DEVELOPMENT AND APPLICATIONS OF SPECTRUM-COMPATIBLE FOURIER AMPLITUDE SPECTRA

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ABSTRACT

This article presents an empirical relationship that allows direct conversions between the 5% damped pseudo-acceleration elastic response spectrum (5% PSA) and the Fourier amplitude spectrum (FAS). The equation was developed based on a highly redundant dataset of synthetic time series covering a wide range of strong motion durations and made compatible to 8 different target response spectra, so that a large variety of dominant frequencies and bandwidths are comprised. The duration-dependent empirical equation is validated using seed-based spectrum-compatible records and actual earthquake records, obtaining results comparable with the more laborious inverse random vibration methodology. Applications on site response analyses and generation of spectrum-compatible time series are presented.

Keywords: Response spectrum; Fourier spectrum; RVT; site response; spectrum compatible earthquake records.

1. INTRODUCTION

Fourier Amplitude Spectrum (FAS) is widely used and constitutes a fundamental tool in seismology and earthquake engineering as it allows extraction of the frequency dependent amplitudes and phases in a ground motion. However, in structural design and assessment applications the preferred representation of seismic hazard continues to be based on the elastic response spectrum (often the 5% damped pseudo-acceleration response spectrum - 5% PSA). Hence, conversions between FAS and 5% PSA are often required. For example, in Random Vibration Theory (RVT) based site response procedures, the input motion is defined as a FAS that needs to be compatible with the prescribed uniform hazard spectrum (UHS) to comply with code requirements. The conversions are usually performed using iterative procedures grounded on RVT. However, while there is a well-established procedure to translate a FAS into a response spectrum, obtaining a FAS from a response spectrum is a more challenging task (Rathje et al. 2005). Moreover, the inversion of the 5% PSA into a FAS requires empirical approximations and/or an iterative scheme to converge. This article summarizes the main aspects of a recently developed duration-dependent empirical equation that allows direct conversions between FAS and 5% PSA (Montejo and Vidot-Vega 2017) and presents applications on site response analyses and in the generation of spectrum-compatible time series.

2. GENERAL ASPECTS OF THE RELATIONSHIP BETWEEN FAS AND 5% PSA.

2.1 Influence of duration

There are many definitions to estimate the strong motion duration of an earthquake record. However, significant duration seems to be the preferred duration definition used in earthquake engineering research and practice nowadays (e.g. Chandramohan et al. 2016, US-NRC 2007). Two different limits

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for the significant duration are investigated, the time intervals at which 5%-95% (SD5-95) and 5%-75% (SD5-75) of the Arias Intensity is reached. We use seed-based spectrum compatible time series to isolate the effects of duration on the FAS from the record-to-record differences in the elastic response spectrum amplitudes. A total of 100 spectrum compatible records were generated for each of 3 different 5% PSA target spectra using the computer code ArtifQuakeLet II (Montejo and Suarez 2013). This code implements a Continuous Wavelet Transform (CWT) based algorithm (Suarez and Montejo 2007) that modifies a seed record in the time-frequency domain so that its response spectrum matches a target spectrum, actual historic records were used as seed. This methodology has been shown to preserve the main characteristics of the seed records when selected properly (Gascot and Montejo 2016, Perez-Rivera and Montejo 2016).

The response spectra for a set of 100 time-series is represented by the gray lines in the 5% PSA plot in Figure 1 along with the target spectrum. The target spectrum is extracted from Section 5 of RG 1.208 (US-NRC 2007) and represents a typical uniform hazard response spectrum (UHRS) at the outcrop rock. It is seen that each time-series share approximately the same response spectrum matching the target. The same cannot be said about the FAS (also shown in Figure 1), despite exhibiting very similar response spectra, the variability in the FAS is significantly large. The influence of duration is assessed by pairing each record duration with the squared FAS in the frequency range 0.1Hz – 30Hz. It is seen that the strongest correlation is obtained for SD5-75, similar results were obtained for the other two target spectra studied (Montejo and Vidot-Vega 2017). SD5-75 is then used for the development of the duration-dependent relationship between FAS and 5%PSA.

Figure 1. Left: 5% PSA and FAS for 100 spectrum compatible records and their average values (red line). Right: correlations between significant durations (SD5-95 and SD5-75) and the integral of the squared FAS.

2.2 FAS contributions to 5% PSA

The sets of seed-based compatible time series previously generated were processed using band pass filters at cut-off frequencies: [0.1Hz 0.33Hz], [0.33Hz 3.33Hz], and [3.33Hz 10Hz]. The 5% PSA is then calculated for each of the filtered motions (Figure 2). It is seen that at low frequencies the 5% PSA amplitudes are mostly correlated to the FAS in that same frequency range. However, at higher frequencies the 5% PSA amplitudes takes contributions from a wider range of frequencies from the FAS, with the peak ground acceleration (PGA) being influenced by the whole FAS. Similar results were found by Bora et al. (2016) using RVT.

Figure 2. FAS contributions to 5% PSA for the time series compatible with RG 1.208 (rock).
3. DEVELOPMENT OF THE EMPIRICAL RELATIONSHIP

8 different response spectral shapes were used as target to generate sets of spectrum compatible synthetic time-series. The records are generated through iterative manipulations of the Fourier coefficients of an initial Gaussian noise. For each target spectrum 100 batches of 100 series are generated, that is, 10000 series were generated per target spectrum for a total of 80000 time series used in the study. Each record in a batch share the same total duration and similar $SD_{5-75}$, the total record durations start at 4s up to 202s in 2s steps, generating average batch $SD_{5-75}$ durations in the interval 1.6s to 68.1s. The response spectra used are presented in Figure 3 normalized to a maximum spectral amplitude of 1 to allow comparisons, it is seen that a large variety dominant frequencies and bandwidths are covered. This figure also shows the spectrum “NGA (mid)” (a deterministic spectrum for a magnitude 6.5 earthquake with a distance to rupture of 50 km) that is not used in the development of the relationship but later employed for validation purposes.

To develop the equation, the average ratios between FAS and 5% PSA are calculated for the 10000 synthetic time-series generated for each target spectrum. Top plots of Figure 5 show, for example, the average ratios between FAS and 5% PSA, per batch and over all, of the synthetic series compatible with RG 1.208. The average ratios from each batch in a target spectrum set is fitted to of the form $a*F^b$, where the coefficients $a$ and $b$ where found to depend on duration:

![Figure 3. Target spectra for the development of the compatible time-series.](image)

![Figure 4. Synthetic time-series compatible with RG 1.208 (rock): response spectra, mean batch FAS and normalized average batch FAS.](image)
where \( F \) is the frequency (in Hz) at which the ratio is evaluated. Suggested values for the coefficients \( a_i \) and \( b_i \) are obtained from the average of the values obtained for all of the 8 target spectral shapes evaluated: \( a_1 = 0.0512, a_2 = 0.4920, a_3 = 0.1123, b_1 = -0.5869, b_2 = -0.2650, b_3 = -0.4580 \). The use of average values for the coefficient is justified in Figure 5 (bottom plots), which show the predicted ratios for two different duration scenarios \( SD_{5-75} = 5s \) and \( SD_{5-75} = 50s \) using the coefficients obtained for each target spectrum as well as the obtained using the suggested average values. While the proposed equation is rather simple considering the complexity of relationship and it exhibits some conceptual problems at extreme frequency values, the following section shows that the results obtained are similar to the obtained using iterative approaches based on RVT.

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\frac{FAS}{5\% PSA} = (a_1 SD_{5-75}^{a_2} + a_3) F^{(b_1 SD_{5-75}^{b_2} + b_3)}
\]

Figure 5. Top: Average ratios between FAS and 5\% PSA, per batch and overall, of the synthetic series compatible with the RG 1.208 spectrum. Bottom: Predicted ratios for two different duration scenarios \( SD_{5-75} = 5s \) and \( SD_{5-75} = 50s \) using the coefficients obtained for each target spectrum and the proposed average coefficient values.

4. EVALUATION OF THE PROPOSED RELATIONSHIP

The proposed relationship is initially evaluated using the sets of 100 seed-based spectrum compatible time-series previously discussed (e.g. Figure 1). Selected results for individual records are presented in Figure 6 for records compatible with the RG 1.208 (rock) spectrum (Figure 3). This figure also shows the FAS obtained using the iRVT-based procedure described in Rathje et al (2005) and implemented in the computer code Strata (Kottke and Rathje 2008). For both procedures the input used consisted of each record PSA and strong motion duration (i.e., although very similar, individual records PSA are used rather than the target spectrum). It is seen that the results obtained using the proposed relationship match the overall behavior of the time-series FAS (presented FAS were interpolated at the same frequencies used to construct the PSA, therefore they may not exhibit the expected pronounced narrow troughs). The characteristic peaking behavior is not captured since the predicted FAS is estimated directly for the 5\% PSA, which shape is naturally smoother than the FAS. When a moving
average is applied to the time-series FAS, the resulting smoothed FAS is in close agreement with the FAS predicted by Equation 1. Moreover, the FAS predicted by the proposed equation are very similar to the predicted using the iRVT approach, with both methodologies exhibiting limited accuracy at high frequencies. This limitation on the iRVT procedure has already been reported in Al Atik et al (2014) when validating the response spectra-compatible FAS generated using iRVT with the FAS generated using single-corner-frequency point-source model.

A summary of the results obtained for all records compatible with RG 1.208 and NGA mid is presented in Figure 7. The level of match between the predicted FAS ($FAS_{p}$) and the actual records FAS ($FAS_{r}$) is measured using an inverse modified RMSE metric $imRMSE$ (Equation 2, Carlson et al., 2016) which values range from 0, for poor matches, to 1, for perfect matches.

$$imRMSE = \frac{1}{e^{\sqrt{\frac{\sum_{i=1}^{n} (FAS_{r_i} - FAS_{p_i})^2}{\sum_{i=1}^{n} FAS_{r_i}^2}}}}$$

(2)

It is seen that the level of match attained is independent of the spectral shape and duration of the records. Moreover, it is confirmed that the proposed relation fits fairly well the smoothed version of the records FAS and the level of match attained is comparable to the obtained using the iRVT approach. The figures on the right in Figure 7 display the average FAS for the 100 time-series in each set and the FAS predicted by Equation 1 and iRVT calculated both using the average 5%PSA and average significant duration of the series in each set. This would be a case similar to the RVT-based linear equivalent 1D site response (EQL) site response scenario, where a single compatible FAS is used as input rather than a set of compatible records as in the traditional time-histories EQL approach (e.g. Chi-Miranda and Montejo 2017a). It is seen that the average time-series FAS is matched better than each individual record FAS by the two approaches evaluated, in part because the peaking behavior tend to vanish when a large number of spectra are averaged.

The equation is also evaluated using actual recorded motions. For this purpose, the 300 historic records used as seed to generate the seed-based compatible time-series are engaged. Figure 8 presents the individual results for 4 of these records with different characteristics and Figure 9 summarizes the $imRMSE$ values obtained for all records. It is seen that the results obtained are very similar to the obtained for the spectrum compatible time-series, suggesting that the developed empirical relationship can also be used for recorded motions.
Figure 7. Left: $imRMSE$ for the FAS estimated via Eq.1 and iRVT using the signal actual FAS and a smoothed version of the FAS as references. Right: Mean of 100 seed-based spectrum compatible time series FAS along with the FAS predicted by Eq.1 and iRVT. Top to bottom: sets compatible with RG 1.208 and NGA (mid) spectra.

Figure 8. An example of the predicted FAS for 4 of the 300 recorded motions used to validate the proposed equation.
Figure 9. Summary of the imRMSE values obtained for the 300 recorded motions used to validate the proposed equation.

5. APPLICATION ON RVT-BASED SITE RESPONSE ANALYSES

Dynamic site response is usually implemented using an equivalent-linear approach through analyses based on time-histories or random vibration theory (RVT). In the RVT approach the input motion is characterized in the frequency domain by means of Fourier Amplitude Spectra (FAS) or Power Spectral Densities so that the need for selecting/developing multiple suitable time-histories is avoided. Nevertheless, the input FAS needs to be compatible with the prescribed uniform hazard spectrum (UHS) to comply with code requirements. The proposed equation can be used to obtain the spectrum compatible FAS from the UHS in a single step. Figure 10 shows the shear velocity profiles and elastic transfer functions for two significant different sites used as example. The amplification functions (AF, ratio between surface and outcrop rock response spectra) for both sites and 3 different intensity levels (PGAs at 0.01g, 0.3g and 0.6g) are presented in Figure 11 using the NGA(mid) spectrum as target input. The AFs presented correspond to the median AF from the time-histories (TH) based approach (using 100 seed-based spectrum compatible records) and the RVT methodology using 4 different methodologies to generate the input FAS: inverse RVT, damping modification factors - DMF (Hatzigeorgiou, 2010), average FAS from the input records and the empirical relationship. If the median from the TH approach is used as “target”, it is seen that, overall, all 4 RVT variations captured the behavior of the AFs independent of the level of inelastic demand. Some overprediction is evident at the sites natural frequencies, which has been shown to be an issue of the RVT methodology itself and not the way the input is defined (Wang and Rathje 2016, Chi-Miranda and Montejo 2017b). Moreover, the AFs obtained from the DMF methodology consistently over-predict the AFs at frequencies larger than ~5Hz.

Figure 10. Shear velocity profiles (a) and elastic transfer functions (b) for the 2 sites studied.
Figure 11. Amplification functions (AF) obtained using inputs compatible with the NGA spectrum. Top figures show the results for the SCH site at PGAs average values of 0.01g (a), 0.3g (b) and 0.6g (c). Bottom figures show the results for the and CC site at 0.01g (d), 0.3g (e) and 0.6g (f).

6. APPLICATION ON THE GENERATION OF SYNTHETIC ACCELERATION TIME SERIES

To address the RVT overprediction issue, Chi-Miranda and Montejo (2017b) proposed the use of a set of synthetic signals developed from Gaussian noise and molded to a FAS compatible to the design spectrum, the resulting signals share the same FAS and a 5% PSA similar to the target design spectrum. In this way, as in RVT-based procedures, the time-consuming and sometimes challenging task of developing a realistic set of seed-based spectrum-compatible time series is circumvented. However, different to RVT-procedures, the input is maintained in the time-domain, providing the possibility of using inverse Fourier transforms to obtain the ground surface response. Thus, avoiding the use of extreme value statistics and approximations known to induce over-predictions in the amplifications at the sites fundamental frequencies.

The procedure for the generation of the signals is as follows (refer to Figure 12, results shown are from the generation of a synthetic signal for the RG 1.208 spectrum): start by generating an initial noise signal (a), the amplitude of the initial noise is modulated using the a trapezoidal window (b), generate the FAS from the windowed noise (c), modify the windowed noise FAS to exactly match the target FAS (d, the target FAS is generated using Equation 1 along with the predefined design spectrum and strong motion duration), the synthetic signal is retrieved via IFFT (e), and finally verify the match with the target/design spectrum (f). It shall be noticed that the length of the signal and shape of the envelope should be defined so that the significant duration of the final signal coincide with the duration used to generate the compatible FAS. Since the signal would be further modified to shape its FAS, setting the length of the signal and envelope is not obvious, Chi-Miranda and Montejo (2017b) presents recommendations on this aspect.
Figure 12. Procedure for the generation of the synthetic signals (SS) with a FAS concurring with a predetermined (spectrum compatible) FAS.

Figure 13 show an example of the surface motions and response spectra obtained when this type of signals is used in EQL analyses. It is seen that while the input (bedrock) and surface time series exhibit non-realistic stationary characteristics, the response spectra obtained from such series behave as expected: that is, the spectra at the bedrock match the design spectrum and the spectra at the surface is close to the average from 20 analyses performed using seed-based spectrum compatible records. An extensive numerical study presented in Chi-Miranda and Montejo (2017b) shows that the amplification functions produced following this methodology are in closer agreement with the ones obtained using the seed-based time series as input, even in cases identified as critical for the RVT-based approach.
7. CONCLUSIONS

The relation between FAS and 5%PSA was found to be highly influenced by the duration of the signal and to correlate well with the strong motion significant duration $SD_{5\%}$. A recently developed duration-dependent empirical relationship between FAS and 5% PSA was presented and evaluated. It was found that the predicted FAS match the overall behavior of the records FAS. However, the characteristic FAS peaking behavior is not captured, as the behavior of the 5% PSA is rather smooth. Nevertheless, the predicted FAS is in closer agreement with the smoothed version of the record FAS and with an average FAS from a set of records sharing the same 5% PSA. Despite being developed based on compatible time-series, it was found that the relationship holds for actual recorded motions, providing results like the obtained using iterative approaches based on RVT theory. However, both methodologies exhibit limited accuracy at high frequencies.

An application was presented in the development of synthetic signals to use as input for EQL site response analyses. The signals are generated to have FAS that coincide with a prescribed spectrum compatible FAS developed using the discussed empirical equation. The results obtained agreed with the obtained using the seed-based time series approach required by most codes. It shall be noticed that the proposed synthetic signals are only intended to be used as input for equivalent-linear analysis, they are just an artifact to obtain random sets of complex Fourier coefficients sharing the same (spectrum-compatible) FAS and may not follow typical progression in characteristics of recorded motions. Therefore, time series developed in this manner are likely not applicable to nonlinear analysis with time varying properties.
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9. REFERENCES


