AN INTRODUCTION TO DESIGN-LED ANALYSIS OF EARTHQUAKE RESISTANT MOMENT FRAMES

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ABSTRACT

The paper presents Design-led Analysis (DLA) as a design philosophy that aims at the minimizing response is considered a natural occurrence rather than numerical result, and structural behavior is looked upon as a function of design rather than analytical output. In the interim, the use of Load-Displacement Interaction Diagrams (LDID) has been introduced as a means of correlating ultimate load combinations to maximum displacements at incipient collapse. The LDID is an upgrading of the classical plastic load interaction plot. The proposed methodology is ideally suited for preliminary design purposes.

Keywords: Moment Frames; Plastic Design; Lateral Displacements; Seismic Loading; Interaction Diagrams.

1. INTRODUCTION

The ability to estimate displacements at collapse can help exercise greater control over the response of MFs at several target drift ratios (Hayman 1961). As implicit in all design codes, the proper estimation of maximum seismic drift at incipient collapse is the focal aspect of performance based design. However, while most steel design codes advocate nonlinear/plastic performance, they do not implicitly relate plastic response to plastic displacements at ultimate loading. While the importance of the strong column-weak beam condition is strongly emphasized the effects of sequences of formation of plastic hinges has not been addressed. The option not to include elastoplastic deformations at incipient collapse can lead to significant underestimation of target based response of any structure beyond first yield. Such procedures may be shown to be deficient in yet another sense; the uncontrolled sequence of formation of the plastic hinges may initiate premature failure mechanisms conditions (Naeim 2001). The elastoplastic deformations of engineering systems depend upon both the strength as well as the stiffness of their members. By the same token the corresponding maximum displacements at incipient collapse depend upon the sequence of formation of plastic hinges as well as the strength and stiffness of the last remaining, stable segments of the structure. The proposed methodologies offer a rational basis for the accurate selection of RBS values for controlling sequences of formations of plastic hinges in all types of MFs (Taranath 1998).

2. BASIC DESIGN CONDITIONS

Consider the MF of Figure 1c. It is required to propose a design strategy that satisfies the prescribed yield criteria, static equilibrium and boundary support conditions, as well as the stipulations, that;

• The structure is expected to undergo relatively large inelastic displacements.

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The sequences of formation of plastic hinges are to be controlled (no hinges at column supports).
The structure is to fail through a purely sway mechanism.
The total weight of the structure is to be a minimum.
The target story level drift at incipient collapse is not exceeded.
Distribution of lateral drift over the structure’s height is to be relatively uniform.
All members shall be designed in such a way as to allow formation of plastic hinges as required.
The strong column-weak beam criterion is to be satisfied throughout the structure.
The racking P-delta effects are to be taken into consideration.

Figure 1 (a) Loading, (b) Drift (c) MF, (d) Modularized frame, (e) Fixed base module, (f) Preferred mode.

The MF of Figure 1d is supported on fixed base portal frames, Figures 1e and 2a. The premise of the concept is that if (Grigorian 2011) each module of the MF can be designed as an optimum weight sub-system, then it should be possible to regenerate the entire structure as a MF of minimum weight. Coordinates \( i = 0, 1, 2 \ldots m \), and \( j = 0, 1, 2 \ldots n \) of Figure 1c are meant to identify the elements of the MF. The approach adapted in this paper is different in that, instead of investigating member suitability on individual basis, the initial design is based on the selection of groups of similar modules such as those shown in Figure 1d that fit into the bays of the original MF, Figure 1c, and satisfy the same conditions as prescribed for the entire structure. The target drift is enforced rather than investigated. The proposed methodology is presented in three steps; First, the elastoplastic response of a fixed base portal frame, Figures 1e and 2a, and a closed loop rectangular module, Figures 1g and 5a, under gravity and lateral forces are studied in terms LDIDs, Figures 2b and 5b. Each LDID and its equations describe the response of the modules through all phases of loading. Next the stipulated number of modules is arranged in horizontal series and merged together to reassemble the sub-frames of the original structure. See Figures 4 and 6. The strength and stiffness of each sub-frame is determined in such a way as to satisfy elastoplastic compatibility along all merging members.

2.1 Plastic response of fixed base portal frames
The relationships between the ultimate capacities and lateral displacements of fixed base portal frames, Figure 2a, under constant gravity load \( W \), constant vertical loads \( P \), and monotonically increasing lateral forces \( V \) can be studied by the use of a generalized LDID such as that shown in Figure 2b. The LDID of Figure 2b is composed of four distinct load-interaction ranges 1, 2, 3 and 4 and three distinct maximum plastic displacement ranges, \( \alpha, \alpha' \) and \( \beta \). Both the load interaction regions and the plastic response ranges are influenced by the relative magnitudes of the applied forces at collapse. Limit state load combination regions 1 and 2, 3 and 4 of Figure 2b describe beam, combined beam and sway and sway and sway mechanisms respectively. The auxiliary range indicators \( \alpha, \alpha' \)
ranges and beam mechanisms respectively. Ranges 1 and 2, from Figure 2a, are capable of rotating through a full reversal of plastic moments from $+M^p$ to $-M^p$ before complete collapse, it can be deducted that; $\delta = \frac{M^p}{EJ} \cdot \alpha$ and $\beta = \frac{2Vh}{M^p}$ at reference points $b$ and $c$ of the LDID respectively. Similarly, $M^B = aM^p$ along ranges 3, 4, and $\overline{\alpha}$ is the ratio of the maximum simply supported bending moment of the beam to the 2/3 of its plastic moment of resistance, then by investigating the free body equilibrium of any combined beam-sway mechanism at incipient collapse, it may be deduced that; $\alpha = \frac{Vh}{f_{cr}M^p} - (2\lambda + 1)$ and $\overline{\alpha} = 3WL/8M^P$

along lines cde and efg of the LDID respectively. Ranges $\alpha$, $\overline{\alpha}$ and $\beta$ correspond to combined, sway and beam mechanisms respectively. Ranges $\alpha = +1$ and $\overline{\alpha} = 0$ coincide at mode transition point $c$ and ranges $\alpha = +1$ and $\overline{\alpha} = 0$ coincide at point $g$. The displacements of the various segments of the LDID in terms of $\alpha$, $\overline{\alpha}$, $\beta$, $f_{cr} = 1 - \mu$ and the limiting load combinations have been derived as follows;

In the purely beam mechanism range 1, from $a$ to $b$; where $\delta_1$ is independent of $\alpha$; as;

$$\delta_1 = \frac{(2\lambda - 1)\beta M^p h^2}{6EJ},$$

$$W = \frac{8M^p}{L},$$

$$V_1 = \frac{(2\lambda - 1)f_{cr}M^p}{h}, \quad 0 \leq \beta \leq 1, \quad \mu = \frac{\beta Ph^2}{3EI} < 1,$$

$$\beta = 1$$  \hspace{1cm} (2)

In the extended beam mechanism range 2, from $b$ to $c$; where $\delta_2$ is still independent of $\alpha$; as;

$$\delta_2 = \frac{(2\lambda + 2)\beta M^p h^2}{6EJ},$$

$$W = \frac{8M^p}{L},$$

$$V_2 = \frac{(2\lambda - 1)M^p}{f_{cr}h}, \quad \frac{V_2}{2\lambda M^p} \leq \frac{2\lambda M^p}{h}, \quad 1 \leq \beta \leq 2$$

and

$$\mu = \frac{(2\lambda + 2)Ph^2}{(2\lambda + 2)EI} < 1,$$

$$V_3 = \frac{2\lambda M^p}{h}, \quad \text{at } \beta = 2$$  \hspace{1cm} (3)

Figure 2, (a) Fixed base portal frame geometry and loading, (b) Load-displacement interaction diagram.
In the combined beam-sway mechanism range 3, from c to e, $\delta_3$ is also independent of $\rho$, thus;

$$\delta_3 = \frac{(2\lambda - \alpha)M^p h^2}{6EJ} \quad \text{for} \quad \frac{8M^p}{L} \geq \frac{W}{4L} \quad \text{and} \quad \frac{2\lambda M^p h}{f_{cr}} \geq \frac{V_1}{f_{cr}} - \frac{V_2 h + WL}{f_{cr}} = \frac{2(2 + \lambda)M^p}{2},$$

$$-1 \leq \alpha \leq 1, \quad \text{and} \quad \left[ \mu = \frac{(2\lambda - \alpha)P h^2}{3(2\lambda + \alpha + 1)EJ} \right] < 1 \quad \text{with} \quad \frac{V_3}{f_{cr}} = \frac{2(\lambda + 1)M^p h}{f_{cr}} = \frac{2(2 + \lambda)M^p}{2} \quad \text{at} \quad \alpha = 1. \quad (4)$$

In the sway mechanism of range 4, corresponding to $\frac{3}{2} \geq \alpha \geq 0$, from e to g, where

$$\frac{4M^p}{L} > W \geq 0 \quad \text{and} \quad \frac{V_4}{f_{cr}} = \frac{2(\lambda + 1)M^p}{h}$$

two distinct possibilities exist. If the last plastic hinge forms at the beam column junction B, then;

$$\delta_{4,l} = \left[ 2 - \lambda + \frac{1}{\rho}(1 + \alpha) \right] \frac{M^p h^2}{6EJ} \quad \text{and} \quad \lambda \leq \lambda_{\min}, \quad \text{with} \quad \mu = \frac{[2 - \lambda + (1 + \alpha) / \rho]P h^2}{6(\lambda + 1)EJ} < 1 \quad (6)$$

where, $\rho = f(t, \lambda_{\min})$ defined by Equation 15 below. For case II, $\lambda \geq \lambda_{\min}$.

$$\delta_{4,ll} = (2\lambda - 1) \frac{M^p h^2}{6EJ} \quad \text{and} \quad \theta_{4,ll} = (\lambda - 1) \frac{M^p h}{2EJ} \quad \lambda > \lambda_{\min}, \quad \text{with} \quad \mu = \frac{(2\lambda - 1)P h^2}{6(\lambda + 1)EJ} < 1 \quad (7)$$

$\theta_{4,ll}$ is the tip rotation for case II. $\lambda_{\min}$, is the value of $\lambda$ that prevents premature formation of plastic hinges at supports. Equation 7 is related to the preferred failure pattern where the last set of plastic hinges form at the column foot. Value of $J$ for drift $q_i = \delta_i / h_i$ may now be computed as;

$$J_i = f(q_i) \quad (8)$$

Putting $\alpha = 0$ in Equation 7 gives the displacements of point g of the LDID. Equating $\delta_{4,l} = \delta_{4,ll}$ with $\alpha = 0$ leads to the mode transition limit $\lambda > \frac{3\rho + 1}{3\rho}$. A proof of conditions (6) and case (7), for the purely shear failure condition, is presented in the forthcoming sections. The LDID of Figure 2 contains the entire mathematical treatment presented above. The use of the LDID can help improve initial selection of $\lambda$ and $\rho$ with respect to conditional use and performance criteria. The tripartite LDID suggests that the use of the MFs is associated with four distinct ultimate load combination categories.

1-Large gravity loading combined with incidental lateral forces, as ranges 1 and 2, Figures 2b and 3b.
2-Moderate gravity and lateral forces, e.g., modeled within range 3, Figures 2b and 3e.
3-Light gravity loading combined with large lateral forces modeled as range 4, Figures 2b and 3f.
4-Small gravity loading combined with large lateral forces, as point g of range 4, Figures 2b and 3g.

While all four loading categories are important the design of category 3 demands greater challenge with respect to economy and drift control. Hence, the present article focuses attention on the study of category 3 loading. If the stated classifications are redefined in terms of their ultimate carrying capacities, then each category may be modified by simply increasing or reducing the ultimate moment of resistance of elements of that category to the corresponding magnitudes of the desired class of frames. Points a, b, c, d, e, f and g of the LDID refer to specific regions and mode transition conditions, Figure 3, where failure patterns (a), and, (f) and (g) correspond to purely beam and purely sway mechanisms respectively. LDID of Figure 2b shows that the limit state combinations of external forces tend to reduce the strength and stiffness of the structure, to different degrees, at incipient collapse. For instance, comparing Equations 2 and 3 for $\alpha = 1$ and $\beta = 2$, and $f_{cr} = \lambda = \rho = 1$, it may be concluded that large gravity loads at $W = 8M^p / L$ (regions 1 and 2), tend to reduce the maximum lateral load carrying and drift development capacities of the module by 50% and 66% respectively.

The bending moment diagram of region 1 of Figure 3a indicates that $W = 8M^p / L$ alone causes the formation of full plastic hinges and half plastified sections at the top and supported ends of the two columns respectively. Application of the lateral force at 25% of its maximum value $V = M^p / h$, Figure 3b, region 2, causes the formation of full plastic hinges at both ends of the leeward columns. From practical design point of view conditions (3a and 3b), are undesirable, particularly if the module
Figure 3. Failure mechanisms and bending moment corresponding to load combination ranges 1 through 4.

is part of the supporting structure of the MF. By contrast, large lateral forces, $V = 4M^p / h$, corresponding to region 4 tends to reduce the gravity load carrying capacity of the system by as much as 50%. While symmetrically disposed gravity forces do not affect lateral displacements of symmetric systems, their relative magnitudes can influence the sequences of formation of plastic hinges, thereby affecting the corresponding displacements at collapse. The observation made at this stage is that the lateral displacements of MFs at incipient collapse can be reduced by controlling the propagation of plasticity within selected groups of members as well as preventing the premature formation of plastic hinges at the fixed ends of the columns. While $M^p$ is independent of $P$, it is influenced by $\lambda$, the collapse load as well as the P-delta effect. Equation 7 may be rewritten in terms of force $V = V_4$, as:

$$\delta = \frac{2\lambda - 1}{\lambda + 1} \frac{Vh^3}{12f_{cr}EJ} = \left( \frac{f_y}{f_{cr}} \right) \frac{Vh^3}{12EJ}$$

Displacement capacity $\delta$ increases with increasing $\lambda$, and that the range of variations of the function $f_y$, in Equation 9, is limited to $1 \geq f_y \geq 2$ for $\infty > \lambda \geq \lambda_{\text{min}}$. A second limiting value of $\lambda$ may now be defined as $\lambda_{\text{max}}$, beyond which the ultimate load-displacement ratio exceeds the economic demand capacity ratio stipulated above. In other words, if the demand imposed by the sway mechanism of Equation 5, is larger than the demand corresponding to combined mechanism, Equation 4, then the maximum over strength, $\lambda_{\text{max}}$ required to impose a sway type mechanism is equal to:

$$\lambda_{\text{max}} = \frac{2Vh}{f_{cr}WL}$$

For $\alpha = 3/2$ and $\rho = 5/6(\lambda - 1)$ Equations 6 and 7 coincide, i.e. the structure may be subjected to over-collapse, (point e). Equations 6 and 7 also coincide at $\alpha = 0$ and $\rho = 1/3(\lambda - 1)$, (point g), indicating that the system can also fail through the simultaneous formation of plastic hinges at both beam ends and column supports. The condition g results in smaller lateral displacements at collapse.

3. PERFORMANCE CONTROL, (PC) AND DESIGN EFFICIENCY

PC is defined as the ability to control the sequences of formations plastic hinges and to design a structure in such a way as to expect predetermined modes of failure, extents of damage and/or drift throughout the history of loading of the system. The response of the MF at incipient collapse may be controlled by proper selection of the strength and stiffness of its members and the sequences of formations of the plastic hinges. In order to minimize drift at incipient collapse the following design strategies may be considered: a- to avoid the formation of plastic hinges along the span of the beam, except at its ends, and b- to avoid the premature formation of plastic hinges at column supports. Here PC is introduced as a design procedure rather than a method of analysis. It may be seen from segments 1 and 2 of the LDID of Figure 2b that the overall stiffness of the module diminishes rapidly due to
early formation of plastic hinges in the beams, an issue that is of great concern when addressing permissible drift limits at incipient collapse. The constituent modules of any MF may be said to be of minimum weight (Foulkes 1954) and drift, if their total weight and drift are minimum at ultimate loading. To this end, it can be shown that small gravity loads \( W \leq 4M_p / L \) have little to no direct effect on the lateral load and drift development potentials of the module at incipient collapse. This implies that for any value of \( W > 4M_p / L \) the strength of the beam may be increased proportionately to shift the failure mode from regions 1, 2 and 3 to region 4. For instance if originally, \( W = 8M_p / L \), and \( V = 2M_p / h \), and the beam is redesigned for an increased moment \( 2M_p \), then the performance of the module at ultimate load, \( V_{\text{max}} = 4M_p / h \) will not be affected by the presence of \( W \) at \( L/2 \). As a general rule, mode shifting for any distribution of gravity loading can be achieved by selecting the ultimate carrying capacity of the beam in excess of its ultimate carrying capacity as if acting as a simply supported element. This in turn raises the query that which one of the two design conditions within the response ranges 2 and 4 is more efficient with respect to lateral resistance? The linearized total weight function of the subject portal frame in terms of its aspect ratio \( r = L / h \) and an arbitrary constant of proportionality \( g \), may be expressed as:

\[
G = gh(2\lambda + r)M_p
\]  

(11)

Then from group of Equations 4:

\[
\frac{G_3}{V_3} = \frac{gh^2}{2} \left( \frac{2\lambda + r}{\lambda} \right) \quad \text{and} \quad \frac{\delta_3}{V_3} = \frac{h^3}{12EJ} \left( \frac{2\lambda + 1}{\lambda} \right)
\]  

(12)

and from group of Equations 5:

\[
\frac{G_4}{V_4} = \frac{gh^2}{4} \left( \frac{\lambda + 2r}{\lambda + 1} \right) \quad \text{and} \quad \frac{\delta_4}{V_4} = \frac{h^3}{12EJ} \left( \frac{2\lambda - 1}{\lambda + 1} \right)
\]  

(13)

Since \( G_4 / V_4 < G_3 / V_3 \) then the sway mechanism of region 4 offers a more economical solution than the combined failure modes of regions 2 and 3. Since \( \delta_4 / V_4 < \delta_3 / V_3 \) for all values of \( \lambda > (3\rho + 1)/3\rho \), then region 4 provides for higher stiffness at incipient collapse than the other regions of the LDID.

3.1 Minimization of lateral displacements of fixed base frames at incipient collapse

The first step in reducing the occurrence of excessive drift at incipient collapse, due to combined mechanisms is to prevent the possibility of formation of a beam mechanism without contributions from the lateral forces. This means increasing the moment of resistance of the beam from \( M_p \) to a minimum of \( 2M_p \) or dropping the magnitude of the gravity force from \( W \) to \( W/2 \). In order to achieve the desired outcome, the elastoplastic properties of the module should satisfy the following inequality;

\[
\left\{ \delta_{4,\ell} = (2\lambda - 1) M_p h^2 / 6EJ \right\} \geq \left\{ \delta_{4,\ell} = 2 - \lambda + \frac{1}{\rho} (1 + \alpha) \right\} \left\{ M_p h^2 / 6EJ \right\}
\]

(14)

which, yields the value of \( \lambda_{\text{min}} \) for the transition of the sequence of formation of the plastic hinges as,

\[
\lambda_{\text{min}} > \left[ \frac{3\rho + 1 + \alpha}{3\rho} \right] = \gamma
\]

(15)

A particular condition of interest occurs at point \( e \) where \( \alpha = 3/2 \) and \( \lambda_{\text{min}} = (6\rho + 5)/6\rho \). Equation 6 corresponds to the condition where full plasticity first develops at the fixed supports. It may be observed from Figure 4 or Equation 16 that for \( \rho = 0 \), \( \lambda = \infty \) and that for \( \rho = \infty \), \( \lambda = 1 \). This ensures that the first and second plastic hinges form at the ends \( C \) and \( B \) respectively of beam \( BC \). Second, to force the last two hinges to develop simultaneously at the fixed ends \( A \) and \( D \) of the two columns. The term \( \alpha \) becomes zero in the absence of the gravity forces, therefore reducing (15) to;

\[
\lambda_{\text{min}} > \left[ \frac{3\rho + 1}{3\rho} \right] = \gamma
\]

(16)

Case I- the first set of plastic hinges form at the feet of the columns and, Case II- at the ends of the beam. The structure is three degrees indeterminate, therefore its final displacements at incipient collapse would necessarily be composed of three or less sequences of formation of plastic hinges.
Figure 4, (a) Generalised multi-span fixed base portal frame, (b) Controlled beam failure prior to collapse

Case 1: \( \lambda \leq \gamma \), region 4, point g of the LDID - Phase I.1- Unless \( \lambda > \gamma \), the first set of plastic hinges occurs at the column feet. The force needed to generate the first set of plastic hinges at supports may be computed as; \( V_{4,1,1} = 2(1 + \gamma) f_c \lambda M^p / \gamma h \); therefore;
\[
\delta_{4,1,1} = (2 - \gamma + \frac{1}{\rho}) \frac{\lambda h^2 M^p}{6\gamma EJ} = (2 - \gamma + \frac{1}{\rho}) \frac{V_{4,1,1} h^3}{12 f_c (1 + \gamma) EJ}
\]  
(17)

Phase I.2 - The additional moment needed to bring about failure is equal to \((\gamma - \lambda) M^p / \gamma h\). The additional force can be computed as \( V_{4,1,2} = 2(\gamma - \lambda) f_c M^p / \gamma h \). The displacements of this phase are equal to;
\[
\delta_{4,1,2} = (2 + \frac{1}{\rho})(1 - \frac{\lambda}{\gamma}) \frac{h^2 M^p}{6EJ} = (2 + \frac{1}{\rho}) \frac{V_{4,1,2} h^3}{12 f_c EJ}
\]  
(18)

The collapse load-moment relationship for the case under study coincides with that of Equation 5, i.e.;
\[
M^p = \frac{\gamma Vh}{2(\lambda(\gamma + 1) f_c + (\gamma - \lambda) f_c)} = \frac{Vh}{2(\lambda + 1) f_c}
\]  
(19)

Case II: \( \lambda > \gamma \), region 4, point g of the LDID- Phase II.1- The lateral shear force, required to generate the first set of the plastic hinges at the ends of the beam, can be computed as; \( V_{4,2,1} = 2(1 + \gamma) f_c M^p / h \). The corresponding lateral displacements can be expressed as;
\[
\delta_{4,2,1} = (2 - \gamma + \frac{1}{\rho}) \frac{h^2 M^p}{6EJ} = (2 - \gamma + \frac{1}{\rho}) \frac{V_{4,2,1} h^3}{12 f_c EJ}
\]  
(20)

Phase II.2 - The additional moment needed at the column supports, may be computed as \((\lambda - \gamma) M^p \), generated by the additional force, \( V_{4,2,2} = 2(\lambda - \gamma) f_c M^p / h \). The corresponding displacements is;
\[
\delta_{4,2,2} = 2(\lambda - \gamma) \frac{h^2 M^p}{6EJ} = \frac{2V_{4,2,2} h^3}{12 f_c EJ}
\]  
(21)

\( \delta_{4,II} + \delta_{4,II} = \delta_{4,II} \) results in the final lateral displacement at incipient collapse for case II, i.e.
\[
\delta_{4,II} = (2 - 1) \frac{h^2 M^p}{6EJ}
\]  
(22)

As expected, \( (V_{4,II,1} + V_{4,II,2}) = V_4 \), leads to the plastic limit state lateral carrying capacity of the MF;
\[
M^p = \frac{Vh}{2[(1 + \gamma) f_c + (\lambda - \gamma) f_c]} = \frac{Vh}{2(1 + \lambda) f_c}
\]  
(23)

3.2 Multi-span continuous portal frames

Consider the plastic limit state design of the multi-bay, continuous sub frame of Figure 4a, under central concentrated forces \( W_j \), with the stipulations that the maximum target drift ratio does not exceed \( \Phi \) and no plastic hinges form prematurely at column supports prior to complete failure of the beams. The nature of Equations 16, 19 and 23 suggests that by using RBS and/or proper selection of

Figure 4, (a) Generalised multi-span fixed base portal frame, (b) Controlled beam failure prior to collapse

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the frame properties \( \lambda_i \) and \( \rho_i \), the formation of the base level plastic hinges can be delayed until complete failure of the beams takes place. This may be achieved by selecting \( M_j^p \geq W_j L_j / 4 \) and \( N_j^p > M_j^p + M_{j+1}^p \). If partial beam failure occurs prior to the development of full plasticity at any column support, then the remaining stable structure may be construed as being composed of a number of fixed base columns connected to each other by means of a series of imaginary pin ended rigid links as shown in Figures 4b and 6d. However, care should be taken not to ignore the possibility of occurrence of a combined mechanism, e.g., Figure 7e. Once the response of the first level sub frame is regulated, the response of the upper levels can be controlled by proportioning the demand capacity ratios of the members of the corresponding modules.

4. RESPONSE OF CLOSED LOOP MODULES

This section discusses the maximum displacements of closed loop modules of uniform strength at incipient collapse. Assuming \( \lambda_{\text{min}} > 1 \), thus forcing all plastic hinges to form within the upper and lower beams of the module. Some of the results pertaining to the current case study are presented in Figure 5b and the group of Equations 24 to 27 without further elaboration

For the purely beam mechanism at point \( a \):

\[
\delta_1 = 0 \quad \text{for} \quad V = 0, \quad M_j^p = W_j L_j / 8, \quad \alpha = -1 \quad \text{and} \quad \alpha > 3 / 2
\]

In range 2, from \( a \) to \( e \), \( \alpha > -1 \), \( \alpha > 3 / 2 \) and \( \delta_2 \) is dependent on \( \rho \), thus;

\[
\delta_2 = \left[ 1 + \frac{1}{\rho} \right] \left( 1 + \alpha \right) \frac{P_f}{12EJ} \frac{8M_j^p}{L} \geq W \geq \frac{4M_j^p}{h} \cdot \frac{V_j}{h} \leq \frac{4M_j^p}{h} \quad \text{and} \quad M_j^p = \frac{1}{8} \left( \frac{V_j}{h} + WL \right)
\]

(24)

\[
\delta_3 = \left[ 1 + \frac{1}{\rho} \left( 1 + \alpha \right) \right] \frac{h^2 M_j^p}{6EJ} \frac{4M_j^p}{L} \geq W \geq 0 \quad \text{with} \quad \alpha = \frac{3}{2} \quad \text{and} \quad \alpha = \frac{3}{2} \quad \text{and} \quad f_{cr} = 1 - \frac{1}{\rho} \frac{P h^2}{12EJ}
\]

(25)

Putting \( \alpha = 0 \) in Eq.26 results in the displacements of the purely shear mechanism point \( g \) of the LDID. \( \alpha \) and \( \alpha \) describe the static equilibrium of the beams of the module at incipient collapse, i.e.

\[
\alpha = \frac{3}{2} \quad \text{and} \quad \alpha = \frac{3}{2} \quad \text{and} \quad M_j^p \geq WL / 4 \quad \text{and} \quad \alpha \leq 3 / 2
\]

(26)

(27)

To prevent combined mechanism, the following conditions should be observed.

\[
M_j^p \geq WL / 4 \quad \text{and} \quad \alpha \leq 3 / 2
\]

\[
4.1 \text{Multi-span continuous closed loop sub-frames}
\]

While the beams of the subject closed loop module (28) under combined loading, the same may not be true for multi-bay frames of similar construction. The latter MF structure may fail through any combination of the same collapse mechanisms. Each distinct stage of loading would then correspond to a corresponding sequence of formation of plastic hinges, such as the simultaneous yielding of the stiffer ends of the top and bottom beams of the stiffest bay at first yield. The next set of plastic hinges would form at the other ends of the same beams. The sequence of formation of the rest of the plastic hinges would then be the same as the deceasing order of the stiffness of the modules of the system. The number of sequences of formations of plastic hinges can then be computed as \( 2(n-1) \). Since the physical response of the subject subframe Figure 6a, can be studied through the response of its modularized counterpart, Figure 6c, then it would be reasonable to expect the sequence of formations...
of the plastic hinges of the prototype, Figure 6b, to follow the same order as the sequences of formations of the corresponding modularized system, Figure 6d. This finding may be used to compute the displacements of the subframe at incipient collapse without resorting to iterative computations. For symmetrically disposed modules both ends of the beams of the last remaining module fail simultaneously. Suppose the last pairs of plastic hinges form at the ends of the beams of the $r^{th}$ bay of the closed loop subframe of Figure 6d, such that $1 < r < n$, then from Eq.25:

$$\delta = \left[ \frac{h}{J_r} + \frac{L_r}{I_r} \right] \frac{(1 + \alpha)hM^p_r}{12E}$$

Equation 26 and the LDID of Figure 6b show that gravity forces affect the sequence of formation of the hinges. The correct sequence results in maximum displacements at collapse. If the last pairs of hinges form at the ends of the beams of the first/last bays of the frame, the corresponding displacement may be computed as:

$$\delta = \left[ \frac{h}{J_r} + \frac{L_r}{I_r} \right] \frac{hM^p_r}{6E}$$

Load-displacement diagrams (2b) and (5b) together with the groups of Equations 2 to 7 and 24 to 28 offer a reliable means of frame classification and member selection for fixed base and grade beam supported portal frames respectively. The same techniques can also be used to select appropriate RBS values for moment and/or displacement control purposes. The ability to estimate maximum lateral displacements at incipient collapse, not only helps improve the limit state design of MFs, but also provides insight into the progressive failure of the subject structure in an orderly, predetermined manner. The next six generic examples have been provided to demonstrate the applications of the proposed methodologies to the study and design of single story sub-frames as component parts of multilevel MFs. The same example frames are then assembled together to reconstruct the original prototype. To this end, it is instructive to view the analysis of example 1 as the basis of PC for laterally loaded MFs. The proposed procedure has been summarized as a self explanatory design template in Figure 10. The drift angle of doubly symmetric closed loop modules changes at the same rate as their interior angle of rotation throughout the history of loading of the system. This implies that the properties of all sub-frames composed of closed loop modules can be manipulated to rotate through the same angle of drift along the height of the structure. The maximum angle of rotation of fixed base portal frames $\Theta_1$ at incipient collapse is comparable in magnitude with the corresponding angle of drift $\Phi_1$. And, as indicated in Figure 1b, this causes the drift angle of the upper levels to change from $\Phi_1$ to $\Phi_1 \pm \Theta_1$. In other words the rigidity of the support level fixed base subframe controls the lateral displacements of the entire frame. Equation 6 gives:

$$\Phi_1 = (2\lambda - 1) \frac{M^p h}{6EJ} \quad \text{and} \quad \Theta_1 = (\lambda - 1) \frac{M^p h}{2EJ}$$

$$\delta = \left[ \frac{h}{J_r} + \frac{L_r}{I_r} \right] \frac{(1 + \alpha)hM^p_r}{12E}$$

$$\delta = \left[ \frac{h}{J_r} + \frac{L_r}{I_r} \right] \frac{hM^p_r}{6E}$$

Figure 6, (a) Closed loop MF, (b) Controlled failure, (c) Modularized sub frame, (d) Controlled failure.

Figure 10. The drift angle of doubly symmetric closed loop modules changes at the same rate as their interior angle of rotation throughout the history of loading of the system.
A practical question comes to mind, under what circumstances, can the differential drift angle be minimized and/or eliminated? The differential drift angle is caused by the applied loading as well as the extreme rigidity of the fixed ends of the support level columns. The desirable condition $\theta_1 < \phi_1$ is associated with $\lambda < 2$. Any value of $\lambda > 2$ will cause the base drift angle to increase from $\phi_1$ to $\phi_1 + \theta_1$. Obviously it is simpler to minimize the differential drift between similarly configured frames, such as two closed loop modules, rather than a closed loop module and a fixed base portal frame. However, comparing the rotations $\theta_1$ and $\phi_1$ of a fixed base module, Equation 31, with a geometrically similar grade beam supported (closed loop) module, i.e. Equation 26, it gives;

$$\frac{J_f}{J_1} = \frac{\lambda + 1}{2(2 - \lambda)}$$

and

$$\frac{1}{\rho_1} = \frac{3(\lambda - 1)}{(1 + \alpha)(2 - \lambda)}$$

as the equivalent grade beam supported base level module, where suffixes $l$ and $f$ refer to closed loop and fixed base modules respectively. In general the properties of the upper module may be selected in such a way as to induce a uniform drift $\phi_1 = \phi_2 = \phi_1$ and $\theta_1 = \theta_2 = \theta_1$ along the height of the structure. Both Equations 10, 31 and 32 indicate that the upper limit of $\lambda$ should be restricted to $\lambda_{\text{max}} < 2$.

4.2 Demonstrative example 1; Single bay portal frame
Consider the plastic limit state design of the fixed base MF of Figure 2a, as component part of the continuous MF of Figure 4a, subjected $W = 40$ kips and $V = 108$ kips, such that the weight of the structure is a minimum and that the drift angle at incipient collapse is not to exceed $\phi = 0.002475$ radians. $h = 15^\circ - 0^\circ$, $L = 30^\circ - 0^\circ$ and $f_{oc} = 0.9$. **Solution:** Here the design strategy prevents the development of beam mechanism and avoids the premature formation of plastic hinges at column feet. Compute $V_1 = 108/0.9 = 120$ kips and $M^p \geq WL/4 = 40 \times 30/4 = 300$ kip-ft. Let $\rho = 1.0$ and $\alpha = 3/2$, which leads to $\lambda_{\text{min}} = 11/6$. Equations 16 and 31 may be used to define the range of $\lambda$ as $2 > \lambda \geq 1.833$. Assuming that $\lambda = 1.85$, Equation 5 gives, $M^p = 315.79$ kip-ft > 300 kip-ft. Equations 6 and 7 give $\delta = 2.65 M^p h^2/6 E J$ and $\delta = 2.70 M^p h^2/6 E J$ respectively. Equation 7 governs and may be used to compute $J = 2.70 \times 15 \times 315.79 \times 12^2 / [6 \times 29000 \times (\phi = 0.002475)] = 4276.53in^4$. With $J$ known, $I$ can also be computed from $\rho = 1$ as $I = 8553.06in^4$. The actual value of $\alpha$ may now be re-evaluated as $\alpha = 3 \times 40/30 \times 2/4 \times 315.79 = 1.425$.

4.3 Demonstrative example 2; Single bay portal frame with RBS
Use RBS to redesign the portal frame of example 1. **Solution:** The prescribed $\phi$ is to remain the same the effective rigidity $\rho$ should be at least as large as that computed for the original frame. However, if $M_{RBS}^p$ and $N_{RBS}^p$ are selected as plastic moments of resistance of the RBS and the columns respectively then the capacity reduction factor $\xi$ may be defined as $\xi = (M_{RBS}^p / N_{RBS}^p)$ such that $\xi < 1$ for all $\rho \geq 1$. If the capacity of the frame is to remain the same then the relationship $\xi = 1/1.85 = 0.5405$ leads to; $N_{RBS}^p = Vh / 2(\xi + 1) = 120 \times 15 / 2(0.5404 + 1) = 584.21$ kip-ft and $M_{RBS}^p = 315.79$ kip-ft. $\xi = 1/\lambda$ both only strong column-weak beam criterion and the sequence of formation of the hinges was controlled.

4.4 Demonstrative example 3; 3-span continuous, fixed base portal frame
Consider the plastic design of the MF of Figure 7a, as the base level imaginary sub frame of the 3 story steel MF of Figure 9a. Design stipulations require that the target drift ratio at incipient collapse, $\phi = 0.002475$ radians. **Solution:** The material and geometric properties of the structure are; $W_{i,j} = W = 40$ kips, $V_{i} = 54 + 108 + 162 = 324$ kips (from Table 1) and $f_{cr,i,j} = f_{cr} = 0.9$. $h_i = 15^\circ - 0^\circ$, $L = 30^\circ - 0^\circ$, $L_{i,j} = L$, $J_{i,1} = J_{1,4} = J$, $J_{1,1} = J_{1,3} = 2J$ and $M_{i,j}^p = M_{j}^p$ for all $j$. Determine $\lambda_{i,j}$, $M_{i,j}$, $I_1$ and $J_1$. 

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Solution: It has been shown, in section 4.5, that the plastic failure of the prototype (7a) can be studied in terms of its equivalent modules (7b) or (7d). Now if for each independent portal frame of Figure 7b \( \lambda_{h,1} \geq \lambda_{1,1} \), then the structure would fail through a purely sway mechanism with the first sets of plastic hinges forming at beam ends as shown in Figure 7c. Therefore if a selection is made such that \( \alpha_{1,1} = 1.425 \), \( \alpha_{1,2} = \alpha_{1,3} = 0 \), then for \( \rho_{1,1} = 1.0 \), \( \rho_{1,2} = 0.5 \), and \( \rho_{1,3} = 1.33 \), the minimum values of \( \lambda \) may be computed as; \( \lambda_{h,1} > 1.833 \), \( \lambda_{h,2} > (1.5 + 1)/1.5 \) and \( \lambda_{h,3} > (4 + 1)/4 > 1.25 \) respectively. The failure loads corresponding to collapse mechanisms (8c) and (8e) can be expressed in the same order as; 

\[
V_{h_1}/f_{cr} = 6(\lambda + 1)M_1^p \quad \text{and} \quad V_{h_2}/f_{cr} + WL/2 = 2(3\lambda + 4)M_1^p
\]

Elimination of \( V_{h_1}/f_{cr} \) gives the mode transfer condition; \( M_1^p = WL/4 \) and leads to \( \lambda_{max} \) for the example frame as; \( \lambda_{max} = (2V_{h_1}/3f_{cr}WL) - 1 = (2 \times 324 \times 15/3 \times 0.9 \times 40 \times 30) - 1 = 2 > (\lambda_{min} = 1.833) \). The stipulation is fulfilled by selecting \( \lambda_1 = 1.85 \) and \( M_{1,\min} = 315.79 \) \( \approx W_{1,1}L_{1,1}/4 = 300 \) kip-ft. In other words \( M_1^p = V_{h_1}/6(\lambda + 1)f_{cr} = 342 \times 15/6 \times 0.9(1.85 + 1) = 315.79 \) kip-ft., Next, from example 6.2 above; \( J_1 = 4276.53 \), \( I_1 = J_1L_{1,1}\rho_{1,1}/h_1 = 8553.06 \) in.\(^4\) and \( \phi_1 = 0.1 = 0.002475 \) radians.

4.5 Demonstrative example 4; 3-span continuous closed loop modules

Consider the plastic limit state design of the three bay, \( n = 3 \), closed loop continuous subframe of Figure 8, as the second level imaginary sub-frame of a \( m = 3 \) story steel moment frame assembled on top of the fixed base MF of Fig.10, with the same stipulations as the preceding example. \( W_{2,1} = 30 \) kips, \( V_2 = 162 + 108 = 270 \) kips and \( f_{cr,2,1} = f_{cr} = 0.9 \). Determine the sequence of formation of plastic hinges, \( \lambda_{2,1}, M_2^p, I_2 \), and \( J_2 \). Solution: The collapse load of the MF can be expressed as; 

\[
V_2h_2/f_{cr} = 12M_2^p \quad \text{and} \quad V_2h_2/f_{cr} + W_2L = 16M_2^p
\]

Once again, elimination of \( (V_2h_2/f_{cr}) \) between pairs of Equation 34 results in the mode transfer condition; \( M_2^p = W_2L/4 \). Following the same rationale as before, selection is made such that

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**Figure 7:** (a) MF, (b) Equivalent MF, (c) Controlled sway, (d) Combined mechanism, (c) Controlled failure.

**Figure 8:** (a) 3-span closed loop frame, (b) Modularized equivalent system, (c) Controlled beam failure.
computed directly in proportion with similar items of the second level sub-frame, i.e.,

\[ M_{2,\text{min}} \geq W_{2,1}L/4 = 225 \text{ kip-ft, } M_2 = V_2h_2/12f_{cr} = 250 \text{ kip-ft, and } M_2^p = [V_2h_2/f_{cr} + W_2L]/16 = 243.75 < 250 \]

kip-ft, then for \( \rho_{2,1} = 1.0 \), \( \rho_{2,2} = 0.5 \) and \( \rho_{2,3} = 1.33 \), \( \lambda_{\text{min}} = \lambda_{\text{max}} \) may be computed as \( \geq (250/250) = 1 \). The over strength ratio may be set at \( \lambda_{2,1} = 1.10 \). Equation 29 gives

\[ \phi_2 = \phi = \left[ 1 + \frac{1}{\rho_{2,2}} \right] \frac{h_2M_2^p}{6EJ_2} = \frac{3.00h_2M_2^p}{6EJ_2} \text{ in Equation } 30 \text{ gives; } \]

\[ \phi_2 = \phi = \left[ 1 + \frac{2.35}{\rho_{2,1}} \right] \frac{h_2M_2^p}{6EJ_2} = \frac{3.35h_2M_2^p}{6EJ_2} \]

The prescribed stipulation can be fulfilled by selecting;

\[ M_2^p = 250 \text{ kip-ft. and } J_2 \text{ and } I_2 \text{ in such a way as to enforce compatible deformations at the common joints of the merging sub-frames, then matching } \phi_2 = \phi_1 \text{ and } \theta_2 = \theta_1 \text{ in Equations 7 and 26, it gives; } \]

\[ \frac{I_2}{J_1} = \frac{(1 + \alpha_2)M_2^p L}{3(\alpha - 1)M_1^p h_1} \quad \text{ and } \quad \frac{J_2}{J_1} = \frac{M_2^p h_2}{(2 - \lambda)M_1^p h_1} \]

Equation 35

\[ \rho_2 = \rho_2 = \rho_3 = 1.0 \text{, } \rho_2 = 0.5 \text{ and } \rho_3 = 1.33 \text{, } W_{2,1} = 20 \text{ kips, } V_3 = 162 \text{ kips, } f_{cr,2,1} = f_{cr} = 0.9 \text{ and } h_2 = 10'. \]

Solution:

Following the exact same steps as for example 4, it gives; \( M_{3,\text{min}}^p \geq W_{3,1}L/4 = 20 \times 30/4 = 150 \) kip-ft, and \( M_3^p = V_3h_3/12f_{cr} = 162 \times 10/12 \times 0.9 = 150 \) kip-ft. \( = M_{3,\text{min}}^p \), or \( M_3^p = [V_3h_3/f_{cr} + W_3L]/16 = 150 \) kip-ft. \( = M_{3,\text{min}}^p \). Therefore, \( M_{3,\text{min}}^p \) governs. Let \( \lambda_{2,1} = 1.1 \). Once

\[ \alpha = 3 \times 20 \times 30 / 8 \times 150 = 1.5 \], Equation 30 gives,

Table 1, Complete solutions to examples 3, 4 and 5, \( J = 4276.52 \text{ in.}^4 \), \( f_{cr} = 0.9 \).

<table>
<thead>
<tr>
<th>( i )</th>
<th>( W_i )</th>
<th>( h_i )</th>
<th>( V_i )</th>
<th>( M_i^p )</th>
<th>( J_i = \bar{J}_i )</th>
<th>( I_i )</th>
<th>( J_i )</th>
<th>( \lambda_i )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>20</td>
<td>180</td>
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<td>10</td>
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<tr>
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<td>50</td>
<td>120</td>
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<td>10</td>
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<td>250.0</td>
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<td>60</td>
<td>360</td>
<td>15</td>
<td>5400</td>
<td>315.7</td>
<td>565.7</td>
</tr>
</tbody>
</table>

Since, \( \theta_3 = \phi_2 = \phi_1 \) and \( \theta_3 = \theta_2 = \theta_1 \), the elastoplastic properties of the roof level subframe may be computed directly in proportion with similar items of the second level sub-frame, i.e.,;

\[ \frac{1}{J_2} = \left[ \frac{M_2^p h_2}{M_2^p h_2} \right] = \frac{150 \times 10}{250 \times 10} = 0.60 \]

\[ \frac{I_1}{I_2} = \frac{(1 + \alpha_3)M_1^p}{(1 + \alpha_2)M_1^p} = \frac{(1 + \alpha_3)M_1^p}{6EJ_3} = \frac{2.5 \times 150}{3.35 \times 250} = 0.6383 \]

(36)
5. CONCLUSIONS

Since the drift profiles of Figure 1b can be used to compute the displacements of the MF at collapse as;

$$\lambda_{i,\min} = \frac{M_i^p}{J_i} I_i$$

The complete solutions to examples 3, 4 and 5, i.e. quantities $M_i^p, I_i, J_i$ and $\lambda_{i,\min}$ have been summarized in Table 1 below.

4.7 Demonstrative example 6; 3-bay, 3 story fixed base moment frame

Figure 9, (a) Example 6.5 fixed base moment frame, (b) Controlled sequential failure, (c) Drift profile.

The sub-frames of examples 3, 4 and 5 can be reassembled to reconstruct the frame of Figure 9a, provided that the loading and the properties of the merging beams are computed as;

$$\bar{W}_i = W_i + W_{i+1}, \quad \bar{I}_i = I_i + I_{i+1} \quad \text{and} \quad \bar{M}_i = M_i^p + M_{i+1}^p$$

Solution: The loading and the solutions to example 6, quantities $F_i, \bar{W}_i, \bar{M}_i^p, \bar{I}_i$ and $\bar{J}_i$, have been summarized in Table 1, where $F_i = 162i/mf_{cr}$ describes the distribution of the external lateral forces. The story drifts are computed electronically for comparison against the solutions of example 6, i.e.

$$\Delta_1 = 0.442 \text{ in.}, \quad \Delta_2 = 0.735 \text{ in.}, \quad \Delta_3 = 1.031 \text{ in.}, \quad \theta_1 = 0.00246 \text{ rad.}, \quad \theta_2 = 0.00244 \text{ rad.}, \quad \theta_3 = 0.002475 \text{ rad.}$$

4.8 Verification of the plastic collapse load

The corresponding equation of virtual work, considering the global failure pattern of Figure 9b, should yield the correct collapse load of the assembled frame, i.e.;

$$\sum_{i=1}^{m} F \times i \times \left( \sum_{i=1}^{l} j_i \bar{h}_j \right) \psi = n \sum_{i=0}^{m} M_i^p \psi$$

Where, $\psi$ is the virtual rotation of the mechanism of Figure 1f. $\sum_{i=1}^{l} j_i \bar{h}_j$ is the height of the $l^{th}$ floor level from the base. Equation 39 gives for the frame of example 6.5, Figure 10a and Table 1;

$$\psi = \left[ 12(150+250) + 6(1+1.85)(315.79) \right] \psi, \quad \text{or} \quad F = 60 \text{ kips}, \quad \text{which is the same as the first level collapse load intensity indicated in Table 1.}$$

4.9 Verification of lateral displacements at incipient collapse

The drift profiles of Figure 1b can be used to compute the displacements of the MF at collapse as;

$$\Delta_i = \phi_i h_i + \sum_{j=1}^{l} (\phi_j + \theta_j - \theta_{j-1}) h_j$$

Since $\phi_i = \phi$ and $\theta_j = 0$, then the corresponding story level lateral displacements can be computed as;

$$\Delta_1 = \phi h_1 = 0.002475 \times 12 \times 15 = 0.445 \text{ in.}, \quad \Delta_2 = \Delta_1 + 0.002475 \times 12 \times 10 = 0.742 \text{ in.}, \quad \text{and} \quad \Delta_3 = \Delta_2 + 0.002475 \times 12 \times 10 = 1.039 \text{ in.}$$

Therefore, proposed solutions are exact and unique.

5. CONCLUSIONS

\[ H_1 = J_3 = 0.6 \times 15047.03 = 9028.18 \text{ in.}^4, \quad J_3 = 0.6383 \times 6240.09 = 3983.05 \text{ in.}^4 \]
A summary of the steps involved in the proposed DLA of regular MFs is presented in Figure 10. If the failure mechanisms of two vertically adjacent modules are the same, then the collapse modes of their common beam would be the same.

If the mechanisms of two vertically adjacent modules are not the same, then the mechanism of their common beam would be the one that undergoes the smaller amount of plastic rotations. No plastic hinges can form along the height of the columns, except at their fixed supports.

The plastic moment of resistance of each closed loop module and support level fixed base portal frame may be computed as \( M_i^p = V_i h_i / 4n \) and \( M_i^p = V_i h_i / 2n(\lambda + 1) \) respectively. The proposed solutions are unique and exact since they satisfy the prescribed yield criteria, static equilibrium as well as the boundary support conditions. The procedure allows the target drift at incipient collapse to be enforced rather than investigated.

6. REFERENCES


