HOW MUCH SUB-SURFACE INFORMATION CAN WE EXTRACT FROM SURFACE RECORDS

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ABSTRACT

For soil sites, the dynamic characteristics of sub-surface soil layers are a critical factor influencing the amplification of earthquake ground motions and structural damage. In-situ field tests (e.g., geotechnical borings, SPT and CPT tests), and earthquake records from vertical downhole arrays are the best way to identify site amplification. However, these tests are expensive and time consuming. In this paper, we show that, in certain conditions (e.g., layered soil media subjected to vertically propagating shear waves) it is possible to identify sub-surface soil characteristics from surface records. The approach is based on the discrete time formulation of wave propagation in layered media and lattice filtering model. For the identification of layer characteristics, we divide soil media into fictional thin layers, where the two-way travel time in each layer is equal to one sampling interval. With this model, we show that the relationship between the excitation and surface response can be represented by an auto-regressive moving-average type discrete time filter. Filter parameters are identified by using tools from the system identification theory. Such filters can be converted into a cascade of lattice filters, each with two inputs and two outputs. Each lattice filter corresponds to a fictional soil layer defined by the reflection coefficient at the upper interface of the layer. Starting from the layer next to the surface, we can identify the reflection coefficients of the layers from the surface record. The reflection coefficients of those fictional layers that do not correspond to an actual interface would be identified as zero. Numerical examples are provided for the methodology.

Keywords: Site amplification; Layered soil media; Lattice filters

1. INTRODUCTION

The amplification (i.e., site amplification) of seismic waves by near-surface soil layers is a critical factor influencing the level of shaking and structural damage experienced on ground surface (Safak, 1997). One of the most accurate ways to identify site amplification is to record ground motions synchronously at different depths below the surface. Such arrays are known as vertical downhole arrays. Downhole arrays are expensive to install because of drilling costs and the cost of downhole sensors.

For soil sites that can be approximated as layered soil media and subjected to vertically propagating shear waves (Ewing, et. al., 1957), it is possible to identify and estimate the characteristics of sub-surface layers from the surface records alone. This paper presents an approach to accomplish this based on discrete-time formulation of wave propagation in layered media via Z-transforms. We show that the surface record can be represented as the response of an IIR (Infinite Impulse Response) filter, subjected to a Gaussian white-noise input. The parameters of the filter, when converted into a lattice form, represent the reflection coefficients of the layers below the surface. The theoretical formulation and a numerical example are given below.

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2. DISCRETE-TIME WAVE PROPAGATION IN LAYERED MEDIA

Consider an \( m \)-layered media with input \( X \) and the output \( Y \), as shown in Figure 1a. It will be assumed that the layer \( m+1 \) has infinite depth such that the reflected waves from interface \( m+1 \) do not come back. The propagation of waves is defined by the wave travel times, \( \tau \), in the layers, and the wave reflection and transmission coefficients, \( r \) and \( t_r \), for the upgoing waves, and \( r' \) and \( t_r' \) for the downgoing waves at each interface. The equations of wave propagation can be represented in terms of upgoing and downgoing waves, \( U \) and \( D \) at the top, and \( U' \) and \( D' \) at the bottom of each layer as shown in Fig. 1b. For the \( i \)'th layer, we can write the following (Claerbout, 1985):

\[
\begin{align*}
D_{i+1}(t) &= r_i^U U_{i+1}(t) + t_{ri} D_i(t) \\
U_i'(t) &= t_{ri}' U_{i+1}(t) + r_i D_i(t)
\end{align*}
\]

Note that the argument \( t \) represents the time. From the equality of displacements and forces at the interface, and accounting for the travel time of the waves in the layer, we can write

\[
\begin{align*}
t_r &= 1 + r, \quad r' = -r, \quad \text{and} \quad t_r' = 1 + r' = 1 - r, \\
U_i'(t) &= U_i(t - \tau_i) = q^{-\tau_i} U_i(t) \quad \text{and} \quad D_i'(t) = D_i(t + \tau_i) = q^{+\tau_i} D_i(t)
\end{align*}
\]

The parameter \( q^\tau \) in the equations represents the time-shift operator, such that \( q^\tau \cdot x(t) = x(t-k) \). The reflection and transmission coefficients, \( r \) and \( t_r \), can be calculated from the layer impedances, and the final expressions can be found in Safak (1995).

For simplicity, we neglected the effects of damping, since its influence on travel times will be small. Using these in the equations for layer \( i \), we can write the following matrix equation for the relationship between the upgoing and downgoing waves at the top of the layer \( i+1 \) and layer \( i \):

\[
\begin{bmatrix}
U_{i+1}(t) \\
D_{i+1}(t)
\end{bmatrix} = \frac{1}{1-r_i} \begin{bmatrix}
q^{-\tau_i} & -r_i \cdot q^{+\tau_i} \\
-r_i \cdot q^{-\tau_i} & q^{+\tau_i}
\end{bmatrix} \begin{bmatrix}
U_i(t) \\
D_i(t)
\end{bmatrix} = T_i \begin{bmatrix}
U_i(t) \\
D_i(t)
\end{bmatrix}
\]

where

\[
T_i = \frac{1}{1-r_i} \begin{bmatrix}
q^{-\tau_i} & -r_i \cdot q^{+\tau_i} \\
-r_i \cdot q^{-\tau_i} & q^{+\tau_i}
\end{bmatrix} = \frac{q^{-\tau_i}}{1-r_i} \begin{bmatrix}
1 & -r_i \cdot q^{+\tau_i} \\
-r_i & q^{2\tau_i}
\end{bmatrix}
\]

2
$T_i$ is known as the transfer matrix for layer $i$, for it transfers the upgoing and downgoing waves from the top of layer $i$ to the upgoing and downgoing waves to the top of layer $i+1$.

Since we do not know the layer interfaces, we will divide the media into superficial layers whose two-way travel time is equal to the sampling interval of the records; that is $2\tau_i = \Delta$, where $\Delta$ is the sampling interval. With this, the transfer matrix for the superficial layer $i$ now becomes:

$$T_i = \frac{q^{-\Delta/2}}{1-r_i} \cdot \begin{bmatrix} 1 & -r_i \cdot q^\Delta \\ -r_i & q^\Delta \end{bmatrix} = \frac{q^{-\Delta/2}}{1-r_i} \cdot \begin{bmatrix} 1 & -r_i / q^{-\Delta} \\ -r_i & 1 / q^{-\Delta} \end{bmatrix}$$
Since the reflection coefficient for the surface (soil-air interface) is -1 for the downgoing waves, and +1 for the upgoing waves, the surface record is twice the upgoing or the downgoing waves at the top of the top layer. Therefore, if the surface record is \( Y \), we can write the upgoing and downgoing waves at the top of the first layer as: \( U_1 = Y/2 \) and \( D_1 = -Y/2 \). Also, noting that at the top of the bedrock we have an upgoing wave, \( X \) and the downgoing wave \( R \) (i.e., the radiation damping), the relationship between the upgoing and downgoing waves at the top of the bedrock and those at the top of first layer can be expressed as the product of the transfer matrices from surface to bedrock (i.e., from layer 1 to layer \( m+1 \)) by the following equation:

\[
\begin{bmatrix}
X(t) \\
R(t)
\end{bmatrix} = T_1 \cdot T_2 \cdots T_m \cdot 
\begin{bmatrix}
Y(t)/2 \\
- Y(t)/2
\end{bmatrix}
\]

3. IDENTIFICATION OF SUB-SURFACE LAYERS

With the assumption that the media is composed of layers with two-way travel time equal to one sampling interval, and inserting the expressions for layer transfer matrices, we can show that the relationship between \( X(t) \) and \( Y(t) \) is equivalent to an IIR (Infinite Impulse Response) filter of the following form:

\[
Y(t) = \frac{b_0}{1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_n q^{-n}} \cdot X(t)
\]

where, for ease of notation, we have assumed that the sampling interval is \( \Delta = 1 \) (i.e., \( q^\Delta = q^1 \)).

It is reasonable to approximate the bedrock input waves, \( X(t) \), as a Gaussian white-noise random process because of broad spectral bandwidth of earthquake motions at bedrock. With this assumption, the equation above represents an auto-regressive polynomial (AR) model for \( Y(t) \). The coefficients \( a_k \) in the denominator can be calculated by various methods available in the literature (e.g., Burg, 1968). It is possible to convert these coefficients into reflection coefficients and represent the AR model in the form of a cascade of lattice filters, defined by reflection coefficients \( r \), as shown in Fig. 2 (Goodwin and Sin, 2009).

Each lattice filter represents the transfer matrix of a layer. By using the lattice filters, we can calculate the upgoing and downgoing waves for each successive layer, starting from layer 1, as shown below.
Figure 2. Lattice filter representation of upgoing and downgoing waves in layered media.
The recorded motion $Y_i(t)$ at interface $i$ is the sum of the upgoing and down going waves at layer $i+1$, that is: $Y_i(t) = U_{i+1}(t) + D_{i+1}(t)$. Therefore at each layer, as we go down calculating the upgoing and downgoing waves at the top of the layers, we also calculate the total response $Y_i(t)$ at each interface, and continue until the upgoing wave at the top the layer below is approximately a white noise.

4. EXAMPLE

We present an example for the methodology presented. Consider the three-layer soil media with properties given in Fig. 3. The input bedrock accelerations and the calculated surface accelerations are shown in Fig. 4, where the sampling interval is 0.005 sec. We match an AR model to the surface accelerations and calculate the reflection coefficients of the corresponding lattice filter. The reflection coefficients are plotted in Fig. 5 against the half-sampling points starting from the surface. The half-sampling points that correspond to large amplitude reflection coefficients (points 80, 100, 140) clearly indicate the layer interfaces, and match the locations given in the table in Fig. 3.

![Diagram of soil media with properties](image)

Figure 3. Properties of 3-layer soil media used in the example.
Figure 4. Bedrock and surface accelerations for the soil media in Fig. 3.

Figure 5. Reflection coefficients that match the surface record of the 3-layer media.
5. CONCLUSIONS

For layered soil media subjected to vertically propagating shear waves, it is possible to estimate the characteristics of sub-surface layers (i.e., layer interfaces, wave travel times, and reflection coefficients at the interfaces) from the surface records. For this, we divide soil media into fictional thin layers, where the two-way travel time in each layer is equal to one sampling interval. With this model, we show that the relationship between the excitation and surface response can be represented by an auto-regressive moving-average type discrete-time filter. Filter parameters are identified by using tools from system identification theory. Such filters can be converted into a cascade of lattice filters, each with two inputs and two outputs. Each lattice filter corresponds to a fictional soil layer defined by the reflection coefficient at the upper interface of the layer. An example shows the application of the methodology. The sensitivity of the methodology to measurement noise and velocity gradients is under investigation.

6. REFERENCES


