TOWARD THE IMPROVEMENT OF THE R-F METHOD FOR THE SEISMIC ANALYSIS OF RECTANGULAR TUNNELS

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ABSTRACT

It is common in tunnelling practice to evaluate the response of rectangular tunnels under transversal ground shaking, using the R-F analysis method. Critical in the efficiency of the method is the racking ratio, \( R \), which is introduced, to account for the soil-structure interaction effects in evaluation of the tunnel response and is commonly related to the soil to tunnel relative flexibility, expressed through the flexibility ratio, \( F \). This paper presents a set of new R-F relations that were developed on the basis of a numerical parametric study, carried out on a wide range of soil-tunnel configurations, in order to elaborate on the experimentally and numerically observed coupled racking-rocking response of rectangular tunnels during transversal ground shaking. The new R-F relations account for the inherent effect of the rotation patterns of the tunnel sections on the computed racking ratios, identified for ‘purely’ elastic soil-tunnels systems. Additionally, the study provides insights on the rocking response of this type of tunnels by quantifying this response on the basis of normalized sectional rotation-flexibility ratio (\( \theta/\gamma_ff - F \)) relations. The efficiency of the R-F method, when the new R-F relations are implemented, is examined by comparing its predictions, in terms of seismic bending moment at critical sections, with the results of rigorous dynamic analysis. The predictions of the simplified analysis compare very well with the dynamic analysis results, especially for rigid tunnels compared to surrounding ground. In this context, the proposed relations may be used in practice, improving the efficiency of the R-F method.

Keywords: tunnels; racking; rocking; static frame analyses; numerical dynamic analyses

1. INTRODUCTION

The response of rectangular tunnels under transversal seismic shaking is commonly evaluated, using the R-F method presented by Wang (1993). The method prescribes a simple static frame analysis for the evaluation of the stress resultants of the lining, when subjected to racking distortion due to seismic movement of the surrounding ground. The structural racking distortion, \( \delta_{str} \), is modelled through an equivalent static load (\( P \)) or pressure (\( p \)) that is imposed on a frame, simulating the lining. This distortion is evaluated by the corresponding to the tunnel axis soil racking distortion at free-field, \( \delta_{ff} \), which is properly adjusted through the so-called racking ratio, \( R = \delta_{str}/\delta_{ff} \), in order to account for the soil-tunnel interaction effects. The racking ratio, \( R \), is usually correlated with the flexibility ratio, \( F \), that describes the soil-tunnel relative flexibility and computed easily, following Wang (1993):

\[
F = \frac{(G_s \times a)}{(S \times b)}
\]

where: \( G_s \) is the strain compatible soil shear modulus, \( a \) and \( b \) are the width and the height of the tunnel section, respectively, and \( S \) is the required force to cause a unit racking deflection of the structure. Several analytical or empirical R-F relations may be found in the literature (e.g. Wang 1993, Penzien 2000, Huo et al. 2006, Anderson et al. 2008).

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Despite its simplicity, the method is based on the hypothesis of pure racking deformation of rectangular tunnels during transversal ground seismic shaking. However, recent experimental and numerical studies (Cilingir and Madabhushi 2011, Tsinidis et al. 2015, Abuhajar et al. 2015, Tsinidis et al. 2016a, Tsinidis et al. 2016b) have demonstrated a coupled racking-rocking mode of deformation for this type of structures, during ground seismic shaking. Along these lines, this study presents a set of new $R$-$F$ relations, developed, for a wide range of rectangular tunnels, on the basis of a comprehensive set of numerical analyses that was conducted to elaborate coupled racking-rocking response of rectangular tunnels. In parallel, insights on the rocking response of the rectangular tunnels during ground shaking are provided. The efficiency of new $R$-$F$ relations is examined by comparing the predictions of the improved $R$-$F$ method with relevant results of rigorous full dynamic analyses.

2. PARAMETRIC ANALYSIS AND NUMERICAL SIMULATION

2.1 Parametric analysis

A wide range of rectangular tunnels, embedded in a uniform soil deposit of 40 m in depth, was examined. The dimensions of the investigated tunnels (width: $a$ × height: $b$) varied, to cover an extended range of typologies (e.g. culverts, subway tunnels, underground arteries, etc), including sections with aspect ratios, $\lambda = a/b$, between 0.33 and 3.0. An internal column was considered at the middle of the span, for tunnels with aspect ratio, $\lambda$, greater than 2, since this concept is more rational from a static design viewpoint. To compare the seismic response of more complex cross-sections to single boxes, additional analyses were conducted for 12 × 6 (m) tunnels ($\lambda = 2$), excluding the internal column (single box analyses). Similarly, additional analyses were conducted for 3 × 9 (m) tunnels ($\lambda = 0.33$), considering an internal slab at the middle height of the side-walls and the response of these cases was compared to the one of the corresponding single box tunnels. Various burial depths, $h$, were examined, including $h = 0$ m, 3 m and 12 m. The tunnel lining was assumed to be made of concrete (elastic modulus, $E = 32$ GPa, Poisson’s ratio, $v = 0.2$, density, $\rho = 2.5$ t/m$^3$).

The analyses were conducted assuming a homogenous, visco-elastic soil deposit, with soil properties, corresponding to soil classes B (rather stiff soils) and C (predominantly soft and loose soils), according to the Eurocode 8. Table 1 summarizes the mechanical and dynamic properties of the investigated soil deposits. A sensitivity analysis was also conducted, including cases of soil deposits with increasing stiffness with depth and different damping ratios $D$.

<table>
<thead>
<tr>
<th>Soil</th>
<th>$V_s$ (m/s)</th>
<th>Poisson ratio, $v_s$</th>
<th>Density, $\rho$ (t/m$^3$)</th>
<th>Damping, $D$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil B</td>
<td>400</td>
<td>0.3, 0.5</td>
<td>1.9</td>
<td>5</td>
</tr>
<tr>
<td>Soil C</td>
<td>250</td>
<td>0.3, 0.5</td>
<td>1.8</td>
<td>5</td>
</tr>
<tr>
<td>Elastic bedrock</td>
<td>1000</td>
<td>0.3</td>
<td>2.2</td>
<td>-</td>
</tr>
</tbody>
</table>
The lining thickness, \( t \), was assumed to be constant for all the structural elements, while it was deliberately changed for each soil-tunnel configuration, so as to achieve flexibility ratios, \( F \), varying between 0.1, i.e. rigid tunnel, and 10, i.e. quite flexible tunnel, compared to surrounding ground. A sensitivity analysis, including variable thicknesses for the lining elements was also conducted, to investigate the efficiency of the new \( R-F \) relations for cases of actual tunnel sections of realistic dimensions.

The analyses were carried out using three real acceleration time histories (Table 2), in order to investigate the effect of the motion characteristics on the deformation patterns of the tunnels. Figure 2 compares the acceleration and displacement response spectra of the selected ground shaking motions. The majority of analyses were performed scaling the records to a peak ground acceleration PGA = 0.35 g (for outcropping conditions), while additional analyses were performed for PGAs = 0.1 g and 0.6 g, within a sensitivity analysis.

**Figure 2. (a) Acceleration and (b) displacement response spectra of selected shaking motions**

<table>
<thead>
<tr>
<th>A/A</th>
<th>Earthquake name</th>
<th>Country</th>
<th>Station Name</th>
<th>Magnitude, M(_w)</th>
<th>PGA (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EQ1</td>
<td>Athens, 1999</td>
<td>Greece</td>
<td>Kipsely District</td>
<td>6.04</td>
<td>0.12</td>
</tr>
<tr>
<td>EQ2</td>
<td>Friuli, 1976</td>
<td>Italy</td>
<td>Tolmezzo-Diga Ambiesta</td>
<td>6.4</td>
<td>0.34</td>
</tr>
<tr>
<td>EQ3</td>
<td>Kobe, 1995</td>
<td>Japan</td>
<td>Takatori</td>
<td>6.9</td>
<td>0.61</td>
</tr>
</tbody>
</table>

### 2.2 Numerical simulation

The analyses were conducted by employing the finite element code ABAQUS (ABAQUS 2012). A typical model layout is presented in Figure 3. The length of the soil grid, i.e. 140 m, was selected on the basis of a sensitivity analysis, carried out to check and reduce potential boundary effects on the computed response of the tunnel. The soil was meshed with quadratic plane-strain elements, while the tunnel lining was simulated with beam elements. The soil element size was selected to ensure the efficient reproduction of all the waveforms of the frequency range under study (i.e. \( f = 0.2 - 10 \) Hz).

**Figure 3. Numerical model of a typical soil-tunnel configuration in ABAQUS**
The analyses were conducted by assuming a full bonding between the lining and the surrounding ground elements, i.e. no-slip conditions. Additional analyses were conducted by introducing a full-slip interface condition for the soil-tunnel interface and precluding or permitting the potential detachment between the lining and the ground elements. In the latter cases, a classical Coulomb friction model was actually implemented, setting the friction coefficient $\mu = 0$. It is worth noticing that the above conditions constitute upper and lower bound limit cases for the actual interface conditions.

A linear elastic model was implemented for the simulation of the lining response, while the soil seismic response was modelled through a visco-elastic model, by introducing the corresponding soil shear modulus and viscous damping. It is worth noticing that the soil shear modulus was purposely kept constant for the various shaking motions studied herein, in order to investigate the effects of other critical parameters on the deformation patterns of the tunnels. The viscous damping was introduced in the form of the frequency dependent Rayleigh type.

The analyses were carried out in steps; the gravity loads were initially introduced. Subsequently, the seismic shaking loading was applied at the base of the numerical model, as vertically propagated shear waves, through appropriate dashpot elements, as per Lysmer and Kuhlemeyer (1969). Kinematic tie constraints were set at the side boundaries of the model (displacements constraints shown in Figure 3), allowing for common lateral displacement patterns, thus imitating the desirable ‘shear beam’ response of the soil at free-field conditions during ground seismic shaking. More details for the numerical analysis may be found in Tsinidis (2017) and Tsinidis and Pitilakis (2018).

3. RESULTS

Representative results of the numerical parametric study are presented in the following sections, focusing on the coupled racking-rocking deformation pattern of rectangular tunnels during shaking and the numerical $R$-$F$ relations proposed for design purposes. The efficiency of the latter is also examined on the basis of comparisons of the predictions of the improved $R$-$F$ method with relevant results of rigorous dynamic analyses.

3.1 Dynamic deformed shapes of rectangular tunnels

Figure 4a illustrates typical dynamic deformed shapes (i.e. deformation patterns due to shaking loading only) of various tunnel sections, i.e. $3 \times 9$ m; $12 \times 6$ m, embedded in soil type B (Table 2). The deformed shapes, computed for the ground seismic motion with record EQ2 (Table 4) scaled to PGA = 0.35 g, correspond to full bonding interface conditions.

![Figure 4. Dynamic deformed shapes of tunnels sections computed for ground shaking EQ2 scaled to a PGA = 0.35 g, assuming (a) full bonding interface conditions and (b) full-slip conditions interface conditions, allowing for potential separation-detachment between the two media](image-url)
In line with recent experimental findings (e.g. Cilingir and Madabhushi 2011, Tsinidis et al. 2015, Tsinidis et al. 2016a, Tsinidis et al. 2016b), a coupled racking-rocking response is identified for the tunnels during the ground shaking. In particular, rigid tunnels compared to surrounding ground (i.e. $F = 0.2$) display a rocking mode of vibration combined with reduced racking distortion. The tunnels that share the same stiffness with the surrounding ground, i.e. $F = 1.0$, are subjected to pure shearing, exhibiting no rocking, while flexible tunnels (i.e. $F > 1.0$) are subjected to significant racking distortion combined with an increased rocking response. Moreover, a particular rotation pattern is identified for the tunnels. Actually, the tunnels are rotating around the section’s centroid, while the rotation depends only on the soil-tunnel relative stiffness. When during the seismic excitation the tunnel moves towards the left side, rigid tunnels ($F < 1.0$) are subjected to a counterclockwise rotation, contrary to flexible tunnels ($F > 1.0$), which are subjected to a clockwise rotation (Figure 4a). The reverse holds when the tunnel moves towards the right side. These observations are consistent for all the investigated cases, as far as the soil behaves in an elastic fashion and a no-slip perfect bonding interface condition is considered.

Different deformed shapes, compared to those presented above, were predicted for the examined tunnel sections, when a full-slip interface condition was considered (Figure 4b). The different deformation patterns should be attributed to the geometrical nonlinearities, i.e. sliding and/or separation, which are taking place along the soil-tunnel interfaces during ground shaking. In particular, a slight movement of the rotation center from the tunnel section centroid is observed for some cases. Additionally, an inversion of the rotation of tunnel sections is observed for some tunnel sections compared to the one predicted for perfect bonding conditions, while a rocking response is also identified for some tunnel sections that share the same stiffness with the ground. These observations are more evident for shallow rigid tunnel sections with higher aspect ratios (i.e. $h < 3.0 \text{ m}$, $\lambda > 2.0$). The increase of the burial depth and the associated increase on the confining ground pressures acting on the tunnel sections resulted in a reduction of the sliding and separation phenomena along the interface, and therefore the deformation patterns of deep tunnels computed for full-slip interface conditions were more similar to those computed for full bonding interface conditions (Tsinidis 2017).

### 3.2 Quantification of the rocking response

The rocking response of tunnel section, observed during ground shaking, was quantified on the basis of an average rocking rotation of the tunnel section, $\theta$, as follows:

$$\theta = \max \left\{a \tan \left[ \frac{u_w^w(t) - u_r^w(t)}{a} \right] \right\} \approx \max \left[ \frac{u_w^w(t) - u_r^w(t)}{a} \right]$$

where: $u_w^w(t)$ and $u_r^w(t)$ are the computed vertical displacement time histories of the side-walls, and $a$ is the width of the tunnel section. The rocking rotation was normalized by an average ground shear strain, $\gamma_{gf}$, computed at free-field conditions and at the corresponding depth with the tunnel. The computed normalized rotations were finally plotted against the flexibility ratios of the soil-tunnel configurations, leading to $\theta/\gamma_{gf}$ - $F$ relations. The following discussion focuses on the $\theta/\gamma_{gf}$ - $F$ relations developed for a full bonding soil-tunnel interface condition, since for these conditions the new $R$-$F$ relations were developed for design purposes. A detailed presentation of the effect of soil-tunnel interface conditions on the rocking response and the $\theta/\gamma_{gf}$ - $F$ relations may be found in Tsinidis (2017) and Tsinidis and Pitilakis (2018).

Generally, the rocking response was more evident for shallow tunnels with low aspect ratios, while it reduced for increasing burial depth, $h$, and increasing aspect ratio, $\lambda$, of the tunnel section. For a perfect bonding interface condition, the $\theta/\gamma_{gf}$ computed for various soil conditions and ground seismic motions were similar, particularly for rigid or relatively rigid tunnels, e.g. $F < 2.0$ (Tsinidis 2017, Tsinidis and Pitilakis 2018). The dispersion was slightly increased for flexible tunnels (e.g. $F > 5.0$) with larger aspect ratios (e.g. $\lambda > 2.0$), as in the latter cases increased curvatures were identified on the flexible linings during seismic shaking. However, the computed $\theta/\gamma_{gf}$ where still quite comparable for the examined soil conditions and shaking motions. This allowed the development of generic ‘average’ $\theta/\gamma_{gf}$ - $F$ relations for rectangular tunnels. Figure 5 summarizes a series of ‘average’ $\theta/\gamma_{gf}$ - $F$ relations,
computed on the basis of the results of this numerical study, highlighting the effects of the aspect ratio, \( \lambda \), and burial depths, \( h \), of the tunnel section, as well as the effect of soil’s Poisson’s ratio, \( \nu_s \). The horizontal axis is displayed in logarithmic scale, to allow for an easier reading of the differences. The physical meaning of the negative \( \theta / \gamma_{FF} \) values is that the tunnels in these cases are rotating in the opposite direction compared to the tunnels with the positive \( \theta / \gamma_{FF} \) values (set clockwise). Regardless the flexibility of the tunnel section, its rotation increases with the decrease of the aspect ratio, \( \lambda \), and the decrease of the burial depth, \( h \). Additionally, the normalized rotation ratios are decreasing with the increase of Poisson’s ratio of the soil, \( \nu_s \). It is worth noticing that the current relations may describe the rocking response of more complex sections (e.g. double boxes) given that the soil-tunnel relative stiffness is properly defined, through the flexibility ratio (Tsinidis 2017, Tsinidis and Pitilakis 2018).

![Figure 5. ‘Average’ \( \theta / \gamma_{FF} - F \) relations computed for a wide range of tunnel sections assuming perfect bonding conditions between the soil and the liner elements.](image)

### 3.3 Numerical \( R_m - F \) relations

A set of racking ratio \( R_m \) – flexibility ratio \( F \) relations, were initially developed on the basis of the results of the extended numerical parametric analysis. The seismic ‘racking’ deformations of the tunnel section, \( \delta_{str} \), and soil under free-field conditions, \( \delta_{ff} \), required for the definition of the presented racking ratios \( R_m \) were estimated by subtracting the horizontal displacements computed on the tunnel and at the free-field soil at the inverted slab elevation from those computed at the same locations at the roof slab elevation (Wang 1993, Hashash et al. 2001, Hashash et al. 2010). This section summarizes representative relations computed for a perfect bonding condition between the lining and ground elements. The effect of soil-tunnel interface conditions on the \( R_m \) racking ratios is discussed in detail in Tsinidis (2017) as well as in Tsinidis and Pitilakis (2018).

Similar to the \( \theta / \gamma_{FF} - F \) relations, the soil properties and the ground seismic motion characteristics, were found to have a negligible effect on the computed \( R_m - F \) relations, when a full bonding interface condition was considered. It is worth mentioning that if strain compatible soil properties were considered for each ground seismic motion, then both the computed racking ratios and the flexibility ratios, would have been affected by the ground motion characteristics, since these affect the soil equivalent properties (Hashash et al. 2010). Additionally, for the particular interface condition, the racking ratios computed for square tunnels of various sizes, assuming perfect bonding interface conditions, were almost identical (Tsinidis 2017), thus allowing for further generalization of the \( R_m - F \) relations based on the aspect ratios, \( \lambda \). Figure 6 illustrates a series of ‘average’ \( R_m - F \) relations, highlighting the effects of the aspect ratio, \( \lambda \), and burial depth, \( h \), of the tunnel section and the Poisson’s ratio of the surrounding ground. The horizontal axis is again displayed in logarithmic scale,
while the computed relations are compared with commonly used analytical and empirical $R$-$F$ relations. Generally, the racking ratios $R_n$ of flexible tunnels ($F > 1.0$) are increasing with the increase of the aspect ratio, $\lambda$, while the reverse tendency holds for rigid tunnels ($F < 1.0$). The racking ratios are decreasing with the increase of the burial depth of the tunnel, as well as with the increase of the Poisson’s ratio of the soil. The analytical and empirical relations (Penzien 2000, Anderson et al. 2008) provide lower racking ratios for the rigid tunnels compared to the numerical relations, with the underestimation being generally higher for lower aspect ratios, $\lambda$. For flexible tunnels, the Anderson et al. (2008) relation either over-predicts or under-predicts the racking ratio compared to the numerically one. Additionally, the Penzien’s analytical relations (Penzien 2000), referring to no-slip interface conditions, seem to provide a maximum envelope for the racking ratios of flexible tunnels for the majority of the examined cases. It is worth noticing that the effect of the damping of the soil on the computed racking ratios, $R_n$, was rather insignificant, for a wide range of lower flexibility ratios (i.e. $F < 5.0$, Tsinidis (2017)).

![Graphs showing Racking Ratios vs. F for different burial depths and Poisson's ratios](image.png)

Figure 6. ‘Average’ $R_n$-$F$ relations computed for a wide range of tunnel sections assuming perfect bonding conditions between the soil and the liner elements, plotted against analytical and empirical $R$-$F$ relations.

### 3.4 Numerical R-F relations for the improvement of the R-F method

The computation of the seismic racking deformation of the tunnel section, $\delta_{str}$, on the basis of a ‘simple’ horizontal relative displacement between the roof and the inverted slabs, provides an accurate estimation of the actual seismic ‘racking’ distortion of the tunnel, only when the rocking response of the tunnel section is negligible. The rotation of the tunnel section due to the rocking motion, which is, inherently, accounted for by the numerical analyses (see for example Figure 4a), affects the computed relative horizontal displacements of the slabs, leading to a different numerical predicted ‘racking’ distortion of the section ($\delta_{str}$) compared to the actual one ($\delta_{str,a}$). This error is illustrated schematically in Figure 7a and 7b, for rigid and flexible tunnels, respectively. The presented deformed shapes are drawn on basis of the particular rotational patterns of the tunnel sections that were identified for the cases of ‘purely elastic’ soil-tunnel configurations, i.e. elastic soil response and perfect bonding between the lining and the soil elements. As seen, the net rotation of rigid tunnels results in an overprediction of the computed seismic ‘racking’ distortion compared to the actual one. This observation explains the high values of the racking ratios, $R_n$, which are identified in case of rigid tunnels with low aspect ratios, $\lambda$ (Figure 6). On the contrary, the particular rotational pattern of flexible tunnels results in an underestimation of the computed seismic ‘racking’ distortion of the section compared to the actual one. Based on the deformation patterns displayed in Figure 7, more accurate
values for the actual seismic ‘racking’ distortions of the tunnel sections may be obtained by correcting the numerically predicted structural seismic distortions, as follows:

\[
\delta_{str,a} \approx \begin{cases} 
\delta_{str} + 2 \times \delta_\theta = \delta_{str} + 2 \times \tan \theta \times b/2 \approx \delta_{str} + \theta \times b & \text{for } F > 1.0 \\
\delta_{str} - 2 \times \delta_\theta = \delta_{str} - 2 \times \tan \theta \times b/2 \approx \delta_{str} - \theta \times b & \text{for } F < 1.0
\end{cases}
\]  

(3)

where \( \delta_\theta \) is half of the horizontal displacement caused by the rocking rotation of the tunnel section, \( \theta \), and \( b \) is the height of the tunnel section. The rocking rotation of the tunnel section may be computed on the basis of \( \theta/\gamma_s F \) relations, similar Figure 5. \( \delta_\theta \) can then be estimated from the highlighted triangles in Figure 7, as follows \( \delta_\theta = \tan \theta \times b/2 \approx \theta \times b/2 \). An alternative method to compute directly the actual seismic ‘racking’ distortion of the tunnel section from the numerical analyses is schematically illustrated in Figure 7c and is based on the geometry of the deformed tunnel section during ground shaking:

\[
\delta_{str,a} = \tan \gamma_s \times b = \left[ \left( d_1^2 - d_2^2 \right)/4 \times a \times b \right] \times b = \left( d_1^2 - d_2^2 \right)/4 \times a
\]  

(4)

where \( \gamma_s \) is the average racking angle of the tunnel section and \( d_1 \) and \( d_2 \) are the lengths of the diagonals of the deformed tunnel section, all referring to the time step of maximum distortion of the tunnel during ground shaking. The actual seismic ‘racking’ distortions, computed by Equations 3 and 4, may be used to develop new improved racking ratios, as follows: \( R = \delta_{str,a}/\delta_{ff} \). For the ‘purely elastic’ tunnel-soil configurations discussed herein, the above equations yield in identical structural seismic distortions for the tunnel sections (Tsinidis and Pitilakis 2018).

Based on the above analysis, a set of new \( R-F \) relations were developed for design purposes, using the actual seismic ‘racking’ distortion of the tunnel sections. Figure 8a portrays the proposed new \( R-F \) relations for a wide range of soil-tunnel configurations, focusing on rigid tunnels \((F < 1.0)\), and (b) flexible tunnels \((F > 1.0)\); (c) association of the actual seismic ‘racking’ distortion of a rectangular tunnel with the diagonal distortions of the section during ground shaking.

![Figure 7](image-url)

Figure 7. Error in the evaluation of the actual seismic ‘racking’ distortion of rectangular tunnels, \( \delta_{str,a} \), by the numerical analyses, due to the rocking response of the sections, for the case of (a) rigid tunnels \((F < 1.0)\), and (b) flexible tunnels \((F > 1.0)\); (c) association of the actual seismic ‘racking’ distortion of a rectangular tunnel with the diagonal distortions of the section during ground shaking.

Based on the above analysis, a set of new \( R-F \) relations were developed for design purposes, using the actual seismic ‘racking’ distortion of the tunnel sections. Figure 8a portrays the proposed new \( R-F \) relations for a wide range of soil-tunnel configurations, focusing on rigid tunnels \((F < 1.0)\), and (b) flexible tunnels \((F > 1.0)\); (c) association of the actual seismic ‘racking’ distortion of a rectangular tunnel with the diagonal distortions of the section during ground shaking. Both axes are displayed in logarithmic scale, allowing an easier view of the differences, while the numerical relations are compared with commonly used analytical and empirical \( R-F \) relations. The dispersion between the new racking ratios, \( R \), computed various aspect ratios, \( \lambda \), is generally reduced compared to the one reported for racking ratios \( R_n \), while the Penzien (2000) and Anderson et al. (2008) relations overpredict the racking ratios compared to the numerical results. The deviations may reach 45 % for shallow quite rigid tunnels. Along these lines, the implementation of the existing \( R-F \) relations may lead to an overdesign of the linings of quite rigid tunnels \((i.e. F < 0.5)\). The \( R-F \) relations, developed for flexible tunnel sections \((F > 1.0)\), are portrayed in Figure 8b. Generally, the racking ratios, \( R \), are...
decreasing with the increase of the burial depth of the tunnel, \( h \), as well as with the increase of the Poisson’s ratio, \( \nu_s \), of the soil. The analytical and empirical \( R-F \) relations underestimate the racking ratios for flexible tunnels compared to the numerical one, with the deviations increasing with the increase of the aspect ratio of the tunnel, \( \lambda \), as well as with the decrease of the tunnel burial depth, \( h \), reaching a maximum of 40 % for very flexible tunnels.

Figure 8. New \( R-F \) relations computed for (a) rigid tunnels and (b) flexible tunnels under perfect bonding between the lining and the soil elements.

3.5 Efficiency of proposed \( R-F \) relations

The efficiency of the new \( R-F \) relations was investigated by comparing the predictions of the improved \( R-F \) method with relevant results of rigorous full dynamic analyses. In particular, a series of static frame analyses were performed, on representative tunnel configurations of the numerical study, as per Wang (1993). The free-field soil racking distortions, \( \delta_{ff} \), were determined on the basis of one-dimensional soil response analyses of the examined soil deposits. The racking ratios, \( R \), used for the
determination of the seismic racking distortions of the tunnel sections, were computed using the new \( R-F \) relations, accounting for the burial depth and shape of the investigated tunnel sections, as well as for the Poisson’s ratio of the examined soil deposits. Simple static frame analyses were then performed, by imposing the seismic racking distortion at the corners of the roof slab of the lining frame, to compute the seismic bending moments at critical sections of the examined tunnels. The results of the static analyses, in terms of seismic bending moments at critical lining sections, were compared with the relevant output of rigorous full dynamic analyses of the investigated soil-tunnel configurations, the latter computed when the seismic ‘racking’ distortion of the tunnel section was maximized.

Figure 9 portrays representative results of this comparative effort, referring to various tunnel sections and various lining flexibilities (i.e. the flexibility ratio \( F \) is ranging between 0.1 and 10). The comparisons indicate a good agreement between the seismic bending moments predicted by the simplified method and the rigorous dynamic analysis. Generally, the comparisons are better for relatively rigid tunnels compared to surrounding ground, i.e. \( F < 2.0 \). This is actually expected, since the linings of quite flexible tunnels exhibit an increased curvature during ground shaking, compared to the rigid tunnels. This response weakens slightly the accuracy of the assumption of a rigid frame rocking rotation, on which the development of the new racking ratios, \( R \), is actually relied. In any case, the deviations between the results of the improved \( R-F \) method and full dynamic analysis do not exceed 30 \%, even for cases of very flexible tunnel linings. It is worth noticing that the efficiency of the \( R-F \) method was significantly reduced when the \( R_n-F \) relations were implemented (Pitilakis and Tsinidis 2018).

The efficiency of the new \( R-F \) relations was examined for cases of tunnels embedded in soil deposits of increasing stiffness with depth, as well as for cases of more complex tunnel sections (e.g. double boxes), or tunnel sections with structural members of various thickness. For all these cases, the improved \( R-F \) method provided quite comparable results with the full dynamic analysis, especially for rigid tunnels compared to the surrounding ground (i.e. \( F < 1.0 \), Tsinidis and Pitilakis 2018).

The results of the \( R-F \) method, presented in Figure 9, refer to the case, where the static analysis is performed by imposing the structural seismic racking distortion at the corners of the roof slab of the frame. According to Wang (1993), this static condition, which is actually identical to the case, where an equivalent concentrated static load is imposed at the top corner of the tunnel section (Figure 1a), may result in more accurate predictions of the seismic bending moments for deep tunnels. For shallow tunnels, Wang (1993) proposed an alternative static analysis configuration, with the structural racking distortion of the tunnel section being simulated as an equivalent static pressure, acting on both side walls, as per Figure 1b. The examination of the two modelling approaches within this study verified this rationale (Tsinidis and Pitilakis 2018). Actually, for the shallow tunnels (i.e. for \( h = 0 \) or 3 m) the implementation of the equivalent static pressure at the side walls led to more comparable results between the \( R-F \) method and the rigorous dynamic analysis.
4. CONCLUSIONS

A set of $R-F$ relations where presented in this study, aiming at the improvement of the $R-F$ method proposed by Wang (1993). The relations were developed on the basis of an extended numerical parametric study that was conducted to investigate the experimentally observed coupled racking-rocking response of rectangular tunnels during transversal ground seismic shaking. The key findings are summarized as follows:

- The soil-tunnel relative flexibility and soil-tunnel interface conditions affected the coupled racking-rocking deformation pattern of rectangular tunnels during ground shaking.
- Particular rotation patterns were identified for elastic soil response and full bonding between the lining and the ground elements (i.e. ‘purely’ elastic systems), regardless the dimensions, shape and burial depth of the tunnel sections. In particular, the rotation pole of the tunnels coincided with the centroid of the sections, while the direction of rotation was affected only by the soil to tunnel relative stiffness. This observation, allowed the development of new $R-F$ relations for design purposes. Along these lines, the discussion focused on these systems.
- The rocking response of the tunnel sections was quantified on the basis of rocking rotation-flexibility ratio ($\theta/γ_{ff} - F$) relations, similar to the $R-F$ relations. The shape, burial depth and stiffness of the tunnel section, along with the Poisson’s ratio of the soil, $v_s$, were found to affect these relations.
- A set of $R_n-F$ relations were developed for a wide range of tunnel sections, following the existing literature. The rocking ratios, $R_n$, were initially evaluated on the basis tunnel ($\delta_{str}$) and soil ($\delta_{ff}$) relative horizontal distortions, the latter estimated by subtracting the horizontal displacements computed on the tunnel and at the soil at the inverted slab elevation from those computed at the same locations at the roof slab elevation. The $R_n-F$ relations were affected by shape, burial depth and stiffness of the tunnel section, as well as the Poisson’s ratio of the soil, $v_s$. Additionally, significant deviations were observed between the $R_n-F$ relations and the analytical and empirical ones (i.e. Penzien 2000 and Anderson et al. 2008) that often used in the tunnelling design practice.
- A set of new $R-F$ relations were developed for design purposes, by correcting the numerically computed structural seismic racking distortions, in order to account for the effect of the rocking rotation of the tunnel section that is inherently considered by the numerical analyses. The ‘corrections’ were made on the basis of the particular rotation patterns observed for elastic soil response and perfect bonding interface conditions. The analytical and empirical relations were found to over predict the racking ratios of rigid tunnels ($F < 1.0$), compared to the new $R-F$ relations, with the differences being as high as 45 % (i.e. for shallow and very rigid tunnels). On the contrary, the analytical and empirical $R-F$ relations underpredicted the racking ratios for flexible tunnels ($F > 1.0$), compared to the new $R-F$ relations, with the differences being as high as 40 %, for quite flexible shallow tunnels with high aspect ratios, i.e. $\lambda > 2.0$.
- The efficiency of the new $R-F$ relations was examined by comparing the predictions of the $R-F$ method with the rigorous dynamic analysis for various soil-tunnel configurations. The comparisons indicated a good agreement, particularly for relatively rigid tunnels compared to the surrounding ground. The comparisons were less satisfactory, when the $R_n-F$ relations were used. Hence, the effect of the rocking rotation of rectangular tunnels on the seismic racking distortion should always be considered, when the evaluation of racking ratios is performed by means of numerical analyses. The numerical $\theta/γ_{ff} - F$ relations, provided herein, may be used to correct the numerical seismic structural deformations and hence the racking ratios.

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6. REFERENCES


