (Matlab® solver ode15s(stiff/NDF), maximum relative tolerance 1e-3, variable step size with upper bound 1e-5 s).

3.2 Optimization Criterion and Optimization PGA

Each CSS type considered in this study is optimized for minimum peak structural acceleration

\[
\text{min}_{\text{max}} \{ |\ddot{u}_s + \ddot{u}_g| \}
\]

using the accelerograms of the El Centro North-South (ECNS) and Loma Prieta earthquakes that are scaled to the PGA of the DBE, which is assumed here as \( 5 \) m/s², in order to obtain maximum reduction of the peak structural acceleration for the DBE. Notice that the definition of the PGA of optimization
is mandatory as the optimization of a CSS with nonlinear damping depends on the damper motion amplitude occurring at the same time instant as the peak structural acceleration whereby the optimization of the CSS damping mechanism depends on PGA.

3.3 PGA Range of CSS Assessment

The goal is to assess the CSS within the entire possible PGA range, i.e. also for earthquakes with far smaller and far greater PGAs than the PGA of optimization. The minimum PGA considered is 0.5 m/s$^2$ to capture the effect at which base shear the CSS relative motion is triggered. The maximum PGA is selected to be 10 m/s$^2$ to assess the CSS behaviour also at PGAs significantly beyond that of the MCE which is approx. 7.5 m/s$^2$ (PGA of the MCE assumed to be 150% of PGA=5 m/s$^2$ of the DBE). Thus, the PGA range used for CSS assessment becomes 0.5 m/s$^2$, 1.0 m/s$^2$, …, 9.5 m/s$^2$, 10.0 m/s$^2$.

3.4 PGA-scaled Accelerograms

The accelerograms used are the measured ground accelerations of the El Centro North-South (ECNS) and Loma Prieta earthquakes. Also other PGA-scaled accelerograms were used by the authors but the results are not shown as they do not give more insights into the PGA-dependency of the isolation performance of the optimized CSS types and describing the results due to more accelerograms would lead to an overlong manuscript.

3.5 Assessment Criteria

The optimized CSS types are assessed in terms of:

- peak structural acceleration $\max(|\ddot{u}_g + \dot{\ddot{u}}_g|)$ which is relevant for occupants,
- peak CSS horizontal force $\max(|f_h|)$ which is relevant for the strength design of the CSS,
- peak CSS horizontal displacement $\max(|u_h|)$ which has a strong impact of the CSS costs, and
- re-centring error $|u_h(max(t))/\max(|u_h(max(PGA))|))$ that must be below 50% as function of PGA (section 3.3).

4. OPTIMIZATION AND ISOLATION ASSESSMENT RESULTS

4.1 Optimization Results of CSS with FD and VD

The optimization curves of the CSS with FD and VD depicted in Figures 5 and 6 show that the optimum parameters $\mu^{opt}$ and $c^{opt}$ significantly differ for the accelerograms considered despite their same PGA-scaling. The peak structural acceleration as function the entire PGA range resulting from the CSS with optimized FD and VD, respectively, is plotted in Figure 7. The following is observed:

- The ratio $\max(|\ddot{u}_g + \dot{\ddot{u}}_g|)/PGA$ is constant for the CSS with linear viscous damping because of its full linear behaviour. This proofs that the optimization of the CSS with VD does not depend on PGA (but on the accelerogram, see Figure 6).
- The ratio $\max(|\ddot{u}_g + \dot{\ddot{u}}_g|)/PGA$ of the CSS with FD is minimized at the PGA of optimization (indicated by the dotted line in magenta colour) while the ratio $\max(|\ddot{u}_g + \dot{\ddot{u}}_g|)/PGA$ is significantly higher at all other PGAs. This clearly points out that nonlinear passive damping can be optimized only in one operating point, i.e. for one damper motion amplitude and consequently one PGA value.
- As expected the peak structural acceleration in (g) of the CSS with FD converges towards the value of the friction coefficient for PGA→0. In contrast, linear viscous damping does not generate any minimum base shear force which enables isolating the primary structure even at very low PGAs.
- The peak structural acceleration due to the CSS with optimized FD worsens at PGA>6 m/s$^2$. 

because $\mu^{opt}$ being optimal for PGA=5 m/s$^2$ is far too small PGAs of and beyond the MCE.

Figure 5. Optimization of CSS with FD for (a) ECNS and (b) Loma Prieta accelerograms scaled to PGA=5 m/s$^2$

Figure 6. Optimization of CSS with VD for (a) ECNS and (b) Loma Prieta accelerograms scaled to PGA=5 m/s$^2$

Figure 7. Peak structural acceleration of CSS with optimized FD and optimized VD for (a) ECNS and (b) Loma Prieta accelerograms scaled to various PGAs
4.2 Optimization Results of CSS with BTF

The BTF is optimized for minimum peak structural acceleration at PGA=5 m/s\(^2\) by setting the friction coefficient \(\mu_0\) at zero slider position to a value slightly below \(\mu_{opt}\) of FD and variation of \(\mu_1\) with the constraint \(\mu_1 > \mu_0\) to guarantee the bow tie shape (see section 2.3). The limit case \(\mu_1 = \mu_0\) describes passive friction damping (FD). Three optimization curves for three reasonably selected values of \(\mu_0\) are depicted in Figure 8. It is seen:

- If \(\mu_0\) is selected to be lower the optimum \(\mu_1\) becomes higher.
- Peak structural acceleration is minimized for the optimum pair \(\mu_0 = 3\%\) and \(\mu_1 = 4.5\%\) (ECNS) and \(\mu_0 = 6.5\%\) and \(\mu_1 = 7.25\%\) (Loma Prieta), respectively, which are the pairs with highest \(\mu_0\) shown in Figure 8. This means that the optimum BTF tuning at PGA=5 m/s\(^2\) is obtained for \(\mu_1 = \mu_0\) (not shown in Figure 8) being equal to \(\mu_{opt}\) of the CSS with FD. Thus, the optimum BTF tuning at one PGA is given by the optimum FD tuning.

Figure 9 shows that optimized BTF improves the structural isolation compared to optimized FD only at PGA\(<2.5\) m/s\(^2\) because \(\mu_0\), which is smaller than \(\mu_{opt}\) of FD, is better tuned for the small damper motion amplitudes at these low PGAs. However, at all other PGAs BTF is not preferable. BTF even worsens the isolation at the PGA of optimization which confirms the finding of the optimization that the optimum BTF tuning at one PGA is given by the optimum FD tuning (see above in bold letters).

Figure 8. Optimization of CSS with BTF for (a) ECNS and (b) Loma Prieta accelerograms scaled to PGA=5 m/s\(^2\)

Figure 9. Peak structural acceleration of CSS with optimized BTF (and optimized FD as benchmark) for (a) ECNS and (b) Loma Prieta accelerograms scaled to various PGAs
4.3 Optimization Results of CSS with APF

The optimization parameter of APF is the “gradient of friction” $\mu_I/U_I$ as the passive and therefore uncontrollable lubricated friction $\mu_0=1\%$ of the curved sliding surface is given (see section 2.4). The optimization curves are plotted in Figure 10 as function of $\mu_I$ with the parameter $U_I$ given in the legend. The discussion of the resulting isolation performance is given in section 4.4.

![Figure 10. Optimization of semi-active CSS with APF for (a) ECNS and (b) Loma Prieta accelerograms scaled to PGA=5 m/s²](image)

4.4 Isolation Assessment Results

All isolation results in terms of peak structural acceleration, peak CSS horizontal force, peak CSS horizontal displacement and re-centring error are depicted in Figures 11 to 14 for all optimized CSS types under consideration and the PGA-scaled accelerograms of the ECNS and Loma Prieta earthquakes. The re-centring error of the CSS with VD is zero and therefore omitted in Figure 14. The observations in addition to the results discussed in previous sections 4.1 to 4.3 are:

- The **CSS with optimized linear viscous damping significantly outperforms all other CSS types in terms 1) peak structural acceleration, 2) peak CSS horizontal force, 3) peak CSS horizontal displacement and 4) re-centring error** (zero) because the optimum tuning of linear viscous damping does not depend on PGA (point 1), the peak viscous force does not coincide with the peak restoring force whereby the total CSS horizontal force is lower than for the CSS with FD (point 2), optimum tuning is achieved independent of PGA and therefore optimum tuning is also present at maximum PGA whereby the maximum CSS horizontal displacement is significantly smaller than that of the CSS with FD (point 3), and any residual displacement does not exist as the viscous force is in proportion to CSS relative velocity whereby the force becomes zero at zero relative velocity and consequently any residual displacement does not occur (point 4) (Weber et al. 2018).

- Bow tie friction may be preferable when the structural acceleration must be strongly reduced at PGAs significantly below (approx. 0%-50%) the PGA of the MCE and when the displacement capacity of the CSS at maximum PGA considered should be reduced compared to conventional CSS with passive friction damping.

- Controlling the actual friction force in proportion to damper motion amplitude (APF) generates a similar but slightly worse performance as linear viscous damping. The additional costs due to the semi-active damper, real-time controller and sensor seem therefore not to be rectified.

- The main drawback of passive friction damping (FD) is that its friction force is constant and therefore independent of damper relative motion amplitude which means that the friction can be optimally tuned only for one damper relative motion amplitude and PGA, respectively, the resulting friction is too high at lower PGAs that evoke smaller damper relative motion amplitudes.
and it is too small at higher PGAs that evoke greater damper relative motion amplitudes.

Figure 11. Peak structural acceleration due to optimized CSS as function of PGA

Figure 12. Peak CSS horizontal force of optimized CSS as function of PGA

Figure 13. Peak CSS horizontal displacement of optimized CSS as function of PGA
5. CONCLUSIONS

Curved surface sliders (CSS) with passive friction damping, bow tie friction as adaptive but passive damping mechanism, linear viscous damping as benchmark and displacement amplitude proportional friction damping that can be realized by real-time control of a semi-active damper. All damping mechanisms are first optimized for minimum structural acceleration response at the peak ground acceleration (PGA) assumed for the Design Basis Earthquake (DBE). Then, the optimized CSS types are assessed in terms of peak structural acceleration, peak CSS horizontal force, peak CSS horizontal displacement and re-centring error as function of a wide range of PGAs between very small values and going up to values of and beyond the Maximum Credible Earthquake (MCE).

The results demonstrate that 1) friction damping can be optimized at only one PGA while the passive friction is too high at lower PGAs and too small at higher PGAs for maximum structural acceleration reduction, 2) bow tie friction improves the structural isolation only at PGAs significantly below the DBE, 3) linear viscous damping performs best and 4) amplitude proportional friction control linearizes the friction damper in terms of cycle energy. From the promising results of the CSS with linear viscous damping it can be concluded that CSS with novel passive friction mechanisms must be found / developed that emulate the behaviour of linear viscous damping as close as possible.

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7. REFERENCES


