Investigating Limit States for Butterfly-Shaped and Straight Shear Links

Alireza Farzampour¹, Matthew Eatherton²

ABSTRACT

Structural fuses can be implemented in buildings as steel plates subjected to shear loading. Strategically engineered cut-outs in these steel plates can be used to create shear links with energy dissipating capability. The links are used to protect the surrounding structure from damage and significant inelastic deformation, and then be replaceable after a major earthquake. Previous studies indicate that structural fuses with shear links improve the initial elastic stiffness, energy dissipation capability and ductility of structures. Butterfly-shaped links that have varying width between larger ends and a smaller middle section have been used to better align bending capacity with moment demand along the length of the link. These links have been shown in previous tests to be capable of substantial ductility and energy dissipation. In this study, the shear and flexural limit states associated with butterfly-shaped links are investigated. The butterfly angle, defined as the angle between intersections of the inclined edges at the middle of the link, is investigated. Using the flexural and shear limit state equations, the critical butterfly angle identifying the transition from the shear to flexure is proposed. The stiffness of a butterfly shape link is developed by dividing the stiffness into four major parts: link flexural deformations, link shear deformations, and shear and flexure deformations of the end zones. Subsequently, finite element models are presented for two configurations to examine the validity of the proposed limit state prediction and stiffness equations. The procedures presented in this paper to predict the controlling limit state, shear strength, and stiffness can be useful in the design of butterfly-shaped links. Design considerations are explored to determine how these concepts could be applied in real structures such as buildings and bridges to improve seismic performance.

Keywords: Structural Fuses, Energy dissipation, Flexural and shear behavior, Buckling limit state, Butterfly shaped links.

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1. INTRODUCTION

While many buildings are designed and built to avoid collapse and save human lives, steel structures can still suffer severe damage during large earthquakes. For limiting the damages and inelastic deformations in the structure, structural fuses have been used to focus inelasticity, protect the surrounding structure from damages, and then be replaceable after a major event (Martinez-Rueda 2002). There are many forms of structural elements with adequate ductility and energy dissipation capability to be implemented as structural fuses. One class of structural fuses consists of steel plates with links cut into them, which yield when the plate is subjected to shear. Slit (straight links) or butterfly-shaped links have indicated advantages in several studies. Butterfly-shaped links (BF), as shown in Figure 1, align moment capacity with the shape of the moment diagram leading to efficient implementation of the steel with distributed yielding. The planar geometry of the butterfly-shaped links make them applicable for space-constrained applications as well (Ma et al. 2011); however, these links are subjected to buckling (Ma et al. 2011, Farzampour and Eatherton 2017).

Hysteretic dampers such as the added damping and stiffness (ADAS) and triangular-plate added damping and stiffness device (TADAS) use tapered links bent about their minor axis (Whittaker et al. 1991, Tsai et al. 1993). Similarly, tapered links can be bent about their major axis (in-plane bending) to create larger stiffness (Ma et al. 2011). In-plane implementation of the links have been shown to have substantial energy dissipation capability, ductility, and large distribution of yielding (Ma et al. 2011, Lee et al. 2015). To control the drift response of high-rise buildings in-plane links are implemented with similar shapes to butterfly shaped fuses for the purposes of appropriate energy dissipation and reducing the demands on the framing members.

Another form of in-plane flexural link is the straight link formed by cutting slits in a steel plate. These links are capable of increasing the ductility of the whole system, reduce the buckling occurrence and have the flexural hinges at the ends as the typical limit states (Ma et al. 2011, Hitaka and Matsui 2006). Slit links have been implemented in structural applications to concentrate the inelasticity far from connection area, which protects the beam-column joints (e.g. Oh et al. 2009).

In this study, equations governing the flexural and shear yielding behavior of the butterfly shaped links as well as the slit links are proposed. The critical angle of butterfly shaped link identifying the transition between the flexure and shear limit states is derived. Similarly, the limiting length of straight links that delineates flexure and shear controlled behavior is developed. Following that, the stiffness of the links is formulated based on the sum of the link flexure, link shear, end zone shear and flexure deformations. Subsequently, finite element models are shown for a few cases with different limits states governing the behavior to compare the proposed critical angle, limit state and stiffness equations with finite element models.

2. INVESTIGATION OF BUTTERFLY-SHAPED LINKS STRENGTH DUE TO DIFFERENT LIMIT STATES

Butterfly-shaped links can experience different limits states such as lateral torsional buckling, shear yielding and flexural yielding. The lateral torsional buckling limit state is previously investigated by Farzampour and Eatherton (2017), and is not explained further in this paper. In this section, the butterfly shaped link shear strength associated with flexure and shear is detailed and formulated. In this derivation and those in the following sections, normal stresses and shear stresses are considered independently (no interaction between flexure and shear), a plane stress condition is assumed, the axial force is taken as zero, and residual stresses are ignored.

2.1 Flexural limit state

To calculate the shear strength of the butterfly links, general geometrical properties and loading condition are defined in Figure 1.
Capacity investigation under flexure means that the whole section would be completely plastic under flexural limit state. It is noted that geometrical properties of the section vary along the length of the butterfly shaped link and are thus a function of the coordinate z. For example, Equation 1 shows the width of each section as a function of z. The force causing the section to develop a flexural plastic hinge is derived by establishing the force associated with flexural plastic stress equations as it is shown in Equation 2 and Equation 3.

\[ w(z) = \frac{2(b-a)z}{L} + a \]  
\[ \sigma_y \frac{tw(z)^2}{4} = VZ \]  
\[ V(z) = \frac{g_y(2(b-a)z+a)^2}{4z} \]

in which \(a, b, L,\) and \(t\) are geometrical properties associated with butterfly link shown in Figure 1. \(V(z)\) is the shear along the length of the link. By taking the derivative of the Equation 2 with respect to \(z\), and equating it to zero, the minimum amount of force associated with flexural capacity would be derived, which is indicated in Equation 4.

\[ V_{p-bf-fx} = \frac{2n(b-a)at\sigma_y}{L} \quad \text{at} \quad z_m = \frac{a}{b-a} \frac{L}{2} \]  

in which \(n\) is the number of links used in a row. Similarly, to get the shear load associated with the first yield, \(V_{y-bf-shr}\), the section modulus is used as it is given in Equation 5.

\[ V_{y-bf-shr} = \frac{4n(b-a)at\sigma_y}{3L} \quad \text{at} \quad z_m = \frac{a}{b-a} \frac{L}{2} \]

in which \(n\) is the number of links used in a row. It should be noted that the moment associated with full plastic flexural hinging of the butterfly-shaped link is 3/2 times greater than the corresponding moment associated with first flexural yielding of the section due to the ratio of plastic section modulus to flexural section modulus. The hinge location is calculated from the middle of the link; therefore, the values of the \(z_m\) should be less than \(L/2\). If the \(a/b\) ratio is less than 0.5, then it means that the inelasticity would be along the length of the link (Equation 6). For the cases with \(a/b>0.5\) it is shown that the hinge would be formed at the end of the butterfly shaped link, similar to straight links (Equation 7).

\[ z_m = \frac{a}{b-a} \frac{L}{2} \quad \text{for} \quad \frac{a}{b} \leq 0.5 \]

\[ z_m = \frac{L}{2} \quad \text{for} \quad \frac{a}{b} > 0.5 \]
The possible areas susceptible to strain accumulation, and subsequently a fracture, are those areas that hinges are developed. In these areas, usually deformation concentrates and a joint of two edges with different curvatures are located.

The butterfly link has a geometrical change at the end of the links as well as the middle section, which would be the potential point with strain accumulation. Therefore, the best area to develop the hinges are those areas, which are far from the sharp angles, indicating that the quarter points, which are between middle section and the end section of the BF links are relatively appropriate points for inelasticity concentration. The appropriate a/b ratio causing the hinges occur in the quarter points \((Z_m=L/4)\), is 1/3 which is shown in Equation 8. It means that if the hinges are located at quarter points, then the a/b ratio should be 1/3 based on the flexural stress estimation.

\[
\frac{a}{b} = \frac{1}{3} \quad \text{to get } \quad Z_m = \frac{L}{4}
\]  

(8)

### 2.2 Shear limit state

The shear limit state is formulated in this section. The geometry of the butterfly link is according to Figure 2. Because the shear is constant along the link length, the section with smallest cross-sectional area is the critical location for maximum shear stress and minimum shear strength as shown in Figure 2. The shear capacity of the butterfly-shaped links associated with shear yielding limit state is obtained from the Equation 9.

\[
V_{p-bf-shr} = n \frac{\sigma_y at}{\sqrt{3}} \quad \text{at } \quad Z_m = 0
\]  

(9)

In which \(n\) is the number of the links. The shear force associated with first yield in shear can also be calculated. The maximum shear stress for a rectangular section is calculated based on the Equation 10. It is indicated that for any rectangular section, maximum shear stress is 3/2 times of the average shear stresses applied to a section (Boresi et al., 1992). Based on the Von Mises yield criterion, the shear yield stress is set equal to the tension yield stress divided by the square root of three as given in Equation 11. In which the maximum values for shear stress is equalized with Von Mises yielding rule leading to Equation 12.

\[
\max [\tau_{shear}] = \frac{3\sqrt{3}}{2\sqrt{w(x)t^2}}
\]  

(10)

\[
\frac{3V_{y-bf-shr}}{2} = \frac{\sigma_y}{\sqrt{3}} \quad \text{at } \quad Z_m = 0
\]  

(11)

\[
V_{y-bf-shr} = \frac{2n\sigma_y at}{3\sqrt{3}} \quad \text{at } \quad Z_m = 0
\]  

(12)
3. DETERMINING WHICH LIMIT STATE CONTROLS FOR BUTTERFLY-SHAPED LINKS

The governing limit state between flexure and shear is investigated in this section and the critical geometry in which the governing limit state transitions from one to the other is identified. It is assumed that the loading condition is as shown in Figure 2. To understand the transition between limit states, Equation 4 and Equation 9 are compared as shown in Equation 13, which is simplified as shown in Equation 14. If the section gets to be yielded fully in shear before flexure, then it means that the total force needed to have the link in shear yielding should be less than the total force for flexural hinge.

\[ \frac{b-a}{L} > 0.28 \quad \text{The link is Shear controlled} \quad (13) \]

\[ \frac{V_{p-bf-shr}}{V_{p-bf-flx}} < The \ link \ is \ Shear \ controlled \quad (14) \]

The exact same inequality results when the shear force associated with first yielding in shear is set less than the shear force associated with first yielding in flexure. Equation 13 is therefore applicable to both first yield and full plastic strength. The right hand side of the Equation 13 could be restated with regard to the butterfly angle. By using Figure 2, the butterfly angle, \( \alpha \), can be established as given in Equation 15.

\[ \alpha = 180^\circ - 2 \tan^{-1}\left(\frac{b-a}{L}\right) \quad (15) \]

The Equation 14 could then be reinterpreted as a function of only one geometrical parameter shown in Equation 16 and Equation 17:

\[ \alpha < 148^\circ \quad \text{Shear controlled} \quad (16) \]
\[ \alpha > 148^\circ \quad \text{Flexure controlled} \quad (17) \]

To summarize, if the butterfly angle, \( \alpha \), is less than 148° then the links yield and develop plastic shear yielding before flexural yielding. Conversely, if \( \alpha \) is more than 148°, then flexural yielding will occur before shear yielding. It is worthy of notice that, the butterfly-shaped links tested in the literature used butterfly angles greater than 148° and flexural yielding was observed (Ma et al. 2011; Lee et al. 2015). Figure 3 demonstrates graphically the types of geometry that are shear or flexure governed.

\[ \begin{align*}
\text{Shear Governed} \\
\text{Flexure Governed}
\end{align*} \]

\[ \begin{align*}
a) \text{Increasing a} & \quad b) \text{Decreasing b} & \quad c) \text{Increasing L}
\end{align*} \]

Figure 3. To be going from flexure governed to shear governed

As it is shown for butterfly shaped links, the angle \( \alpha \) is significantly important parameter for determining whether shear yielding or flexure yielding will control. It is expected that flexural yielding will produce more deformation capacity and energy dissipation because the yielding will be
spread out over a larger area of the links. To produce the largest deformation capacity, therefore, the butterfly angle, $\alpha$ should be set larger than the limit required to achieve flexural hinging ($\alpha > 148^\circ$) and the geometrical ratio should be set $a/b=1/3$ to locate the flexural hinges as far as possible from points of stress concentration.

**4. STRENGTH INVESTIGATION ON STRAIGHT LINKS**

The slit fuses’ shear yielding and flexural yielding limit states are investigated in this section. The geometry the slit link is shown in Figure 4. Hinges will form at the ends of the link because the section is constant and the maximum moment demand is at the ends. Equation 18 and Equation 19 indicate the shear force associated with first yield in flexure, and plastic hinging strength, respectively.

![Figure 4. The slit panel with straight links](image)

\[ V_{y-sr-flx} = \frac{nh^2t}{3L} \sigma_y \]  
\[ V_{p-sr-flx} = \frac{nb^2t}{2L} \sigma_y \]  

Along the same lines, equations can be developed for shear yielding. The shear force associated with first yield and plastic strength for the limit state of shear are obtained as given in Equation 20 and Equation 21, respectively.

\[ V_{y-sr-shr} = \frac{2\sigma_y}{3\sqrt{3}} bt \]  
\[ V_{p-sr-shr} = \frac{\sigma_y}{\sqrt{3}} bt \]  

By equating the two equations associated with flexure and shear limit states, a geometrical limit is obtained that defines the transition between shear and flexure as shown in Equation 22 and Equation 23. The same equations results regardless of whether the first yield or full plastic strength equations are considered.

\[ \frac{b}{L} > 1.15 \quad Shear \ controlled \]  
\[ \frac{b}{L} < 1.15 \quad Flexure \ controlled \]  

Therefore, the width of the link, $b$, should be less than 1.15 times the length to have flexure govern over shear. Most of the previous testing has used link geometry that was in the flexure controlled range and flexural hinges were observed at the ends of the straight links (Ma et al. 2011, Lee et al. 2011).
5. STIFFNESS INVESTIGATIONS:

5.1 Butterfly-shaped links:

The shear stiffness of a structural fuse with butterfly-shaped links is obtained using the virtual work method. The force is assumed to be applied on the top edge of the butterfly-shaped links. It is noted that the total displacement is combination of the displacement associated with flexural displacement of butterfly-shaped link ($\delta_b$), shear displacement of butterfly-shaped link ($\delta_v$), and the flexural ($\delta_c$) and shear ($\delta_d$) displacements associated with end zone. The end zone is the plate with depth of $c$ attached to the butterfly shaped link. The stiffness of the butterfly-shaped link includes the end zone and the link itself (Figure 5).

The flexural displacement of one link is obtained as given in Equation 24.

$$\delta_b = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{M(z)m(z)}{EI(z)} \, dz = \frac{3VL^3}{2EB^2t} \left[ \frac{2L \ln \left( \frac{a}{b} \right)}{b} \left( \frac{a}{b} - 1 \right) \left( \frac{a}{b} - 3 \right) \right]$$

(24)

where $n$ is the number of links in a row as indicated Figure 5.a and $E$ is modulus of elasticity. $M(z)$ is the moment along the length of butterfly shaped link, and $m(z)$ is the virtual moment along the length of the link subjected to unit load. $L_n$ is the natural logarithm. For shear deformations of the link, Equation 25 is developed.

$$\delta_v = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\nu(z)p(z)}{\sigma A(z)} \, dz = \frac{6VL}{5Gt} \left[ \ln \left( \frac{b}{a} \right) \right]$$

(25)

where $V(z)$ is the shear along the length of the butterfly shaped link, and $v(z)$ is the virtual shear along the length of link subjected to unit load. Therefore, the stiffness associated with a group of $n$ butterfly-shaped links can be described as given in Equation 26 and 27 for flexural deformations and shear deformations, respectively.

$$K_b = n \left( \frac{V}{\delta_b} \right) = \frac{2nEb^3t}{3L^3} \left[ \frac{2L \ln \left( \frac{a}{b} \right)}{b} \left( \frac{a}{b} - 1 \right) \left( \frac{a}{b} - 3 \right) \right]$$

(26)

$$K_v = n \left( \frac{V}{\delta_v} \right) = \frac{5Gt}{6\pi} \left[ \ln \left( \frac{b}{a} \right) \right]$$

(27)

The effect of deformations in the end zone is described with the help of defining the rotation at the bottom of the butterfly-shaped link, which is shown in Equation 28 and Equation 29. It is noted that
flexural displacement of the end zone (δ_c) over the link length is due to γ, the rotation of the end zone, and displacement of the end zone itself is neglected (Figure 5.b).

\[
\gamma = \frac{\delta_c}{L} = \frac{M}{EI} = \frac{V}{E \left( \frac{L^3}{12} \right)} C
\]

\[
\delta_c = \frac{6PL^2C}{Et^2(L-h)^2} \rightarrow K_c = n \frac{P}{\delta_c} = \frac{nEtL_b^3}{6C(L)^2}
\]

The shear stiffness of the end zone is based on the Equation 30, and it is shown in Figure 5.c.

\[
\delta_d = \frac{6VL}{Gtt_b} \rightarrow K_d = \frac{Gtt_b}{6L}
\]

in which n is the total number of links, L_b is the total length of n number of links. The total stiffness is equal to summation of the flexural and shear of links as well as the end zone flexural and shear stiffness. It is noted that the total stiffness is calculated based on Equation 31.

\[
K_T = \frac{K_bK_cK_d}{K_bK_aK_c+K_bK_pK_d+K_bK_pK_cK_d+K_bK_cK_d}
\]

The effect of the end zone on the stiffness is suggested to be considered in deriving of the stiffness equations (Ma et al. 2011).

5.2 Straight links:

To derive the stiffness of the shear panel with straight links, a shear load is assumed to be applied along the top edge of the plate. The total displacement is comprised of flexural deformation of the link, shear deformation of the link and shear deformation of the end zone. Equation 32 and Equation 33 show the flexural and shear stiffness of the slit link.

\[
\delta_b = \frac{VL^3}{12EI} \rightarrow K_b = \frac{nEtb^3}{L^3}
\]

\[
\delta_d = \frac{VL}{Gtt_a} \rightarrow K_d = \frac{Gtt_b}{6L}
\]

For the flexural stiffness of the end zone associated with the slit link, Equation 34 could be utilized.

\[
\gamma = \frac{\delta_c}{L} = \phi C \rightarrow \delta_c = \frac{M}{EI} = \frac{V}{Et^2L_b^2/12} \rightarrow CL = \frac{6cL^2}{Et^2} \rightarrow K_c = \frac{EtL_b^3}{6cL}
\]

The shear deformations for the end zone are calculated as shown in Equation 35.

\[
\delta_d = \frac{VL}{Gtt_b} \rightarrow K_d = \frac{Gtt_b}{6L}
\]

Therefore, the total stiffness of the shear panel with shear links could be obtained from Equation 36.

\[
K_T = \frac{K_bK_aK_cK_d}{K_bK_aK_c+K_bK_pK_d+K_bK_pK_cK_d+K_bK_cK_d}
\]

5. FINITE ELEMENT MODELING AND VERIFICATIONS

In order to investigate the accuracy of the proposed equations, two example configurations are evaluated, one panel with butterfly-shaped links that are flexure controlled and one that is shear
controlled.

5.1 Validation of Modeling Approach

The finite element software ABAQUS is used and the modeling approach was validated against the experimental specimen B10-36 tested by Ma et al. (2011) which is shown in Figure 6. The bottom edge of the plate is fixed and the upper edge is connected to a reaction frame similar to that used in the tests. A displacement controlled loading history is applied at the top of the reaction frame. A four node shell element with reduced integration is used (S4R) with five integration point through the thickness as it is recommended in previous studies for steel structures (Farzampour et al. 2017, Farzampour et al. 2015, Farzampour and Yekrangnia 2014). The dynamic explicit solver is used to capture the push over curves. A mesh sensitivity analysis was conducted (not shown here) resulting in approximately 10 mm element size (or less) in the shear plate. The material constitutive model is calibrated to match the average values from the coupon tests conducted on the shear plate material. The material had a yield stress of 273 MPa, ultimate stress of 380 MPa, and linear kinematic hardening parameters are evaluated based on the given coupon test data (Ma et al., 2011). The typical geometry of the flexural dominated BF link and shear dominated BF link models are shown in Figure 7.

Figure 6. Verification of Finite element modeling methodology in ABAQUS

Figure 7. The meshed butterfly shaped link
5.2 Example 1 – Flexure Controlled

This model had properties of $a=10\text{cm}$, $b=30\text{cm}$ $L=100\text{cm}$, $t=20\text{mm}$ which is shown in Figure 7.a. This model is chosen to have the flexural limit state as the governing limits state. To check whether flexure limit state would occur before shear, the proposed transitional Equation 15 and Equation 17 are used, which is confirmed in Equation 37. Therefore, the flexure hinges are expected to dominate. As shown in the left inset picture of Figure 8, flexural hinges are forming at approximately the quarter points when the model starts yielding. This observation supports both Equation 16 which predicts flexure will control, and the relationship discussed at the end of Section 3 for which an $a/b$ ratio of 1/3 will result in flexural hinges at the quarter points.

$$[\alpha = 157^\circ] > 148^\circ$$  \hspace{1cm} (37)

Table 1 shows that that Equation 4 predicts the shear strength associated with flexural hinging of the butterfly shaped link with less than 5\% error. It is also shown that the Equation 31 predicts the stiffness with less than 3\% error. Figure 8 shows that there is an increase in strength after the first limit state is reached. This increase is associated with geometric hardening as the link experiences tension at larger displacements. Figure 8 also shows a peak strength is reached at which point the stress distribution suggests a shift from flexure to shear dominated mechanism. The shear strength associated with this second limit state matches well with the predicted strength for shear-dominated shear strength as given in Table 1.

Table 1. The comparison between equations and FE model for flexural dominated BF.

<table>
<thead>
<tr>
<th>Flexure BF</th>
<th>$V_{p-bf-flx}$ (KN)</th>
<th>$V_{p-bf-shr}$ (KN)</th>
<th>STIFFNESS (KN/MM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation 4</td>
<td>230</td>
<td>368</td>
<td>34300</td>
</tr>
<tr>
<td>FE</td>
<td>240</td>
<td>346</td>
<td>33500</td>
</tr>
<tr>
<td>Error (%)</td>
<td>4.2</td>
<td>6.3</td>
<td>2.3</td>
</tr>
</tbody>
</table>

5.3 Example 2 – Shear Controlled

A second example as shown in Figure 7.b, is analyzed for a model expected to experience the shear limit state first; see Figure 7 and Figure 9. The model had dimensions of $a=10\text{cm}$, $b=50\text{cm}$ $L=100\text{cm}$, $t=20\text{mm}$. For this model, the shear yielding would govern over the flexure, which is verified with Equation 38.

$$[\alpha = 136^\circ] < 148^\circ$$  \hspace{1cm} (38)

Table 2 shows that Equation 9 and Equation 31 predict the shear strength and stiffness within 1\% and 7.9\% error, respectively. The observed stress distribution shown in the inset diagram indicates that shear yielding was the first limit state supporting the accuracy of Equation 17 in predicting the shear limit state before flexure for this configuration. Similar to the previous example, geometric hardening
occurs after the first limit state and then the stress distribution at the peak strength appears to be associated with the other limit state (flexure in this case).

Figure 9. Pushover analysis for shear dominated butterfly shaped link in ABAQUS

Table 2. The comparison between equations and FE model for shear dominated BF.

<table>
<thead>
<tr>
<th>Shear BF</th>
<th>$V_{p-bf-shr}$ (KN)</th>
<th>$V_{p-bf-flx}$ (KN)</th>
<th>STIFFNESS (KN/MM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation 9</td>
<td>390</td>
<td>517</td>
<td>76900</td>
</tr>
<tr>
<td>FE</td>
<td>388</td>
<td>482</td>
<td>83500</td>
</tr>
<tr>
<td>Error (%)</td>
<td>0.5</td>
<td>7.2</td>
<td>7.9</td>
</tr>
</tbody>
</table>

6. CONCLUSIONS

In this study, equations for shear strength associated with flexural and shear limit states are proposed and a critical butterfly angle is derived to delineate the transition from the shear to flexure controlled behavior. Similar equations are developed for straight links and a critical length is defined for the transition between shear and flexure controlled behavior. In addition, the stiffness of panels with either butterfly shaped or straight links are derived consisting of three contributors to deformation: link flexure, link shear, and end zone shear and flexure. To validate the proposed equations, two example configurations, one flexure-dominated and one shear-dominated butterfly link were investigated using FE models. The shear strength and stiffness from FE models were compared with the derived equations and it was shown that for two examples, the first limit state and stiffness were predicted within 3% and 5% error, respectively.

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7. REFERENCES


