SEISMIC RESPONSE OF FREE-STANDING ROCKING MASONRY WALLS CONSIDERING TOE CRUSHING

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ABSTRACT

Rocking mode of response is repeatedly observed during earthquakes and shaking table tests of masonry buildings. Limited and controlled rocking response of structural masonry piers and walls usually results in reduced inertial forces and collapse prevention of inherently brittle unreinforced masonry elements. However, toe crushing is an inevitable consequence of rocking motion in members without toe crushing protection measures. This behavior reduces the effective lever arm of re-centering moment and consequently exacerbates the stability of the rocking wall under earthquake ground motions. The aim of this study is to evaluate rocking response of free-standing rigid masonry rocking walls under seismic ground motions that result in toe crushing during consecutive impacts of the wall with its rigid base. The effective re-centering lever arm is reduced with a reasonable assumption based on the finite element analysis and the mechanical properties of masonry materials. The results show that considering toe crushing may significantly alter the seismic response and stability of rocking masonry walls.

Keywords: Unreinforced masonry walls; Rocking response; Toe crushing; Impact load

1. INTRODUCTION

Review of past earthquake damage shows that unreinforced masonry (URM) structures are more susceptible to failure compared to other construction types. However, recent researches show that the performance of low-rise URM buildings might be acceptable in regions of low to medium seismicity. In-plane response of URM walls has been investigated both experimentally and theoretically. These studies have been primarily focused on four discrete failure modes which describe in-plane behavior of URM walls. But the combination of failure modes under seismic loads has not been considered as it deserves. Among them, one can refer to the consecutive occurrence of flexible rocking mode of response and brittle crushing failure at the wall toes.

Rocking mode of response has been considered to be appreciable, especially for inherently brittle URM walls which results in reduced inertial forces, acting like an isolator system for the building. On the other hand, it is sometimes accompanied by the brittle crushing failure of the wall toes. As an actual example, Figure 1 shows an unreinforced brick masonry wall which experienced crushing at its toes after several cycles of rocking motion (Dizhur et al. 2011). Crushing of a reinforced concrete wall under in-plane cyclic loading is represented in Figure 2 which resulted in the concrete cover to be spalled off (Almeida et al. 2016). Other similar cases can be easily found showing that how rocking response of structural walls is changed or occasionally prevented by such brittle failures.

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Hence, there is a need for a thorough investigation of the actual response of rocking walls. In this study, we will introduce a simple and efficient method to consider the effects of toe crushing on the rocking response of rigid walls. The numerical procedure is performed with standard ODE solvers available in MATLAB (2015) to solve and modify the nonlinear equations of motion. The solution accounts for the verification of the fundamental parameters of rocking motion (i.e. the frequency parameter, $p$ and the angle of block, $\alpha$) due to crushing failure. This paper reveals that considering toe crushing may have significant effects on the seismic response and stability of rocking URM walls.

### 2. ROCKING RESPONSE OF FREE-STANDING BLOCKS

#### 2.1 Equations of motion

The rigid block shown in Figure 3 with slenderness $\alpha = \tan^{-1}(B/H)$ is set to rocking and pivots on the centers of rotation $O$ and $O'$. The coefficient of friction is assumed sufficiently large so that the block will rock with no sliding. If the block is exposed to a sufficiently large positive horizontal acceleration,
If \( \ddot{u}_g(t) \), it will start to rotate with a negative rotation, \( \theta < 0 \). This condition can be described as follows:

\[
m\ddot{u}_g(t) \frac{H}{2} > mg \frac{B}{2}
\]

which is equivalent to: \( \ddot{u}_g(t) / g > \tan^{-1}(a) \), where \( m \) is the mass of the block and \( g \) is the acceleration of gravity.

![Figure 3. Schematics of a rocking rigid block](image)

The equations governing rocking motion under a horizontal ground acceleration \( \ddot{u}_g(t) \) are (Makris 2001) (Housner 1963):

\[
I_0 \ddot{\theta}(t) + mgR \sin(\alpha - \theta) = -m\ddot{u}_g(t)R \cos(\alpha - \theta) \quad \theta > 0
\]

and

\[
I_0 \ddot{\theta}(t) + mgR \sin(-\alpha - \theta) = -m\ddot{u}_g(t)R \cos(-\alpha - \theta) \quad \theta < 0
\]

where \( I_0 \) is the moment of inertia of the block (for rectangular blocks \( I_0 = \frac{4}{3}mR^2 \)) and \( R \) is the distance from the center of gravity to the center of rotation. The compact form of the two above equations can be expressed as:

\[
\ddot{\theta}(t) = -p^2 \left\{ \sin\left( \alpha \text{sgn}(\theta(t)) - \theta(t) \right) + \frac{\ddot{u}_g(t)}{g} \cos\left( \alpha \text{sgn}(\theta(t)) - \theta(t) \right) \right\}
\]

In which \( p = \frac{3g}{4R} \) is the frequency parameter of the block (rad/s).

The impact is assumed to be inelastic, so the rotation continues about the toes with no bouncing and the conservation of momentum about \( O' \) is confirmed:

\[
I_0 \dot{\theta}_1 - 2bm\theta R \sin \alpha = I_0 \dot{\theta}_2
\]
where $\dot{\theta}_1$ and $\dot{\theta}_2$ are the angular velocities just before and after impact, respectively. The amount of the kinetic energy reduced during each impact is:

$$r = \left(\frac{1}{2} I_o \dot{\theta}_2^2\right) / \left(\frac{1}{2} I_o \dot{\theta}_1^2\right) = \left(\frac{\dot{\theta}_2}{\dot{\theta}_1}\right)^2$$

(6)

Which together with Equation (5) gives:

$$r = \left(1 - \frac{3}{2} \sin^2 \alpha\right)^2$$

(7)

The solution of Equation (4) can be determined numerically with the condition defined in Equation (1) to initiate rocking and the impact condition expressed in Equation (7).

### 2.2 Vertical impact load

Based on investigations carried out by Toranzo-Dianderas (2002), the forces developed at the corners of a wall at impact are the largest forces during the rocking process. Using a simplified energy method, the vertical impact load, $N$ can be defined by an impact amplification factor, $f_{\text{imp}}$ applied to the equivalent static solution, $N_{\text{static}}$. The system is assumed to be rigid with flexible contact springs restrained to the base. The stiffness of the horizontal and vertical springs at the base of the wall are $k_x$ and $k_y$, respectively. The process of calculations can be found in Toranzo-Dianderas (2002) and will not be repeated here. Accordingly, one can write:

$$N = f_{\text{imp}} \cdot N_{\text{static}}$$

(8)

where $N_{\text{static}}$ is the equivalent static vertical load described below with regard to Figure 3:

$$N_{\text{static}} = W + mR\ddot{\theta} \sin(\alpha - \theta) - mR\dot{\theta}^2 \cos(\alpha - \theta)$$

(9)

and $f_{\text{imp}}$ is the impact amplification factor:

$$f_{\text{imp}} = \frac{1}{2} \left[1 + \sqrt{1 + \frac{(2v \sin \alpha)^2}{g \cdot \delta_{st}}}\right]$$

(10)

In which $v = R\dot{\theta}$ is the velocity of the center of mass at impact and $\delta_{st} = \frac{W}{k_y}$ is the static deformation of the vertical springs. $W$ is the weight of the wall and $k_y$ is the half-space spring constant in the vertical direction defined in the literature for a rigid footing as follows (Villaverde 2009):

$$k_y = \frac{8G}{\pi(1 - \nu)} F_y$$

(11)

where $G$ indicates the half-space’s shear modulus of elasticity, $\nu$ is the Poisson ratio and

$$F_y = \frac{L}{2} \ln\left(\frac{\pi}{4} + \frac{1}{2} \tan^{-1} \frac{B}{L}\right) + \frac{B}{2} \ln\left(\frac{\pi}{4} + \frac{1}{2} \tan^{-1} \frac{L}{B}\right)$$

(12)
B and L are the length and width of the footing, respectively. In this study, L represents the thickness of
the wall and B could approximately be estimated as the length of the equivalent uniform compressive
stress due to the weight of the wall:

\[ B = \frac{W}{a f_w' \times t_w} \]  \hspace{1cm} (13)

Thus, we can obtain the vertical impact load, N by substituting the aforementioned parameters in
Equation (8).

2.3 Calculating the crushing length

Consider the wall section shown in Figure 4 which is set to rock about its toes. the strain and stress
distribution at the contact surface of the wall are shown in Figure 4 (b) and (c), respectively. As the
lateral forces developed in the wall increase, the contact area becomes smaller. The uniform compressive
stress \( f_m' \) develops on the entire contact surface. Using simple equilibrium at this stage, one can derive:

\[ N = (\alpha f_m') \times (a \times t_w) \]  \hspace{1cm} (14)

where \( N \) is the vertical impact load from Equation 8, \( f_m' \) is the compressive strength of masonry, \( \alpha \) is
a constant equal to 0.85, \( t_w \) is the wall thickness, and \( a = \beta_c \) is the entire contact length of the wall
just before crushing. \( \beta_c = 0.85 \) is assumed based on ACI considerations for concrete elements. c is the
distance from extreme compression fiber to the neutral axis.

![Figure 4](image)

Figure 4. (a) Wall section (b) Strain limit before crushing (c) Stress distribution before crushing

Crushing process is assumed to propagate through a length of \( B_c \) where the maximum compressive
strength is reached. This length corresponds to the distance between a strain range of 0.002 to 0.004, as
demonstrated in Figure 4 (b):

\[ \frac{c - B_c}{c} = 1 - \frac{B_c}{c} = \frac{\epsilon_{ult}}{\epsilon_{su}} \]  \hspace{1cm} (15)
this equation can be simplified as:

\[ B_c = c(1 - \frac{\varepsilon_{su}}{\varepsilon_{sw}}) = c(1 - \frac{0.002}{0.004}) = 0.5c \]  \hspace{1cm} (16)

substituting the value of \( c \) from Equation (14) into the above equation and assuming \( a_i = \beta_i = 0.85 \) gives:

\[ B_c = 0.5(\frac{1}{\beta_i} \frac{N}{a_i f_w t_w}) = 0.692 \frac{N}{f_w t_w} \]  \hspace{1cm} (17)

The crushing length, \( B_c \), is the fundamental parameter of crushing failure which is calculated through a simple process. Based on equation (17), as the vertical impact load increases, the crushing length will increase.

2.4 Analysis Procedure

To obtain rocking response of an effective wall (a wall with crushed toes), the fundamental equations of rocking motion (Equations (2) and (3)) should be modified. Once rocking starts, these equations are confirmed until crushing occurs. From now upwards, the center of rotation will change and the effective re-centering lever arm and the angle of the block are reduced (Figure 5). The block will not rock on its entire length anymore and the actual length of the block will be \( B' = B - B_c \).

![Figure 5. Fundamental parameters of (a) Rocking wall before crushing and (b) effective wall after crushing](image)

The fundamental parameters of the block should be substituted as:

\[ \alpha' = \tan^{-1}\left(\frac{B - 2B_c}{H}\right) \]  \hspace{1cm} (18)

\[ R' = 0.5\sqrt{(B - 2B_c)^2 + (H)^2} \]  \hspace{1cm} (19)

\[ p' = \sqrt{\frac{3g}{4R'}} \]  \hspace{1cm} (20)

Thus, Equation (4) will be changed as follows:
The numerical procedure to solve the later equation and consider the abovementioned changes in rocking response of walls is performed with standard ODE solvers available in MATLAB (2015).

3. RESULTS

Rocking response of walls has been analyzed under one-cosine/ sine pulses and earthquake ground motions. Different crushing schemes can be assumed, among them are: crushing at the initiation of rocking, crushing after first significant impact, and crushing after response decreases. Also, one can adopt a scheme based on which crushing occurs at consecutive impacts. But the most advantageous way to consider toe crushing is to calculate crushing length after first considerable impact in terms of the vertical dynamic impact load. This would be beneficial to be adopted as a design strategy, as other schemes seem to be highly conservative or not functional.

The rocking response of a rigid wall with $B = 1 \text{ m}$ and $H = 2.5 \text{ m}$ ($R = 1.35 \text{ m}$ and $p = 2.34 \text{ rad/sec}$) subjected to one-cosine and one-sine pulses, shown in Figure 6 is compared to the effective wall response.

![Figure 6. One-Cosine pulse and One-Sine pulse accelerations with $T_p = 2\text{ sec}$](image)

The acceleration functions can be determined as follows:

$$\ddot{a}_{g}(t) = a_{max} \cos\left(\frac{2\pi}{T_p} t\right), \quad 0 \leq t \leq T_p$$

(22)

$$\ddot{a}_{g}(t) = a_{max} \sin\left(\frac{2\pi}{T_p} t\right), \quad 0 \leq t \leq T_p$$

(23)

where $T_p$ is the period of pulse adopted here as 2 sec and $a_{max}$ is assumed to change between three different levels of maximum acceleration.

The Rocking responses of ideal and effective wall under one-cosine pulse are shown in Figure 7 and Figure 8 in terms of normalized rotation $\theta / \alpha$ for different maximum acceleration values. The crushing values under each maximum acceleration together with corresponding crushing half-cycle are
Figure 7. Rocking response of the ideal wall under one-cosine pulse acceleration

Figure 8. Rocking response of the effective wall under one-sine pulse acceleration

Table 1. Calculated crushing length under one-cosine pulse with different maximum accelerations

<table>
<thead>
<tr>
<th>Maximum acceleration, $a_{\text{max}}$</th>
<th>Crushing length, $B_c$ (mm)</th>
<th>Crushing half-cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.471g</td>
<td>68</td>
<td>5 and 6</td>
</tr>
<tr>
<td>0.472g</td>
<td>79</td>
<td>5 and 6</td>
</tr>
<tr>
<td>0.473g</td>
<td>89</td>
<td>5 and 6</td>
</tr>
</tbody>
</table>

A similar procedure is repeated and Rocking responses of the ideal and effective wall under one-sine pulse are shown in Figure 9 and Figure 10. The crushing values are reported in Table 2.
Comparing the values of crushing length shows that as maximum acceleration increases, crushing length normally increase. However, the crushing length depends as much on the amount of rotation that the wall has experienced just before impact. That is why the crushing length under one-sine pulse with a maximum acceleration of 0.464g is greater than that under one-cosine pulse with a maximum acceleration of 0.472g. So this parameter plays a more effective role in determining the crushing length. It is of interest to compare rocking response of walls with different slenderness ratios under earthquake
ground motion. Figure 11 shows the East-West component of the acceleration history of the 1995 Kobe earthquake with a maximum acceleration of 0.83g. Selected walls and their inherent characteristics in terms of slenderness, α and natural period, $T = 2\pi/p$ are summarized in Table 3.

![Figure 11. East-West component of the 1995 Kobe earthquake acceleration decreased by 85%](image)

**Table 3. Selected walls with different slenderness ratios**

<table>
<thead>
<tr>
<th>Wall size (m)</th>
<th>Slenderness, α (degree)</th>
<th>Natural period, T (sec)</th>
<th>Crushing length, $B_c$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 × 7.3</td>
<td>15</td>
<td>4.5</td>
<td>430</td>
</tr>
<tr>
<td>4 × 14.6</td>
<td>6.4</td>
<td>6.4</td>
<td>240</td>
</tr>
<tr>
<td>1.2 × 3.24</td>
<td>20</td>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>5.3 × 14.31</td>
<td>6.4</td>
<td>6.4</td>
<td>320</td>
</tr>
<tr>
<td>1.5 × 3.15</td>
<td>25</td>
<td>3</td>
<td>185</td>
</tr>
<tr>
<td>4 × 8.4</td>
<td>5</td>
<td>5</td>
<td>670</td>
</tr>
</tbody>
</table>

![Figure 12. Rocking response of walls with different slenderness ratios considering toe crushing](image)
Rocking response of effective walls is shown in Figure 12 and calculated crushing length corresponding to each one is reported in Table 3. The results show that with decreasing the slenderness of a wall (which means increasing wall angle, \( \alpha \)), its overall stability increase. It can also be concluded from Figure 12 that by increasing the natural period of a wall (which means increasing wall size), its behavior becomes more stable; even if the crushing length is considerable. The wall with \( B = 1.2 \) m and \( H = 3.24 \) m has the least crushing length among all walls, but shows the most instability compared to others. On the other hand, the wall with \( B = 4 \) m and \( H = 8.4 \) m has the biggest crushing length, but its behavior remains stable. This reveals the important role of wall aspect ratio and also the self-weight of the wall which together with other important parameters discussed earlier determine its overall rocking response.

4. CONCLUSION

A simple and efficient method is introduced to consider the effects of toe crushing on the rocking response of rigid walls. The numerical procedure is performed with standard ODE solvers available in MATLAB (2015) to modify the nonlinear equations of motion. The solution accounts for the alteration of the fundamental parameters of rocking response after crushing occurs. A simple and reasonable approach is adopted which assumes that crushing occurs after the first significant impact. The results show that as the acceleration of ground motion increases, crushing length increase. However, the maximum rotation after which crushing occurs, has greater influence on crushing length. In addition, as the weight of the wall increases, crushing length extremely increase. Smaller walls with high slenderness ratio are in danger of overturning at the most, even if their crushing length is very small. However, the larger ones behave more stable even in cases of more crushing lengths.

5. REFERENCES


