GLRC_HEGIS GLOBAL CONSTITUTIVE MODEL FOR RC WALLS AND SLABS FOR SEISMIC NONLINEAR STRUCTURAL ANALYSES

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ABSTRACT

The new GLRC_HEGIS model for reproducing the RC walls and slabs mechanical nonlinear cyclic behavior is presented. The model is formulated in the framework of Thermodynamics of Irreversible Processes and is obtained by means of an analytical homogenization which takes into account four nonlinear physical phenomena governing the reinforced concrete mechanical behavior: concrete cracking, concrete damage, steel-concrete relative slip and steel yielding. As a result of these choices, the constitutive model is formulated at the global scale, so the constitute law is a relationship between the stress resultant forces and the generalized strains. Therefore, the model can be applied to efficient shell finite elements of relatively big size, adapted to large-scale structural seismic computations, as pushover or transient nonlinear dynamic analyses. 18 scalar internal variables describe the nonlinear mechanical behavior and allow obtaining local information of the RC state as a direct result of the finite element computation: crack width and slip, steel plastic strain, concrete stiffness loss… The validity of the model is shown by its application and comparison to experimental tests on different RC structures: a good fit both at the global (force-displacement curves) and the local (crack width) scales is observed.

Keywords: constitutive model; RC walls; RC slabs; cracking; nonlinear analysis

1. INTRODUCTION

Computationally efficient constitutive models able to accurately reproduce the nonlinear cyclic mechanical behavior of RC structures are needed when assessing the seismic margin of existing Reinforced Concrete (RC) buildings of large structures (as nuclear power plants facilities). Pushover or transient dynamic analyses which use this type of models are a good tool not only for the assessment of the seismic capacity of existing structures but also for an optimization of the reinforcement design of new facilities.

In this framework, the cyclic nonlinear constitutive model GLRC_HEGIS for RC walls and slabs is presented in chapter 2 of this paper. In order to ensure an accurate description of the mechanical behavior of RC 2-D elements, four different nonlinear physical phenomena are considered in the formulation of the model: concrete cracking, concrete damage, steel yielding and the relative slip between steel reinforcement bars and the surrounding concrete. These mechanisms are taken into account by means of an original analytical multi-scale analysis, so that the mechanical behavior of a RC plate is fully described at the global scale in the framework of Thermodynamics of Irreversible Processes in section 2.1. Therefore, the model is described in terms of a stress resultant – generalized strains relationship (depending on 18 scalar global internal variables) and it has been implemented in shell Finite Elements (FE) with only one integration point in the height of the section (monolayer elements).

The local laws retained for the description of the four nonlinear mechanisms are presented in section 2.2. Two different families of cracks are considered, aiming at reproducing the typical crack pattern

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found in RC walls submitted to seismic cyclic loads. Each family of cracks is characterized by an average crack orientation and spacing, which are determined by the stress state at cracking onset. After cracking, these two parameters remain fixed and the crack displacements in normal (crack width) and tangential direction evolve following the retained laws for the bridging and the aggregate interlock effects respectively. Concrete isotropic damage reproduces the loss of concrete stiffness and the post-elastic behavior when submitted to high compressive stress. Bond stresses are supposed to appear at the interface between steel reinforcement bars and the surrounding concrete when a relative slip occurs. As a result of this stress transfer, tension stiffening effect is reproduced by the model and the maximum steel stress is located at the intersection with cracks. Therefore, steel yielding is assumed to be concentrated at this location and an elastic – perfectly plastic law is retained.

In chapter 3, the performance of GLRC_HEGIS model is shown by means of three numerical applications: the evaluation of the capacity of a RC section at the Ultimate Limit State (ULS), the RC shear wall n°3 of the CEOS.fr experimental program and the transient dynamic analysis on the SMART benchmark building. At the global scale, the comparison of the force-displacement curves and crack pattern onset and evolution shows a good agreement between the experimental and numerical results. At the local scale, good results are also found for important parameters for the RC structures design and assessment as steel yielding and crack width: the limitation of the maximum crack width is mandatory by the civil engineering design codes as Eurocode 2 (CEN 2005) –EC2- and Model Code 2010 (fib 2011) –MC10- so a good assessment of this parameter is of great importance for civil engineers.

2. GLRC_HEGIS MODEL FORMULATION

The GLRC_HEGIS model is formulated in the framework of Thermodynamics of Irreversible Processes. First, in section 2.1, the Helmholtz free energy surface density is presented, which depends on the state variables. Then, in section 2.2, the evolution of the internal variables is defined using threshold functions which reproduce the four nonlinear physical phenomena taken into account by the constitutive model in order to reproduce the nonlinear behavior of RC plates: concrete cracking, concrete damage, steel yielding and the relative steel-concrete slip. Finally, the evolution of the 18 internal variables of the model is defined in section 2.3.

2.1 Helmholtz free energy surface density

2.1.1. Helmholtz free energy surface density as a function of the state variables

The Helmholtz free energy surface density defines the elastic part of the material behavior and depends on the state variables of the model:

- Observable variables, which define the linear elastic behavior:
  - The generalized membrane strain $\epsilon$, composed of three components $\epsilon_{xx}, \epsilon_{yy}$ and $\epsilon_{xy}$
  - The generalized curvature $\kappa$, composed of three components $\kappa_{xx}, \kappa_{yy}$ and $\kappa_{xy}$

- Internal variables, which describe the nonlinear behavior of the model:
  - The crack displacement $\mathbf{w} = (w_n, w_t)$, in the normal (crack opening) $w_n$ and in the tangential $w_t$ directions with respect to the crack:
    - For the first family of cracks of the top layer: $\mathbf{w}_1^t$
    - For the second family of cracks of the top layer: $\mathbf{w}_2^t$
    - For the first family of cracks of the bottom layer: $\mathbf{w}_3^b$
    - For the second family of cracks of the bottom layer: $\mathbf{w}_4^b$
  - The steel-concrete inelastic slip $\mathbf{v}^p = (v_x^p, v_y^p)$ in the x $v_x^p$ and y $v_y^p$ reinforcement directions:
    - For the top layer: $\mathbf{v}^{pt}$
    - For the bottom layer: $\mathbf{v}^{pb}$
  - The steel plastic strain $\mathbf{e}^{ps} = (e_x^{ps}, e_y^{ps})$ in the x $e_x^{ps}$ and y $e_y^{ps}$ reinforcement
directions:
- For the top layer: $\varepsilon^{pst}$
- For the bottom layer: $\varepsilon^{psb}$
- Concrete damage variable $d$:
  - For the top layer: $d^t$
  - For the bottom layer: $d^b$

Thus a total of 18 internal scalar variables, together with the observable strain variables, define the Helmholtz free energy surface density, which can be expressed as the sum of three different terms:

$$
\psi^o = \psi^{lin}(\varepsilon, \kappa, w_j^i, w_j^b, \varepsilon^{pst}, d^t, w_t^b, \varepsilon^{psb}, d^b) + \psi^{nl,t}(w_j^i, w_j^b, \varepsilon^{pst}, d^t) + \psi^{nl,b}(w_t^b, w_b^b, \varepsilon^{psb}, d^b)
$$

with $\psi^{lin}$ depending on the observable variables

$$
\psi^{lin} = \frac{1}{2} \begin{pmatrix} \varepsilon \end{pmatrix} \left( \begin{pmatrix} A_{mm} & A_{mf} \end{pmatrix} \begin{pmatrix} \varepsilon \\ \kappa \end{pmatrix} - \sum_\beta \left( \sum_\gamma \left( \begin{pmatrix} B_{\gamma\beta} \\ C_{\gamma\beta} \end{pmatrix} \cdot w^\beta - \begin{pmatrix} \varepsilon^{m\beta} \\ \varepsilon^{ps\beta} \end{pmatrix} \right) \right) \right)
$$

and $\psi^{nl,t}$ and $\psi^{nl,b}$ (one for each $\beta = top, bottom$ layer) determining the coupling between the internal variables:

$$
\psi^{nl,\beta} = \sum_\gamma \frac{1}{2} w^\beta_y \cdot D^\beta_y \cdot w^\beta_y + \frac{1}{2} w^\beta_y \cdot v^\beta_y \cdot v^\beta_y + \frac{1}{2} w^\beta_y \cdot F^\beta_y \cdot \varepsilon^{ps\beta} - \sum_\gamma \left( \begin{pmatrix} w^\beta \cdot G^\beta_y \cdot v^\beta + w^\beta \cdot H^\beta \cdot w^\beta \end{pmatrix} \right)
$$

where $\gamma = 1, 2$ denotes the first and second family of cracks for each layer.

It is remarked that:
- The fourth order tensor $A_{mm}$, $A_{mf}$ and $A_{ff}$ depend on $d^t$ and $d^b$
- The three order tensors $B_{\gamma\beta}$ and $C_{\gamma\beta}$ depend on $d^\beta$
- The two order tensors $D^\beta_y$, $E^\beta_y$, $G^\beta_y$ and $H^\beta$ depend on $d^\beta$

and all of them, together with the three order tensors $C_{\gamma\beta}$ and $C_{\gamma\beta}$ and the two order tensors $F^\beta$ are determined uniquely from:
- Geometrical properties of the section: section height, reinforcement layers position in the height and the steel reinforcement section.
- Material elastic properties: concrete and steel Young’s modulus and concrete Poisson’s ratio.
- Crack pattern geometry: crack spacing and orientation for each of the two crack families of both top and bottom layers. These parameters are automatically calculated by the model at the cracking onset, following the indications given in 2.2.1.

The exact expressions of these tensors can be found in Huguet (2016).

2.1.2. Thermodynamic forces

The constitutive law of the model is the relationship between the generalized efforts:
- in-plane stress resultant $N = (N_{xx}, N_{yy}, N_{xy})$
- out-of-plane generalized bending moment $M = (M_{xx}, M_{yy}, M_{xy})$

and the observable variables: the generalized membrane strain $\varepsilon$ and the generalized curvature $\kappa$. By definition, this relationship is obtained by derivation of the Helmholtz free energy surface density:

$$
\begin{align*}
N &= \frac{\partial \psi^o}{\partial \varepsilon} = \frac{\partial \psi^{lin}}{\partial \varepsilon} = A_{mm} \varepsilon + A_{mf} \kappa - \sum_\beta \left( \sum_\gamma \left( \begin{pmatrix} B_{\gamma\beta} & \cdot w^\beta + \begin{pmatrix} C_{\gamma\beta} \cdot \varepsilon^{ps\beta} \end{pmatrix} \right) \right) \\
M &= \frac{\partial \psi^o}{\partial \kappa} = \frac{\partial \psi^{lin}}{\partial \kappa} = A_{mf} \varepsilon + A_{ff} \kappa - \sum_\beta \left( \sum_\gamma \left( \begin{pmatrix} B_{\gamma\beta} \cdot \varepsilon^\beta + \begin{pmatrix} C_{\gamma\beta} \cdot \varepsilon^{ps\beta} \end{pmatrix} \right) \right)
\end{align*}
$$

The thermodynamic forces associated to each of the internal variables are also obtained by derivation
of the Helmholtz free energy surface density:

\[ y^\beta = - \frac{\partial \psi^\beta}{\partial d^\beta} \quad q^\beta_r = - \frac{\partial \psi^\beta}{\partial \omega^r} \quad q^\beta_\gamma = - \frac{\partial \psi^\beta}{\partial \Omega^\gamma} \quad q^\beta = - \frac{\partial \psi^\beta}{\partial \xi^m} \quad (5) \]

Since \( \partial A_{mm}^m / \partial d^\beta \), \( \partial A_{ff}^m / \partial d^\beta \), \( \partial A_{ff}^m / \partial d^\beta \), \( \partial A_{mm}^f / \partial d^\beta \) and \( \partial A_{ff}^f / \partial d^\beta \) only depend on \( d^\beta \), it can be proven that the previous four thermodynamic forces on the layer \( \beta \) only depend on the internal variables of the same layer \( \beta \).

The evolution of the internal variables is governed by the threshold functions defined with respect to these thermodynamic forces, as presented in paragraph 2.2.

### 2.2 Threshold functions: local constitutive laws

The evolution of the 18 scalar internal variables presented in the previous section is governed by 18 scalar threshold equations depending on the thermodynamic forces of the model. These threshold functions are the description of each of the four nonlinear physical phenomena retained for the description of the RC plates behavior: concrete cracking, concrete damage, steel yielding and the relative steel-concrete slip.

#### 2.2.1 Concrete cracking

The good modelling of concrete cracking is of great importance since it is an important source of nonlinear mechanical behaviour and also the civil engineering design codes as EC2 and MC10 demand to assess the maximum crack opening in order to preserve the durability, tightness and aesthetics of RC buildings.

In a RC plate submitted to any loading originating tension stress in at least the first principal direction, cracking onset occurs when the stress reaches the concrete tensile strength:

\[ F_{ct}(\sigma^c) = \sigma_i^c - f_{ct} \leq 0 \quad (6) \]

The crack pattern evolves with the increasing load (not changing its direction but only the magnitude of the crack width) until reaching a stabilized crack pattern. Typically, in a RC wall submitted to a
non-reversed horizontal load which creates a global shear stress state, a quite regular diagonal cracking pattern appears as in Figure 1 (a), characterized by an average spacing \( s_r \) and orientation \( \theta_r \). If the applied load is cyclic, as observed in Figure 1 (b), two different family of cracks can be distinguished, characterized by the average crack orientations \( \theta_{r1} \) and \( \theta_{r2} \) and spacings \( s_{r1} \) and \( s_{r2} \), see Figure 1 (d). Since a new crack family needs the development of tensile stress in the orthogonal direction with respect to the existing crack family, the latter appear with a significant difference of orientation with respect to the first one. In the model, a minimum difference in crack orientations of 60° is imposed in order to have only 2 crack families.

When a crack family appears following Equation (6), the crack orientation \( \theta_r \) is fixed as the orthogonal direction to the first principal stress direction. Moreover, the average crack spacing \( s_r \) is calculated from the crack orientation with the expression that was first given by Vecchio and Collins (1986):

\[
s_r = \frac{\sin|\theta_r|}{s_{rx}} + \frac{\cos\theta_1}{s_{ry}}
\]

where \( s_{rx} \) and \( s_{ry} \) are the theoretical average crack spacings in the equivalent RC tie-beams in the \( x \) and \( y \) directions (steel reinforcement directions), and which can be calculated with the expressions given by EC2 or MC10 for example.

Once cracks have appeared, crack spacing and orientation remain constant, while concrete displacement both in the normal (crack width) and tangential direction evolve following the associated laws concerning concrete stress transfer at cracks:

- In the normal direction with respect to the crack, a law representing the bridging stress effect is retained. Figure 2 (a) shows the linear post-peak behavior (after attainment of the concrete tensile strength \( f_{ct} \)) with respect to the crack opening; the chosen slope of \( f_{ct}/(2G_f) \) ensures that the concrete fracture energy dissipation \( G_f \) is satisfied. Unloading is done at constant crack width until compressive stress is reached, where crack recloses linearly with the stress up to a minimum residual crack width defined as the \( \alpha_r \) ratio of the maximum historic value \( w_n^{max} \). Reloading is done under constant crack width under compressive strength and then a linear curve is retained to recover the last point on the envelope curve.

- In the tangential direction with respect to the crack, the aggregate interlock phenomenon is modeled with the law shown in Figure 2 (b). No tangential slip occurs until the \( T_0 \) tangential stress at cracks is reached; then a linear curve with a constant slope \( T_1 \) represents the increase of the aggregate interlock with the tangential slip at low \( w_t \) values.

![Figure 2. Retained cyclic laws for (a) bridging stress and (b) aggregate interlock phenomena](image)

The suitability of this approach for representing the cracking phenomena on RC walls is analyzed in Huguet et al. (2016).

### 2.2.2 Concrete damage

In GLRC_HEGIS model, the onset and development of homogeneous diffuse micro-cracking resulting
in a concrete stiffness reduction is modeled by using an isotropic concrete damage formulation. It is considered that the degradation of concrete stiffness only takes place at high stress levels, above the tensile strength \( f_{\alpha} \). Therefore, concrete damage evolution is supposed to evolve when concrete is submitted to high compressive stress, although it affects the mechanical behavior for all types of stress state. The associated constant threshold \( k_0 \), which limits the energy release rate in concrete \( Y^d \), is defined to correspond to the nonlinear behavior onset of concrete in compression:

\[
f_d = Y^d - k_0 \leq 0
\]  
(8)

Concrete damage is governed by the rational function \( 0 \leq \zeta(d) \leq 1 \) which depends on the scalar damage variable \( d > 0 \) and accounts for an isotropic damage acting on the concrete stiffness tensor \( C_c \) so affecting the relationship between concrete strain \( \varepsilon_c \) and stress \( \sigma_c \):

\[
\sigma_c = \zeta(d)C_c\varepsilon_c = \frac{1 + \gamma d}{1 + d} C_c\varepsilon_c
\]  
(9)

where it is noted that this function differs from the classical damage function \( 1 - d \) (so that that damage variable \( d \) is not limited to 1) and that \( \gamma \) parameter reads for the tangent stiffness in the damage evolution phase.

2.2.3 Steel-concrete slip and tension stiffening effect

Cracking originates a relative slip between the steel reinforcement bars and the surrounding concrete (displacement discontinuity in concrete at cracks vs. steel reinforcement displacement continuity) and bond stresses appear at the steel-concrete interface to oppose to this slip. An elastic perfectly-plastic law for these stresses in both \( x \) and \( y \) reinforcement directions is retained, with the local elastic stiffness \( K_t \) which can be measured in pull-out tests and the internal variable \( \xi^p \) acting as the inelastic slip.

Between two cracks, bond stresses transmission form steel bars to concrete originates the tension stiffening effect. Since a part of steel stress is still carried by concrete between cracks, cracked RC element submitted to a monotonic load has a stiffer behavior than the reinforcement bars without the concrete contribution. Equivalently, when the RC element is unloaded in a cyclic load history, this effect reverses its sign and the structural response becomes less stiff than the steel bars response. This phenomenon is represented by the retained elastic-plastic law with a constant threshold of \( k_t f_{\text{y,ct}} \) for the average tension stiffening stress in concrete for both loading and unloading, which is in accordance with the civil engineering codes EC2 and MC10 (where relatively conservative values of the tension stiffening coefficient \( k_t \) can be found).

2.2.4 Steel plasticity

The steel reinforcement bars are supposed to carry only longitudinal stress. For both \( x \) and \( y \) steel reinforcement layers, the steel mechanical behavior is modeled using an elastic–plastic constitutive law. The constant threshold is equal to the steel yielding stress \( f_{\text{y,ct}} \), and the internal variable \( \xi^{ps} = (\xi^p_x, \xi^p_y) \) acts as the scalar strain plastic variable in both \( x \) and \( y \) reinforcement layers.

In reason of the tension stiffening effect, which concentrates the highest steel stresses at cracks, steel plastic strains are considered to be only located at the crack crossings. Therefore, between two consecutive cracks, the mechanical behavior of steel reinforcement is linear elastic.

2.3 Evolution of internal variables

Contrary to most homogenized models, which need a numerical homogenization of the problem implying an important numerical time cost, GLRC_HEGIS model is obtained by means of an analytical multi-scale analysis detailed in Huguet et al. (2017). Two main results are obtained:
- The exact expression of the 4th, 3rd, and 2nd order tensors which define the Helmholtz free energy surface density in section 2.1.1.
- The expression of the retained local constitutive laws of section 2.2 as functions of the thermodynamic forces (given by Equation (5)) of the model: \( f_{g,n} f_{g,t}, f_{d}, f_{v} \) and \( f_{s} \) account for the bridging stress, aggregate interlock, concrete damage, steel-concrete slip and steel yielding local law expressed as functions of the thermodynamic forces.

With the second result, the internal variables evolution can be easily defined in the framework of Thermodynamics of Irreversible Processes. In particular, with the choice of the normality rule for each of the nonlinear physical phenomena, a Generalized Standard Material formulation results for GLRC_HEGIS model which ensures a high robustness of the convergence of the internal variables evolution problem:

\[ \dot{\beta} = \dot{\lambda}_n \frac{\partial f_\alpha}{\partial \psi_n} \]  
\[ \dot{\psi}_n = \dot{\lambda}_n \frac{\partial f_\alpha}{\partial \psi_n} \]  
\[ \dot{\psi}_{y,n} = \dot{\lambda}_{y,n} \frac{\partial f_\alpha}{\partial \psi_{y,n}} \]  
\[ \dot{\psi}_{v,t} = \dot{\lambda}_{v,t} \frac{\partial f_\alpha}{\partial \psi_{v,t}} \]  
\[ \dot{\psi}_{x,a} = \dot{\lambda}_{x,a} \frac{\partial f_\alpha}{\partial \psi_{x,a}} \]  
\[ \dot{\psi}_{y,a} = \dot{\lambda}_{y,a} \frac{\partial f_\alpha}{\partial \psi_{y,a}} \]  
\[ \dot{\psi}_{v,a} = \dot{\lambda}_{v,a} \frac{\partial f_\alpha}{\partial \psi_{v,a}} \]  
\[ \dot{\psi}_{x,a} = \dot{\lambda}_{x,a} \frac{\partial f_\alpha}{\partial \psi_{x,a}} \]  

where the indexes account for the internal variables in the \( \alpha = x,y \) reinforcement directions, \( \beta = top, bottom \) layers and \( \gamma = 1,2 \) families of cracks, so the previous equation accounts to 18 scalar equations (one for each scalar internal variable); the Kuhn-Tucker conditions are satisfied for each of the 18 internal variable evolution definitions.

### 3. NUMERICAL APPLICATION OF GLRC_HEGIS MODEL

The GLRC_HEGIS model is implemented in DKTG shell finite elements in Code_Aster (EDF, 2018) and is applied to three different case studies:
- Evaluation of the capacity of a RC section at the ULS, in section 3.1.
- Pseudo-static cyclic non-reversing global shear loading on a RC wall, in section 3.2.
- Transient dynamic analysis on a RC building with RC walls and slabs modeled with GLRC_HEGIS, in section 3.3.

#### 3.1 Interaction Diagram at the Ultimate Limit State

#### 3.1.1 Test description and modeling

Although the main objective of GLRC_HEGIS model is to represent the nonlinear cyclic mechanical behavior of RC walls and slabs (see section 1), in this section the capability of the model to represent the strength of a RC section with respect to the coupled action of an axial force \( N \) and a bending moment \( M \) is analyzed. This will show whether the model is able or not to limit the efforts in the section to reasonable values. The so-called N-M interaction diagram can be obtained by calculating several couples of axial forces and bending moments originating the failure of the section (steel reinforcement and/or concrete failure). In particular, the strain states of Table 1 are considered as the most representative at the ULS (for example, points 1 and 6 account for pure compression and tension respectively) and are used to calculate 6 points of the N-M interaction diagram.

<table>
<thead>
<tr>
<th>Point</th>
<th>Lower/Upper steel bars strain</th>
<th>Upper/Lower surface concrete strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-45 %0</td>
<td>-45 %0</td>
</tr>
<tr>
<td>2</td>
<td>-45 %0</td>
<td>3.5 %0</td>
</tr>
<tr>
<td>3</td>
<td>-10 %0</td>
<td>3.5 %0</td>
</tr>
<tr>
<td>4</td>
<td>-3.5 %0</td>
<td>3.5 %0</td>
</tr>
<tr>
<td>5</td>
<td>0.35 %0</td>
<td>3.5 %0</td>
</tr>
<tr>
<td>6</td>
<td>2 %0</td>
<td>2 %0</td>
</tr>
</tbody>
</table>

Table 1. Considered strain states for defining the N-M interaction diagram

The considered RC element is a shell of 1mx1m with 0.5m thickness. The steel bars are defined by
two layers of 25cm² section located at 10mm from the concrete border. The considered mechanical properties are: concrete Young’s modulus $E_c = 32.8 \, GPa$, steel Young’s modulus $E_s = 210 \, GPa$, mass density $\rho = 2500 \, kg/m^3$, concrete tensile strength $f_{ct} = 2 \, MPa$, concrete compressive strength $f_c = 16.8 \, MPa$ and steel yield stress $f_{sy} = 500 \, MPa$.

3.1.2 Results

The obtained numerical results obtained with GLRC_HEGIS model are compared in Figure 3 with the N-M diagram obtained with the EC2 calculation hypotheses.

![Interaction diagrams obtained with GLRC_HEGIS and EC2](image)

Figure 3. Interaction diagrams obtained with GLRC_HEGIS and EC2

It is observed that the tensile and compressive strengths are perfectly simulated by GLRC_HEGIS. The composite bending in compression is also well reproduced. However, the capacity of the section when submitted to pure bending and bending coupled with traction is overestimated. This difference is originated by the theoretical formulation of the model: constant values of the internal variables (specially the crack width and the damage variable) are considered in the two halves of the section height. In a nonlinear global model as this one, intended for reproducing the cyclic mechanical behavior of RC walls and slabs, a better representation of the bending strength is hardly achievable.

3.2 Shear RC wall number 3 of CEOS.fr project

3.2.1 Test description and modeling

In the CEOS.fr (Comportement et Evaluation des Ouvrages Spéciaux. Fissuration – Retrait / Behavior and Assessment of Special Structures. Cracking – Shrinkage) experimental program (IREX, 2018), four RC shear walls have been tested and a lot of results concerning the global force-displacement curves and the crack pattern and widths values (maximum and residual crack widths at the end of a cyclic loading) are obtained (Bisch et al. 2018).

In this paper, the GLRC_HEGIS model is applied to the test concerning the RC wall number 3, which represents a standard RC wall used in nuclear facilities with an assumed geometrical scale factor of 1/3. The mock-up has the following dimensions: 4200mm of length, 1050mm of height and 150mm of thickness. Cracking due to global bending is prevented by vertical rebars of 25mm and 32mm diameter at the extremities and by a height/length ratio of 1/4 which ensures a low slenderness of the wall. Two highly reinforced RC beams at the top and the bottom of the wall assure that the applied (by an actuator) horizontal force on the top beam is distributed as a global shear effort in the RC wall. As a consequence, a relatively uniform shear crack pattern is expected to appear in the wall.

Figure 4 (a) shows the steel frame that contains the RC walls, whose vertical displacements are restrained by two couples of Dywidag bars, and which form together a self-equilibrated system. LVDT (Linear Voltage Differential Transformers) sensors C5, C6 and C7 measure the evolution of three crack widths, while the relative displacement between sensors C9 and C10 defines the global force-displacement curve which is used for the numerical comparison. The actuator on the right of the top
beam applies the cyclic (without reversing the sign) loading history shown in Figure 4 (b) as a pseudo-
static force since the scale of time in the experimental test is very large.

![Figure 4. RC wall 3 of CEOS.fr program: (a) scheme of the steel frame structure, load application and instrumentation, and (b) applied loading history](image)

The wall is reinforced in both layers with a 15.7 cm$^2$/m$^2$ steel section (1.05% reinforcement ratio) with steel bars of 10mm of diameter spaced by 100mm, with a cover of 10mm and 20mm for the horizontal and vertical rebars respectively. The measured steel mechanical properties are: Young’s modulus $E_s = 200$ GPa, mass density 7850 kg/m$^3$ and yield stress $f_{sy} = 555$ MPa. Concrete experimental properties are: Young’s modulus $E_c = 27.4$ GPa, Poisson’s ratio $\nu_c = 0.13$, mass density 2200 kg/m$^3$, compressive strength $f_c = 40.0$ MPa, tensile strength $f_{ct} = 3.5$ MPa and fracture energy $G_f = 158$ J/m$^2$; actually the concrete size effect between the sample of the tensile test and the wall is estimated to 2/3, so $f_{ct} = 2.33$ MPa and $G_f = 71.1$ J/m$^2$ are retained in computations in order to maintain the post-peak slope.

Concerning the rest of the nonlinear model parameters needed by GLRC_HEGIS, concrete damage is supposed to appear in compression at $f_c/4 = 10$ MPa and the asymptotic damage slope is set to $\gamma_d = 0.3$, so the associated damage threshold is $k_0 = 1277$ Pa. The theoretical average crack spacings are estimated as $s_{rx} = 124$ mm and $s_{ry} = 138$ mm, which differ due to the different cover in each direction. Between cracks, a relatively low tension stiffening coefficient $k_t = 0.2$ is supposed to be developed in reason of the cyclic loading which degrades the steel-concrete bond. In reason of the cyclic loading, the values of the cyclic bridging stress parameters are retained as $\alpha_r = 0.02$ and $\alpha_u = 0.05$. As no extremely large crack widths are expected, aggregate interlock parameters are retained as $T_0 = 0.05$ MPa and $T_1 = 10$ GPa/m.

### 3.2.2 Results

The comparison between experimental and numerical results concerning the global force-displacement curve is shown in Figure 5. Both the envelope curve and the unloading-reloading cycles are relatively well represented: the model shows a relatively satisfactory representation of the permanent strains (related to the remaining crack widths, governed by the crack reclosing ratio $\alpha_r$) and the hysteretic loops (partially governed when unloading by $\alpha_u$ parameter), specially for the first load cycles.

![Figure 5. Experimental vs. numerical global force-displacement curve](image)
The numerical and experimental crack pattern evolution, one of the main sources of the nonlinear behavior of the wall, is shown in Figure 6 for the different load steps: 1500, 2400 and 3600 KN, the last one corresponding to the experimentally observed stabilization of crack pattern. The numerical crack pattern is represented by the zones where the crack width internal variable \( w_n \) is greater than 0 (color different from blue). It is noted that the constitutive model predict correctly that the first cracks appear in the lower right corner of the wall and then cracking propagates gradually to the rest of the wall. Moreover, far from the wall edges, the experimental average crack orientation of \( \theta_t \approx 28^\circ \) is found.

Figure 6. Experimental vs. numerical crack pattern evolution

Figure 7 presents a more local analysis concerning the evolution of crack widths. The experimental measure of LVDT sensors C5, C6 and C7 are compared with the internal variable \( w_n \) average within the finite element in which each sensor is located. It is observed that cracking onset is well represented as well as the crack width evolution with the increasing load. One part of the slight difference is originated by the fact that the punctual measures of crack width values made by LVDT sensors are in some cases not too much representative of the average crack width in a zone (or finite element).

Figure 7. Experimental vs. numerical crack width evolution at LVDT sensors C5, C6 and C7

3.3 RC building of SMART benchmark

3.3.1 Test description and modeling

The RC building of the benchmark SMART (‘Seismic design and best-estimate Methods Assessment for Reinforced concrete buildings subjected to Torsion and non-linear effects’), supported by EDF and CEA (and partially endorsed by the International Atomic Energy Agency), is considered in this section (see Richard et al. 2016). The considered structure is the asymmetric 1/4 scale mock-up of a RC multi-story building representative of NPP buildings, see Figure 8 (a).

The experimental mock-up consists of RC slabs, walls, beams and columns. Only the two first types of structural elements are modeled with GLRC_HEGIS for the FE analysis, while beams and columns are assumed as linear elastic. The mechanic and geometric properties of each of the walls is specified in (Richard et al. 2016), and the nonlinear parameters identification is done following the principles
used in section 3.2.1. As the material parameter identification is different for each of the RC walls and slabs, the details are not shown here but can be found in Huguet (2016).

The three-component seismic signal of Figure 8 (b) is applied at the basis of the building and a full transient dynamic analysis is performed in Code_Aster FE software.

3.3.2 Results

One of the main goals of the development of GLRC_HEGIS is to be able to do an advanced crack assessment. In this sense, at each load step of the transient dynamic analysis, the model gives as a direct result the field distribution of all the state variables, and among them the crack orientation, spacing (which remain fixed after the first cracking) and the crack displacement in the normal (width) and tangential direction.

Figure 9 (a) shows that the crack width distribution at the load step where the crack width is maximized at the basis of the left wall, while Figure 9 (b) accounts for the residual crack width field (when the oscillations of the building have stopped). It is observed that the mains cracked zones are at the connection of the walls with the foundation and at the corners of the walls openings. Equivalently, the crack tangential slip at the same maximization load step shown in Figure 9 (c) and the residual crack tangential slip in Figure 9 (d) highlights than this phenomenon appears at the same zones as crack opening but there is less recover, because no tangential slip reclosing is adopted in the retained aggregate interlock law of Figure 2 (b).

The steel plastic strain distribution of Figure 10 (a) shows that steel yielding is concentrated at the wall connection with the foundation, where the maximum crack widths are also found. The total energy dissipation surface density (the sum of the contributions of the four considered nonlinear phenomena) at the end of the test of Figure 10 (b) is non-zero in many zones of the building; it attains a maximum at the same location where crack width, tangential slip and steel plastic strain attain their maximum values.
4. CONCLUSIONS

The GLRC_HEGIS model for RC plates (walls and slabs) submitted to in-plane forces and out-of-plane bending moments is presented in this paper. The Helmholtz free energy surface density, the threshold functions and the evolution of the 18 internal variables of the model are detailed. A great care is dedicated to explain the local nonlinear physical phenomena that govern the global nonlinear mechanical behavior of the model. This model is implemented in shell finite elements in Code_Aster software and is applied to a pseudo-static and a transient dynamic structural analyses corresponding to experimental test. The first one consists in a RC wall submitted to a cyclic (non-reversing) loading history while the second one is a non-symmetrical three-story RC building. Comparison with experimental results shows a relatively good fitting both at the local (e.g. crack width) and global (structural behavior) scales. The capabilities of the model to show the evolution of variables (crack width, steel reinforcement plastic strain, energy dissipation…) fields at every load step, and to estimate in a relatively accurate manner the RC section strength, are also demonstrated.

5. REFERENCES


