16TH EUROPEAN CONFERENCE ON EARTHQUAKE ENGINEERING THESSALONIKI 18 - 21 JUNE 2018

EVALUATION OF A MULTIMODE PUSHOVER PROCEDURE FOR TORSIONALLY FLEXIBLE R/C BUILDINGS UNDER BIAXIAL SEISMIC EXCITATION

Grigorios MANOUKAS[1]

ABSTRACT

Recently, a new multimode pushover procedure for the approximate estimation of the seismic response of asymmetric in plan buildings under biaxial seismic excitation has been developed. Its main idea is that the seismic response of an asymmetric multi-degree-of-freedom (MDOF) system with N degrees of freedom under biaxial excitation can be related to the responses of N 'modal' equivalent single-degree-of- freedom (E-SDOF) systems under uniaxial excitation. The preliminary evaluation of the proposed methodology for single- and multi-storey buildings led to quite satisfactory results. However, the evaluation studies conducted up to date are limited to torsionally stiff buildings. Hence, the objective of this paper is the further evaluation of the procedure for torsionally flexible systems. Firstly, the theoretical background and the assumptions of the proposed methodology are briefly outlined. Secondly, the sequence of steps to be followed for its implementation is systematically presented. The accuracy of the methodology is evaluated by a parametric study which comprises applications to single-storey asymmetric in plan R/C buildings. The whole investigation shows that, in general, the proposed methodology provides a reasonable estimation of the seismic response quantities.

Keywords: Pushover Analysis; Asymmetric Systems; Directional Superposition; Nonlinear Dynamic Analysis

1. INTRODUCTION

In recent years many researchers tried to establish simplified nonlinear analysis methods for the approximate estimation of the inelastic performance of buildings under seismic excitations. As a result of these efforts, Static Pushover Analysis (SPA) has been born. Initially, SPA has been developed in some more or less similar variants called 'conventional' procedures. SPA was shortly adopted by several seismic codes and prestandards (ASCE/SEI 41-06, ATC-40, Eurocode 8. etc.) under the name 'Nonlinear Static Procedure' (NSP) and became a very popular and useful tool for the earthquake resistant design of new, as well as the seismic rehabilitation of existing buildings.

However, this procedure involves many shortcomings and limitations. This is mainly due to the fact that the determination of the structure's response is based on several rough assumptions. In the case of asymmetric spatial systems under biaxial excitation additional errors occur due to the implementation of simplified directional combination rules in order to take into account multidirectional seismic effects. However, these rules are based on the superposition principle, while it is well known that this approach lacks a theoretical basis in the domain of inelastic response.

Recently, a new multimode pushover procedure for the approximate estimation of the seismic response of asymmetric in plan buildings under biaxial seismic excitation has been developed (Manoukas et al. 2012). Its main idea is that the seismic response of an asymmetric multi-degree-of-freedom (MDOF) system with N degrees of freedom under biaxial excitation can be related to the responses of N 'modal' equivalent single-degree-of-freedom (E-SDOF) systems under uniaxial

[1]Civil Engineer, PhD, Thessaloniki, Greece, grman7@otenet.gr

excitation. The whole procedure is quite similar to the well-known Modal Pushover Analysis (Chopra and Goel 2001) as extended for asymmetric buildings (Reyes and Chopra 2011a) (Reyes and Chopra 2011b). However, the establishment of the E-SDOF systems is based on an essentially different concept. In particular, the properties of the E-SDOF systems are determined by proper equations which take into account bidirectional seismic effects. The proposed methodology does not require independent analysis in each direction of excitation, so directional combination is avoided. The preliminary evaluation of the proposed methodology for single- and multi-storey buildings led to quite satisfactory results (Manoukas et al. 2012) (Manoukas and Avramidis 2014) (Manoukas and Avramidis 2015). However, the evaluation studies conducted up to date are limited to torsionally stiff buildings. Hence, the objective of this paper is the further evaluation of the procedure for torsionally flexible systems.
Firstly, the theoretical background and the assumptions of the proposed methodology are briefly outlined. Secondly, the sequence of steps to be followed for its implementation is systematically presented. The accuracy of the methodology is evaluated by a parametric study which comprises applications to single-storey asymmetric in plan R/C buildings. The whole investigation shows that, in general, the proposed methodology provides a reasonable estimation of the seismic response quantities.

2. THEORETICAL BACKGROUND

In principle, the proposed methodology is based on the following fundamental assumptions (Manoukas and Avramidis 2015):

- The seismic response of a MDOF system can be expressed as superposition of the responses of appropriate SDOF systems just like in the linear range.
- Each SDOF system corresponds to a vibration ‘mode’ i with ‘modal’ vector $\boldsymbol{\varphi}_i$ (the quotation marks indicate that the application of the superposition principle is not strictly valid).
- The displacements $\mathbf{u}_i$ and the inelastic resisting forces $\mathbf{F}_{si}$ are supposed to be proportional to $\boldsymbol{\varphi}_i$ and $\mathbf{M}\boldsymbol{\varphi}_i$, respectively (where $\mathbf{M}$ is the mass matrix).
- The ‘modal’ vectors $\boldsymbol{\varphi}_i$ are supposed to be constant, despite the successive development of plastic hinges.
- It is supposed that Rayleigh damping is present.

Of course, such assumptions violate the very logic of nonlinearity, as the superposition principle does not hold for nonlinear systems. However, keeping always in mind that our main intention is the development of an approximate simplified procedure, the recourse to these assumptions is inevitable. They must be thought as a fundamental postulate, which constitutes the basis on which many simplified pushover procedures are built (Manoukas et al. 2011).
The only additional assumption introduced is that the two horizontal seismic components $\ddot{u}_g(t)_X$ and $\ddot{u}_g(t)_Y$ are proportional to each other, i.e.:

$$\ddot{u}_g(t)_Y = \kappa\ddot{u}_g(t)_X = \kappa\ddot{u}_g(t) \tag{1}$$

where κ is a constant factor. Of course, this is not true for recorded ground motions. However, this approximation is in accordance with the very common assumption adopted by seismic codes which specify that - within the framework of NSP as well as the linear analysis methods - the two horizontal seismic components are represented by the same design spectrum, while directional combination may be conducted using the percentage combination rule (e.g., ASCE/SEI 41-06, Section 3.2.7.1) which implies a constant factor (0.3) similar to κ. Obviously, the evaluation of this assumption, as well as the definition of specific values of κ is beyond the objective of the present study.
Given the aforementioned assumptions, the nonlinear response of an L-story MDOF system with N degrees of freedom (in the usual case of rigid diaphragms N = 3L) to a biaxial earthquake ground motion ($\ddot{u}_g(t)_X$ and $\ddot{u}_g(t)_Y = \kappa\ddot{u}_g(t)_X = \kappa\ddot{u}_g(t)$ along X and Y axes, respectively) is described by the following equation (for the sake of simplicity (t) is left out in all following expressions) (Manoukas et al. 2012):

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{F_s} = -\mathbf{M}(\boldsymbol{\delta}_{,X} + \kappa\boldsymbol{\delta}_{,Y})\,\ddot{u}_g \Rightarrow \mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{F_s} = -\mathbf{M}\boldsymbol{\delta}_{,XY}\,\ddot{u}_g \quad (2)$$

where $\mathbf{u}$, $\dot{\mathbf{u}}$, $\ddot{\mathbf{u}}$ are the displacement, velocity and acceleration vectors of order N, $\mathbf{M}$ is the $N\times N$ diagonal mass matrix, $\mathbf{C}$ is the $N\times N$ symmetric damping matrix, $\mathbf{F_s}$ the resisting forces vector and $\boldsymbol{\delta}_{,X}$, $\boldsymbol{\delta}_{,Y}$ are the influence vectors that describe the influence of support displacements on the structural displacements for independent uniaxial horizontal seismic excitations along X and Y axes, respectively. Vector $\mathbf{u}$ is written as follows:

$$\mathbf{u} = [\mathbf{u}_X, \mathbf{u}_Y, \boldsymbol{\theta}_Z]^T \quad (3)$$

where $\mathbf{u}_X$, $\mathbf{u}_Y$, $\boldsymbol{\theta}_Z$ are the vectors of order L of displacements along X axis, along Y axis and rotations around Z (vertical) axis, respectively. The influence vectors $\boldsymbol{\delta}_{,X}$ and $\boldsymbol{\delta}_{,Y}$ are:

$$\boldsymbol{\delta}_{,X} = [\mathbf{I}, \mathbf{0}, \mathbf{0}]^T \quad (4)$$

$$\boldsymbol{\delta}_{,Y} = [\mathbf{0}, \mathbf{I}, \mathbf{0}]^T \quad (5)$$

where $\mathbf{I}$, $\mathbf{0}$ are vectors of order L with each element equal to unity and zero, respectively. Due to the aforementioned assumptions, vectors $\mathbf{u}$ and $\mathbf{F_s}$ can be expressed as the sum of the 'modal' contributions (Anastassiadis 2004), (Chopra 2007):

$$\mathbf{u} = \sum_{i=1}^{N} \mathbf{u}_i = \sum_{i=1}^{N} \boldsymbol{\varphi}_i q_i\,, \quad (6)$$

$$\mathbf{F_s} = \sum_{i=1}^{N} \mathbf{F}_{si} = \sum_{i=1}^{N} \alpha_i \mathbf{M}\boldsymbol{\varphi}_i \quad (7)$$

where α_i is a hysteretic function that depends on the 'modal' co-ordinate q_i and the history of excitation (Anastassiadis 2004). By substituting Equations 6 and 7 into Equation 2 and applying well-known principles of structural dynamics, N uncoupled equations can be derived, each one corresponding to an E-SDOF system (Manoukas et al. 2012):

$$M^*_{XYi}\,\ddot{D}_i + 2\,M^*_{XYi}\,\omega_i\zeta_i\,\dot{D}_i + V_{XYi} = -\,M^*_{XYi}\,\ddot{u}_g \quad (8)$$

where $D_i = q_i / \nu_{XYi}$, $\dot{D}_i$, $\ddot{D}_i$ the displacement, velocity and acceleration of the i^{th} (i = 1…N) E-SDOF system, ω_i and ζ_i are the natural frequency and damping ratio of the elastic vibration mode i and:

$$V_{XYi} = V_{Xi} + \kappa V_{Yi} \quad (9)$$

$$M^*_{XYi} = M^*_{Xi} + \kappa(\nu_{Xi}\,L_{Yi} + \nu_{Yi}\,L_{Xi}) + \kappa^2 M^*_{Yi} \quad (10)$$

$$\nu_{XYi} = \nu_{Xi} + \kappa\nu_{Yi} \quad (11)$$

where V_{Xi}, V_{Yi} are the 'modal' base shears parallel to X and Y axes respectively, M^*_{Xi}, M^*_{Yi} and ν_{Xi}, ν_{Yi} are the effective modal masses and the modal participation factors of the elastic vibration mode i due to independent uniaxial excitations along X and Y axes respectively, while $L_{Xi}=\boldsymbol{\delta}_{,X}^T\mathbf{M}\boldsymbol{\varphi}_i$ and $L_{Yi}=\boldsymbol{\delta}_{,Y}^T\mathbf{M}\boldsymbol{\varphi}_i$.

Equation 8 shows that, due to the aforementioned assumptions, the nonlinear response of a MDOF system with N degrees of freedom subjected to a biaxial seismic excitation $\ddot{u}_{gX}$ and $\ddot{u}_{gY} = \kappa\ddot{u}_{gX} = \kappa\ddot{u}_g$ along X and Y axes, respectively, can be expressed as the sum of the responses of N SDOF systems under uniaxial excitation $\ddot{u}_g$, each one corresponding to a vibration 'mode' having mass equal to

M^*_{XYi}, displacement equal to D_i and inelastic resisting force equal to V_{XYi}, i.e. the sum of 'modal' base shear parallel to X axis plus 'modal' base shear parallel to Y axis multiplied by κ (see Equation 9) (Manoukas et al. 2012).

3. THE PROPOSED METHODOLOGY

The application process of the proposed methodology resembles the one of MPA. However, the definition of the E-SDOF systems is essentially different, in order to take into account multidirectional seismic effects. In Table 1 the properties of the i^{th} 'modal' E-SDOF system are tabulated, along with the properties that it would have in case of uniaxial excitation (parallel to X axis).

Table 1. Properties of the i^{th} E-SDOF system

Property	Uniaxial excitation $\ddot{u}_{gX}$	Biaxial excitation $\ddot{u}_{gX} + \kappa\ddot{u}_{gY}$
Mass	M^*_{Xi}	$M^*_{XYi} = M^*_{Xi} + \kappa(\nu_{Xi} L_{Yi} + \nu_{Yi} L_{Xi}) + \kappa^2 M^*_{Yi}$
Resisting force	V_{Xi}	$V_{XYi} = V_{Xi} + \kappa V_{Yi}$
Displacement	$D_i = u_{Ni} / \nu_{Xi}\varphi_{Ni}$ (roof displacement u_{Ni})	$D_i = u_{Ni} / \nu_{XYi}\varphi_{Ni} = u_{Ni}/(\nu_{Xi} + \kappa\nu_{Yi})\varphi_{Ni}$ (roof displacement u_{Ni})
Damping factor	$2 M^*_{Xi}\omega_i\zeta_i$	$2 M^*_{XYi}\omega_i\zeta_i$

The proposed methodology should be implemented for all possible combinations of the seismic components. In particular, the following four combinations should be examined:

$$\ddot{u}_{gX} + \kappa\ddot{u}_{gY} \tag{12}$$

$$\ddot{u}_{gX} - \kappa\ddot{u}_{gY} \tag{13}$$

$$\ddot{u}_{gY} + \kappa\ddot{u}_{gX} \tag{14}$$

$$\ddot{u}_{gY} - \kappa\ddot{u}_{gX} \tag{15}$$

The equations derived by the process presented in the previous paragraphs have to be modified proportionately for each combination. It can be easily proved - by simple implementation of the process - that the consideration of the four combinations with opposite sign (e.g., $-\ddot{u}_{gX} - \kappa\ddot{u}_{gY}$ instead of $\ddot{u}_{gX} + \kappa\ddot{u}_{gY}$) leads to identical properties for the E-SDOF systems, so they can be skipped.

The steps needed for the implementation of the proposed methodology are as follows (Manoukas et al. 2012):

Step 1: Create the structural model.

Step 2: Calculate ν_{XY1} (Equation 11) and M^*_{XYi} (Equation 10) of the fundamental elastic vibration mode 1 for the first combination of seismic components ($\ddot{u}_{gX} + \kappa\ddot{u}_{gY}$).

Step 3: Apply to the structural model a set of lateral incremental forces (and moments) proportional to the vector $\mathbf{M}\boldsymbol{\varphi}_1$ of the fundamental elastic vibration mode 1 and determine the (resisting force)-(displacement) curve V_{XY1}-u_{N1} of the MDOF system. u_{N1} can be chosen to correspond to any degree of freedom, but usually the roof displacement parallel to X or Y axis is used.

Step 4: Divide the abscissas of the V_{XY1}-u_{N1} diagram by the quantity $\nu_{XY1}\varphi_{N1} = u_{N1}/D_1$ and determine the (resisting force)-(displacement) curve V_{XY1}-D_1 of the E-SDOF system.

Step 5: Idealize V_{XY1}-D_1 to a bilinear curve using one of the well known graphic procedures (e.g., ASCE/SEI 41-06, Section 3.3.3.2.5) and calculate the period T_1 and the yield strength reduction factor R_1 of the E-SDOF system corresponding to mode 1, from the following equation:

$$T_1 = 2\pi \sqrt{\frac{m_1 D_{y1}}{V_{y1}}} \rightarrow S_{a1}(T) \rightarrow R_1 = \frac{m_1 S_a(T)}{V_{y1}} \quad (16)$$

where $m_1 = M^*_{XYi}$, D_{y1}, V_{y1} are the mass, the yield displacement and the yield strength of the system, respectively, and $S_{a1}(T)$ is the spectral acceleration.
Step 6: Calculate the target displacement of mode 1 using one of the well known procedures of displacement modification (e.g., ASCE/SEI 41-06, Section 3.3.3.3.2 / FEMA 440, Section 10.4). If the procedure is applied for research purposes using recorded earthquake ground motions, it is recommended to estimate the inelastic displacement of the E-SDOF system by means of nonlinear dynamic analysis, instead of using the relevant coefficients (e.g., C_1 in ASCE/SEI 41-06 and FEMA 440). This is due to the fact that the coefficient values given by codes are based on statistical processing of data with excessive deviation and, therefore, great inaccuracies may result (Manoukas et al. 2006).
Step 7: Calculate the 'modal' values of the other response quantities of interest (drifts, plastic rotations, etc.) of mode 1 by conducting pushover analysis up to the already calculated target displacement.
Step 8: Repeat steps 3 to 7 applying the incremental forces (and moments) in the opposite direction.
Step 9: Repeat steps 2 to 8 for an adequate number of modes.
Step 10: Calculate the extreme values of response parameters by utilizing one of the well established formulae of modal superposition (SRSS or CQC).
Step 11: Repeat steps 2 to 10 for all possible combinations of the two horizontal components of the seismic excitation (Equations 12, 13, 14 and 15).

4. EVALUATION STUDY

4.1 Structural Models

The implementation of the proposed methodology to torsionally stiff buildings produced quite satisfactory results (Manoukas et al. 2012) (Manoukas and Avramidis 2014) (Manoukas and Avramidis 2015). In the present study a parametric study is carried out comprising applications to torsionally flexible and torsionally similarly stiff buildings too. In particular, three models are designed according to the Greek codes for the seismic hazard level zone I of the Greek territory (peak ground acceleration equal to 0.16g):

- Model 'ts' (torsionally stiff): An asymmetric reinforced concrete single-storey building with plan given in Figure 1. Its structural system consists of moment frames and the storey height is 3m. The concrete is of class C20/25 (f_{ck}=20 MPa) and the steel bars S500 (f_{yk}=500 MPa) according to the Greek standards. The building posses a mass of 58 t and a mass moment of inertia equal to 720.17 tm^2. Lateral displacements dominate the motion in the first two vibration modes, so this building is characterized as 'torsionally stiff' (Chopra and Goel 2006).
- Model 'tss' (torsionally similarly stiff): A single-storey building identical to model 'ts' except that the mass moment of inertia is multiplied by a factor of 2. In the first two vibration modes of this building lateral and torsional motions are strongly coupled, so it can be characterized as 'torsionally similarly stiff' (Chopra and Goel 2006).
- Model 'tf' (torsionally flexible): A single-storey building identical to model 'ts' except that the mass moment of inertia is multiplied by a factor of 4. Torsion dominates the motion in the first mode, so this building can be characterized as 'torsionally flexible' (Chopra and Goel 2006).

The natural periods of the three examined models are tabulated in Table 2, while the modal participating mass ratios are shown in Table 3.

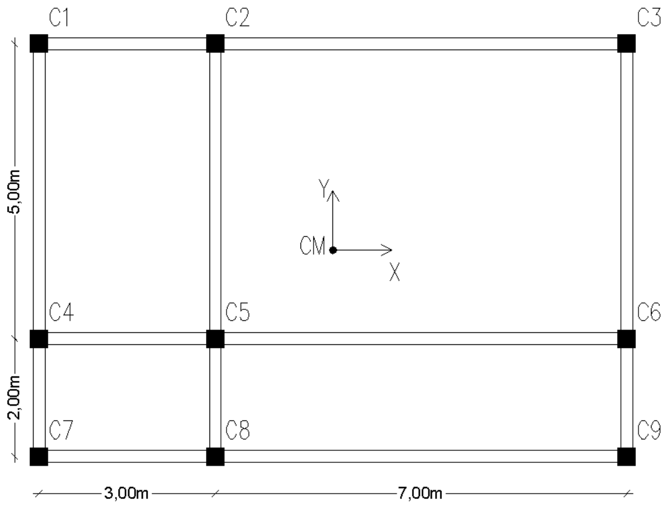

Figure 1. Plan of the analyzed buildings

Table 2. Natural periods of the examined models

Building models	Natural periods (sec)		
	T_1	T_2	T_3
Model 'ts'	0.186	0.182	0.122
Model 'tss'	0.195	0.182	0.165
Model 'tf'	0.253	0.183	0.179

Table 3. Modal participating mass ratios of the examined models

Building models	Modal participating mass ratios (%)					
	Mode 1		Mode 2		Mode 3	
	X	Y	X	Y	X	Y
Model 'ts'	57.1	39.5	41.7	58.1	1.2	2.4
Model 'tss'	27.3	35.8	61.3	38.3	11.4	25.9
Model 'tf'	2.1	3.5	85.7	13.9	12.2	82.6

All analyses are performed using the program SAP 2000. The modeling of the inelastic behaviour is based on the following assumptions:

- Shear failure is precluded.
- The inelastic deformations are concentrated at the critical sections, i.e. at the ends of the frame elements (plastic hinges).
- Plastic hinges are modeled by bilinear elastic-perfectly plastic moments-rotations diagrams with practically unlimited available plastic rotations and yield moments calculated automatically by the program.
- The moment-axial force interaction is taken into account by appropriate interaction surface incorporated in SAP 2000.

4.2 Earthquake Excitations

The whole investigation conducted here comprises a number of 20 accelerograms, which is considered adequate to obtain concrete conclusions. These accelerograms are obtained from the PEER strong motion database and tabulated in table C-3 of Appendix C of FEMA 440 project. The accelerograms are scaled to peak ground acceleration equal to 0.32g in order to ensure that the analyzed buildings undergo extensive nonlinear deformations for all excitations. It is considered that each ground motion acts simultaneously along the two horizontal axes of the buildings with the same intensity.

4.3 Analysis Process

For each building two sets of pushover analyses are performed:

- One based on the proposed methodology (PM). Given that each ground motion acts simultaneously along the two horizontal axes with the same intensity, i.e. $\kappa=1$ and $\ddot{u}_{gX} = \ddot{u}_{gY}$, the possible combinations of the seismic components are only two: $\ddot{u}_{gX} + \ddot{u}_{gY}$ (PM+) and $\ddot{u}_{gX} - \ddot{u}_{gY}$ (PM-).
- A second similar to MPA (conventional procedure - CP), which comprises pushover analyses of the buildings for independent uniaxial excitations along X and Y axes and directional combination of the response quantities using the percentage combination rule. The assumptions and steps of the second procedure are nearly identical to those of the proposed method, except that step 11 is obviously skipped and in steps 2 to 4 v_{Xi}, M^*_{Xi}, V_{Xi} or v_{Yi}, M^*_{Yi}, V_{Yi} are used in place of v_{XYi}, M^*_{XYi}, V_{XYi}.

In both sets of pushover analyses all the three vibration modes are taken into account. The ‘modal’ superposition is conducted by applying the CQC formula. The maximum ‘modal’ response of each E-SDOF system is calculated by means of nonlinear dynamic analysis for each excitation. Then, the target roof displacement is estimated by multiplication of the resulting response by the quantity $v_{XYi}\varphi_{Ni}$ (PM) and $v_{Xi}\varphi_{Ni}$ or $v_{Yi}\varphi_{Ni}$ (CP). For each building, the storey displacements at the center of mass (CM), at the flexible side (C3) and at the stiff side (C7) of the plan are determined.

The response values obtained by the two variants of pushover analysis are compared to the results of nonlinear dynamic analysis, which is considered as the reference solution. For the latter analysis, each accelerogram is considered acting simultaneously along the two horizontal axes in all possible combinations ($\ddot{u}_{gX} + \ddot{u}_{gY}$, $\ddot{u}_{gX} - \ddot{u}_{gY}$, $-\ddot{u}_{gX} + \ddot{u}_{gY}$ and $-\ddot{u}_{gX} - \ddot{u}_{gY}$). For each response parameter $R_{j,s}$ estimated by the two applied variants of pushover analysis for an excitation j, the error with regard to the nonlinear dynamic analysis results E_j is determined by the following relation:

$$E_j\,(\%) = 100\frac{R_{j,s} - R_{j,d}}{R_{j,d}} \tag{17}$$

where $R_{j,d}$ is the value of the response parameter obtained by the nonlinear dynamic analysis. Furthermore, the mean error ME for the 20 excitations used in this study is determined using Equation 18:

$$ME\,(\%) = \frac{1}{20}\sum_{j=1}^{20} E_j = 100\frac{1}{20}\sum_{j=1}^{20}\left(\frac{R_{j,s} - R_{j,d}}{R_{j,d}}\right) \tag{18}$$

4.4 Results

In Tables 4, 5 and 6 the mean errors of PM and CP for the 20 excitations (referring to the maximum values obtained by nonlinear dynamic analysis) of storey displacements at the selected points (CM – center of mass, C3 – flexible side, C7 – stiff side) along X and Y axis are shown. Notice that the positive sign (+) means that response parameters obtained by PM or CP are greater than those obtained by nonlinear dynamic analysis. Conversely, the negative sign (-) means that the response parameters are underestimated. It is apparent that the two combinations of PM (PM+ and PM-) provide an upper bound and a lower bound value for each response parameter. The exact value (nonlinear dynamic analysis) in most cases lies in this range. For the vast majority of response parameters PM leads to conservative results. The mean errors of the more conservative combination range between -4% and 74%. In comparison with CP, the absolute values of mean errors resulting from PM are smaller for all response parameters. Mean errors of CP range between 51% and 146%. Comparing the three models, it seems that no particular trend for the accuracy of the proposed procedure can be identified, i.e. in general the proposed procedure can provide quite satisfactory results regardless the torsional stiffness of buildings.

Table 4. Mean errors of displacements resulting from pushover analyses – Model 'ts'

Points	Mean errors (%)					
	PM+		PM-		CP	
	X	Y	X	Y	X	Y
CM	-23.72	16.47	23.21	59.24	87.11	145.72
C3	-31.27	-2.12	71.70	73.85	120.96	128.79
C9	5.73	4.72	-3.53	-21.95	96.42	81.98

Table 5. Mean errors of displacements resulting from pushover analyses – Model 'tss'

Points	Mean errors (%)					
	PM+		PM-		CP	
	X	Y	X	Y	X	Y
CM	-4.20	-0.65	-28.05	16.45	52.95	91.03
C3	-10.28	-15.28	19.49	43.18	90.80	99.14
C9	20.97	-3.74	-24.25	21.86	66.75	78.27

Table 6. Mean errors of displacements resulting from pushover analyses – Model 'tf'

Points	Mean errors (%)					
	PM+		PM-		CP	
	X	Y	X	Y	X	Y
CM	33.69	15.58	-2.96	39.62	59.61	73.89
C3	24.53	1.07	-11.64	17.01	50.64	55.88
C9	48.48	13.87	30.11	71.53	90.54	91.17

5. CONCLUSIONS

In this paper, a recently developed multimode pushover procedure for the approximate estimation of the seismic response of asymmetric buildings under biaxial excitation is evaluated for buildings with different torsional properties. The main idea of the procedure is that the seismic response of an asymmetric building under biaxial excitation can be related to the responses of a series of "modal" E-SDOF systems under uniaxial excitation. The whole procedure is quite similar to the well-known MPA. However, the establishment of the E-SDOF systems is based on an essentially different concept. From the evaluation of the methodology some conclusions derived from previous studies (Manoukas et al. 2012) (Manoukas and Avramidis 2014) (Manoukas and Avramidis 2015) are verified:

- The proposed methodology provides for each response parameter an upper limit and a lower limit which in the vast majority of cases envelope the corresponding value obtained by nonlinear dynamic analysis.

- In general, the mean errors with regard to the nonlinear dynamic analysis results are smaller than those resulting from a multimode pushover procedure comprising independent analysis along two horizontal axes and directional combination of the results.

Furthermore, it seems that there is not any particular diversification of the accuracy of the methodology when it is applied to buildings with different torsional properties.

Finally, it is worth noticing that despite the fact that no restrictions are set to the development of the proposed methodology, generalization of the above conclusions for all types of asymmetric buildings requires further investigations, comprising application to a large variety of spatial structures and using an adequately high number of earthquake ground motions.

6. REFERENCES

American Society of Civil Engineers (2008). Seismic rehabilitation of existing buildings, ASCE/SEI 41-06 Standard.

Anastassiadis KK (2004). Approximate methods for the estimation of the inelastic response of buildings. Lecture notes (in Greek). Aristotle University, Thessaloniki.

Applied Technology Council (ATC) (1996). Evaluation and Retrofit of Concrete Buildings. Vol. 1, ATC-40, Applied Technology Council, Redwood City, California.

Chopra AK (2007). Dynamics of Structures – Theory and Applications to Earthquake Engineering. 3rd ed. Pearson Prentice Hall, New Jersey.

Chopra AK, Goel RK (2001). A Modal Pushover Analysis Procedure to estimating seismic demands of buildings: theory and preliminary evaluation. PEER Report 2001/03. Pacific Earthquake Engineering Research Center, University of California, Berkeley.

Chopra AK, Goel RK (2006). Evaluation of the Modal Pushover Analysis Procedure for unsymmetric-plan buildings. *Proceedings of the1st European Conference on Earthquake Engineering and Seismology,* September 3-8, Geneva, Switzerland.

European Committee for Standardization (2004). Eurocode 8: Design of structures for earthquake resistance. EUS, Brussels.

Federal Emergency Management Agency - Applied Technology Council (ATC) (2004) Improvement of Nonlinear Static Seismic Analysis Procedures. Report No. ATC-55 (FEMA 440).

Manoukas GE, Athanatopoulou AM, Avramidis IE (2006). Comparative evaluation of static pushover analysis' variations according to modern codes, *Proceedings of the 15th Hellenic Conference on R/C structures*, October 25-27, Alexandroupoli, Greece.

Manoukas GE, Athanatopoulou AM, Avramidis IE (2011). Static Pushover Analysis Based on an Energy-Equivalent SDOF System. *Earthquake Spectra*, 27(1): 89-105.

Manoukas GE, Athanatopoulou AM, Avramidis IE (2012). Multimode Pushover Analysis for Asymmetric Buildings under Biaxial Seismic Excitation Based on a new Concept of the Equivalent Single Degree of Freedom System. *Soil Dynamics and Earthquake Engineering*, 38: 88-96.

Manoukas GE, Avramidis IE (2014). Evaluation of a Multimode Pushover Procedure for Asymmetric in Plan Buildings under Biaxial Seismic Excitation. *Bulletin of Earthquake Engineering*, 12(6): 2607-2632.

Manoukas GE, Avramidis IE (2015). Improved Multimode Pushover Procedure for Asymmetric in Plan Buildings under Biaxial Seismic Excitation – Application to Tall Buildings. *The Structural Design of Tall and Special Buildings*, 24(6): 397-420.

Pacific Earthquake Engineering Research Centre (PEER) (2003). Strong Motion Database http://peer.berkeley.edu/smcat/. Accessed 1 July 2017

Reyes JC, Chopra AK (2011a). Three dimensional modal pushover analysis of buildings subjected to two components of ground motion, including its evaluation for tall buildings. *Earthquake Engineering and Structural Dynamics*, 40: 789-806.

Reyes JC, Chopra AK (2011b). Evaluation of three-dimensional modal pushover analysis for unsymmetric-plan buildings subjected to two components of ground motion. *Earthquake Engineering and Structural Dynamics*, 40: 1475-1494.