SOME ADVANCES IN THE UNDERSTANDING AND ESTIMATION METHODOLOGY OF KAPPA RELATED TO SIGMA-1

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ABSTRACT

This contribution summarises developments in the understanding and estimation of κ, which were related to the Sigma-1 project. 1. One of the key problems with using κ is that existing values exhibit a very large range, even for similar Vs values, and come from a variety of estimation methods. We compiled a state of the art discussing scatter in κ, and categorising available approaches for κ estimation into a taxonomy for consistency. 2. κ is most often considered as the effect of damping. We used data from a Greek accelerometric borehole array to show that measured κ comprises components of intrinsic material damping as well as scattering, and to suggest that hard-rock κ values may reach regional-dependent asymptotic values. 3. κ uncertainty is often seen even between horizontal components of the same record. We used rock data from New Zealand to propose a method of deriving orientation-independent κ values. 4. κ estimation is particularly challenging in low-seismicity regions with uncertain stress drop. Using NGA-East rock data, we proposed a methodology for scanning large datasets and applying considerations of usable bandwidth and stress drop to identify records for which κ can be computed with different methods. 5. In the challenging case of the US Transportable Array, we used a combination of methods to account for large uncertainties and data paucity and estimated κ exclusively from severely band-limited data (<16 Hz). Work on κ is planned within Sigma-2 to address remaining issues, including the trade-off between site amplification and attenuation, through better characterisation of hard-rock sites, and improved procedures for κ-Vs corrections for GMPEs.

Keywords: kappa; near-surface attenuation; high frequencies; rock response

1. INTRODUCTION

One of the most challenging and uncertain parameters in hazard estimation for PSHA, especially in low- to moderate seismicity contexts, is kappa. At high frequencies, the amplitude of the Fourier acceleration spectrum decays rapidly. Anderson and Hough (1984) introduced the spectral decay factor (κ) to model the rate of decay in log-linear space. Its site-specific component, κ0, comes from the top few km of the crust. κ0 is an important input parameter in the simulation and prediction of ground motion. It is the principal site parameter controlling the limitation of high frequencies (>5 Hz) at close-in distances (out to ~50 km). Thus, its range of values is important in characterising strong ground motions for engineering design, particularly in regions of sparse seismicity. Current uncertainty in the estimation of κ0 is very high. In practice, this can have significant implications on seismic risk for certain critical
infrastructures, namely the safety-related equipment in nuclear facilities (e.g., Bandyopadhyay and Hofmayer, 1986), and for the seismic behaviour of small concrete dams. Over the past few years and in relation to the Sigma-1 project, some advances have been made with respect to its physical meaning and components, its methods of estimation, and approaches targeted at low-seismicity regions and bandlimited data. This paper presents an overview of these advances.

2. THE TAXONOMY OF KAPPA

The $\kappa$ parameter has received a certain degree of notoriety over the past decade, due to two reasons: 1. its importance for the high-frequency response of certain critical infrastructures; and 2. the fact that its physics are still partly a subject of debate, and its estimation follows several different methods which may not always be consistent.

An effort was made in Ktenidou et al. (2014; 2017) to review all known methods used for estimating $\kappa$ and consider some of their features in order to group them into generic approaches. Some of the features considered were: 1. the frequency range within which $\kappa$ is estimated (this yielded a categorisation of broadband and bandlimited approaches, and in the latter case, above or below the source corner frequency); 2. how the effect of path attenuation ($Q$) is addressed (it may be present and require correction, it may have been removed in a previous step, or it may be ignored through choice of appropriate short-distance data).

![Table](image)

Figure 1. The taxonomy of kappa according to method of estimation (reprinted from Ktenidou et al., 2014).
The methods identified are the following (see Figure 1): the widely used acceleration spectrum method, corresponding to the original Anderson and Hough (1984) definition; the broadband inversion method, which was the second one to be defined (Anderson and Humphrey, 1991); the response spectrum method, that uses normalised response spectral shapes and corresponds to the third estimation method defined (Silva and Darragh, 1995); the displacement spectrum method, introduced by Biasi and Smith (2001) for small magnitudes; the source spectrum method; the inverse random vibration theory method; and the transfer function method.

In addition to the different existing methods of estimation, this work also identified the different underlying regional properties in the crust, and the different frequency bands used in estimation of κ as potential factors behind the large observed uncertainty in κ estimates, e.g. when considering the large scatter in κ0-Vs30 empirical correlations that are often resorted to when no data are available for ad hoc estimation.

### 3. THE PHYSICS OF KAPPA

One of the factors behind the large observed scatter in κ values was identified in Ktenidou et al. (2014) as possible regional differences in the upper crust. In Ktenidou et al. (2015), borehole records were used to investigate this further to improve the physical understanding of κ0. In this work, κ was estimated for the EUROSEISTEST valley, a geologically complex and seismically active region in Greece, with a permanent strong motion array consisting of 14 surface and 6 downhole stations. Site conditions in that location range from soft sediments (Vs30<200 m/s) to hard rock (Vs30>1500 m/s). The classical approach (AS) was used to separate local and regional attenuation and measure κ0. A new conceptual model of κ0 with Vs was proposed, comprising two new concepts: regional stabilization and the effect of scattering.

Observing attenuation at different locations in and around the valley, it was found that κ0 decreased for records farther away from the valley centre (surface stations) and at deeper downhole stations. The lowest κ0 values were for the two downhole stations in the hard rock material underlying the basin. However, these values were significantly higher than those typically found in literature from existing correlations at high Vs30 (>1500 m/s), and the κ-Vs relation showed a tendency for κ0 to stabilise at high Vs rather than continue to sharply drop (Figure 2 left). This observation was combined with another dataset on Swiss hard rock, to suggest that κ0 may possibly stabilise for high Vs values, after the effect of the sedimentary column is removed, and these may be regionally bound and related to the regional nature of the crust and its faults/sources (Figure 2 right). Borehole measurements, hardly ever used prior to this study for κ0, may be useful in determining such region-dependent values.

![Figure 2. Left: Global κ0-Vs30 correlations (top) and the stabilising trend for κ0 in our data (bottom). Right: The conceptual model for regional stabilisation of hard-rock κ0 (reprinted from Ktenidou et al., 2015; Figs 10,11).](image)
Moreover, the borehole data were used to investigate the components of attenuation in the sedimentary column above the hard rock. It was found that material damping, as expressed through travel time (t*), did not suffice to account for the total k0 measured at the surface, and that uncertainties in the quantities involved could not justify this discrepancy. Simulations were then used, which considered two alternative soil profiles from the literature with increasing complexity in the layering (7 and 20 layers). It was found that for the more complex profile, which accounted also for small Vs inversions and hence stronger stratigraphic filtering, use of both the classical AS and the TF approach on the simulated results yielded higher k0 values, which were closer to the values estimated from the recordings (Figure 3). This finding led to the suggestion that, apart from material damping, additional site attenuation may be caused by scattering from small-scale variability in the profile.

The final conceptual model proposed considers that geotechnical damping measurements may not suffice to infer the overall crustal attenuation under a site; but starting with a regional value (possibly from a borehole, as mentioned above) and adding damping from the layers in the sedimentary column above, a lower bound for site-specific k0 may be defined. For the total site attenuation to be estimated, however, records are needed. A later study used coda waves to investigate regional values (Mayor et al., 2017), while another introduced krigging to map them continuously (Van Houtte et al., 2017).

![Figure 3. Left: The two profiles considered in simulations, with 7 and 20 layers. Middle: k0 values from TF approach on simulations showing increase with profile complexity. Right: kr values from AS approach on simulations showing increase with profile complexity. (reprinted from Ktenidou et al., 2015; Figs 13, 14).](image)

4. THE EFFECT OF COMPONENT ORIENTATION AND USABLE FREQUENCY RANGE ON KAPPA

The Canterbury earthquake sequence was used in Van Houtte et al. (2014) to investigate some further factors that may contribute to the large uncertainty and scatter in k0 values. This work studied 7 hard-site stations and showed that, for some of these sites, k (when estimated using the classical AS approach) exhibited significantly larger uncertainty due to component orientation (Figure 4). These sites had been identified previously (Van Houtte et al., 2012), as exhibiting their own amplification pattern, which included differences between the components due to 2D response, e.g. from topography. In order to minimise the effect on k of such site effects at high frequencies, this work proposed an orientation-independent definition of k that averages over all possible sensor orientations. By rotating components successively and averaging, a robust mean is estimated for k, along with an estimate of its scatter due to component orientation. Later studies also found large within-station variability of k0, likely related to azimuth (Bora et al., 2017) and often larger even than between-station variability (Bora et al., 2017; Van Houtte et al., 2017).

The study further considered the constraints placed on the estimation of k given the available usable frequency, when earthquake magnitude decreases. As the AS method is applied above the corner frequency, the smaller the event, the higher the frequency range that needs to be used to estimate k.
Given the constraints on the highest usable frequency based on noise and anti-alias filtering, this effectively decreases the range available. For the magnitude range studied in the Canterbury sequence (about M1-M5), different upper bounds were assumed for the frequency range used (about 20-40 Hz). Figure 5 illustrates the onset of bias in the estimation of $\kappa$ for each assumed upper bound, which is due to shorter frequency windows being used, and the effect of the source causing trade-off between the corner frequency and $\kappa$. The values found in this study are specific to the region, instrument characteristics, noise level, etc., but this test clearly shows that increasing this upper bound (e.g. through increasing sampling rate) can significantly decrease the cut-off magnitude that can be successfully used, hence increasing the number of usable data considerably. Edwards et al. (2015) also demonstrated the sensitivity of $\kappa$ to the chosen frequency band; they found that wider ranges (e.g. where the FAS shape can be bilinear) may yield lower values, depending on the frequency dependence of the path component.

![Figure 4](Image)

**Figure 4.** The sensitivity of $\kappa$ to the orientation of the two horizontal components used to estimate it for 7 New Zealand rock sites (reprinted from Van Houtte et al., 2014).

![Figure 5](Image)

**Figure 5.** Left: The frequency range used to estimate $\kappa$ with decreasing magnitude. Right: The effect of the maximum usable frequency on $\kappa$ estimates with decreasing magnitude (reprinted from Van Houtte et al., 2014).

### 5. DERIVING KAPPA FROM LARGE DATASETS (NGA-EAST)

Most studies of $\kappa$ until recently used rather restricted datasets, which permitted the selection of data to be made manually. The recently released NGA-East dataset (Goulet et al., 2014) for Central-Eastern North America (CENA) contains over 21,500 recordings, which necessitates a new approach to selecting data which are appropriate for estimating $\kappa$. In Ktenidou et al. (2016), a methodology was introduced to scan large datasets for the information that is most pertinent when it comes to measuring $\kappa$.

Several issues with this dataset make the task challenging: 1. Due to the lack of large-magnitude events in this dataset, the two main bandlimited methods mentioned above, AS and DS, were considered (the former applying to larger magnitudes above the corner frequency, and the latter to smaller ones, below
the corner frequency), along with the two main broadband methods, the BB and the RESP method. Due to the large uncertainty in stress drop in CENA, which has a strong impact on corner frequency, and hence the frequency band that can be used to measure $\kappa$, it was decided to consider the lowest and highest plausible values.

For each record in the dataset, the theoretical frequency bands for which each bandlimited method should be used were estimated, based on each record’s usable bandwidth considering noise, filtering, etc., and the assumed source corner frequencies. Where this theoretical band was adequate (positive and $>10$ Hz), the record was used for $\kappa$ estimation. By varying the stress drop, a subset of data was created for which $\kappa$ could be computed with both the AS and DS approaches (Figure 6). An additional constraint applied was that the epicentral distance should be less than 200 km, to ensure the $\kappa$ values contain adequate site attenuation and not be dominated by the path attenuation ($Q$) component.

The proposed methodology was applied to rock sites in CENA. Mean $\kappa_0$ values are 8 msec for the AS approach and 27 msec for the DS approach. Stacking all spectra together led to mean $\kappa_0$ values of 7 and 29 msec, respectively. Overall, the DS approach yields 2–3 times higher values than the AS. The AS approach worked consistently down to M3, but not below. The two broadband approaches, BB and RESP, yielded similar results, with mean $\kappa_0_{BB}$ of $5\pm0.5$ msec across all NEHRP class A sites and $\kappa_0_{RESP}$ for selected sites of 5–6 msec. From literature, the average value of $\kappa_0$ in CENA at hard-rock sites ($Vs30>$3000 m/s) is $6 \pm 2$ msec.

The bandlimited AS method also yielded negative $\kappa$ estimates for some sites. When these were included, then the mean bandlimited $\kappa$ results were similar to the broadband estimates and the literature. When only positive $\kappa$ values were considered, then the mean value of $\kappa_{AS}$ increased by a factor of 2–3 and deviated strongly from the broadband and literature values. Negative values were suggested to contain amplification effects: although $\kappa$ is considered to be caused solely by damping in the shallow crust, bandlimited measurement techniques may not be able to separate the effects of damping and site amplification, and thus represent the net effect of both phenomena.

Figure 6. Top left: The upper and lower bound of stress drops assumed. Top right: The available frequency band
6. DERIVING KAPPA FROM BANDLIMITED DATA (TRANSPORTABLE ARRAY)

The Transportable Array (TA; http://www.usarray.org/researchers/obs/transportable) has significantly added to the ground-motion data available for the US, and for low-seismicity regions, it often yields the vast majority of short-distance records. For critical structures in regions of sparse seismicity, it is important to estimate region-specific seismic hazard with whatever sparse data are available. In this context, Ktenidou et al. (2017) proposed a framework for resolving \( \kappa_0 \) using bandlimited data such as the TA data in regions of low seismicity.

The case study was an application to Southern Arizona. The challenging conditions imposed were: 1. the low seismicity (limited, poor-quality, distant records); 2. the severe band limitation of the TA data (maximum usable frequency of 16 Hz); and 3. the magnitude range (ML 1.2 – 3.4), which was somewhat low for the AS method and somewhat high for the DS method; and 4. the large uncertainty in stress drop (corner frequency, \( f_c \)). A combination of the AS, DS, and BB approaches was used to overcome the various issues and limitations and address uncertainties. In the proposed scheme, as in NGA-East above, stress drop could not be resolved, so lower and upper regional bounds for it were considered. Several estimates of the regional Q attenuation were found in the literature to validate the path-correction of \( \kappa \) estimates for the (mostly distant). Given most of the few available records were at long distances, it was impossible to use only close-in recordings.

Due to the paucity of records per station, all stations irrespective of site conditions were eventually combined to provide a robust estimate of \( \kappa \) for the average site conditions. However, some stations exhibited resonance, which affected the bandlimited (AS/DS) \( \kappa \) estimates according to whether the available bandwidth for \( \kappa \) estimation was within the resonance peak (Figure 7a) or not (Figure 7b).

\( \kappa_0 \) based on the combined bandlimited (AS/DS) and the BB approach were similar in terms of mean (0.033 s) and range (standard deviation of 0.5 in ln units), and Q estimates (900-300 at 9–16 Hz) were compatible between approaches and with literature. The two bandlimited approaches, however (AS, DS) exhibited a strong discrepancy of the order of 3. This is a known discrepancy, though usually observed as a factor nearer 2 (Kilb et al., 2012). This study proposed a reason explaining this discrepancy that is related to the bias due to the \( f_c \) when using DS (and, in the general case, also AS). The problem is that when \( \kappa_{DS} \) is measured close to the \( f_c \), there is an additional apparent decay measured not due to attenuation but to the spectral shape near the \( f_c \). This is more prominent as the stress drop decreases and as the magnitude increases (Figure 8a,b). We suggest a quantification of this bias (or ‘spectral droop’) as a function of these two parameters (Figure 8c). The bias can assume significant values, comparable
to those of the actual damping-related $\kappa$.

Our analyses also indicate that the stress drop in this region may be higher than what was found in other studies using TA data. This is likely due to the possibility of $f_{\text{max}}$ (Hanks, 1982) masking the true source $f_c$, and leading to smaller apparent $f_c$ and hence stress drops. This was observed by Frankel (1982) for small earthquakes. In analyzing bandlimited data such as those of the TA, this potential issue should be taken into account.

Finally, for this dataset, the within-approach uncertainty is much larger than the between-approach uncertainty, and it cannot be reduced if the data quality is not improved. The challenges found here are pertinent to any region where data come primarily from the TA. Increasing the sampling rate would greatly contribute towards avoiding such issues.

![Graphs](image)

Figure 8. (a) and (b). The effect of stress drop and magnitude on ‘spectral droop’ (bias in $\kappa_{\text{DS}}$). (c) Quantification of this bias (or ‘spectral droop’) as a function of these two parameters (reprinted from Ktenidou et al., 2017).

7. CONCLUSIONS

Several advances were made in recent years in relation to the Sigma project concerning the nature and estimation of $\kappa$, including: a taxonomy of its estimation methods; a new conceptual model of $\kappa_0$ with $V_s$ comprising two new concepts, namely regional stabilisation of rock $\kappa_0$ and the effect of scattering in addition to damping; an orientation-independent definition of $\kappa$ that yields a robust mean for $\kappa$ and an estimate of its scatter due to component orientation; a methodology to scan large datasets for the information most pertinent to $\kappa$ measurement that accounts for stress drop uncertainty; a methodology for estimating $\kappa$ from sparse and severely bandlimited data, such as those of the TA; a quantification of the bias in the DS method that may explain the known discrepancy between AS- and DS-derived $\kappa$ values. Work on $\kappa$ is planned within Sigma-2 to address the trade-off between site amplification and attenuation, through better characterisation of hard-rock sites, and to improve procedures for $\kappa$-$V_s$ corrections for GMPEs.
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9. REFERENCES


