EXPERIMENTAL INVESTIGATION OF THE STABILITY OF COLONNADES UNDER HARMONIC EXCITATION

Mazen TABBARA¹, Gebran KARAM²

ABSTRACT

The dynamic response of large historical block structures, as manifested by the response of the free-standing variable size drums colonnade of Baalbek due to strong ground shaking during earthquake events, is investigated experimentally for the purpose of predicting damage or collapse due to future earthquake events. Reduced single column multi drum models of scales 1/20 and 1/40 were subjected to harmonic excitations of varying amplitudes and periods on a 1D shaking table for up to 25 cycles noting collapse or survival. The results are compared to two dimensional simulations on UDEC as well as to the predictions of the Housner formula for single rigid block overturning by a half-cycle sinusoidal acceleration pulse. It is shown that the experimental results agree qualitatively with computer simulations for the reduced scale but cannot be directly extrapolated to full scale.

Keywords: Rocking; Harmonic excitation; Shaking table; Distinct element method; Multi-drum column;

1. INTRODUCTION

Seismic events represent one of the major hazards that endanger the conservation of large masonry structures such as encountered in archeological and historical sites. These monuments represent an important part of the world cultural and historical heritage requiring protection. Lebanon is particularly rich in these heritage sites such as Baalbek, Tyr, Saida, Tripoli, and Byblos. In addition to the immediate objective of strengthening and conservation, studying the damage and structural collapse caused to these archeological structures by historically recorded seismic events can help estimate the magnitude and nature of these events making contributions to archaeoseismology.

1.1 Colonnade at Baalbek

A UNESCO World Heritage site since 1984, the temple of Jupiter at Baalbek in the Bekaa valley of Lebanon is the largest temple complex that the Romans ever built relative to its size, its complexity and the height of its colonnade (Ito and Sueyasu, 2009). The temple was surrounded by a colonnade constituted of 54 external multi-drum Corinthian columns of variable size drums supporting a layered lintel structure. The present ruins belong to the temple constructed during the mid-1st century and probably completed around AD 60 presumably to replace an earlier one using the same massive megalithic stone foundation. The temple stones were quarried at the same limestone quarries from which the original foundation blocks were obtained a couple thousand years earlier. Under the reign of Justinian, eight of the complex's Corinthian columns were disassembled and shipped to Constantinople for incorporation in the reconstruction of Hagia Sophia sometime between 532 and 537 giving the cathedral its present 6th-century form.

The site has been devastated by earthquakes over the last ten centuries notably the three major earthquakes that occurred in the 12th century, in 1139, 1157, and 1170. The last destructive earthquake to hit Baalbek was in 1759. Historical records from travelers to the region report that

¹Associate Professor, Lebanese American University, Byblos, Lebanon, mtabbara@lau.edu.lb
²Associate Professor, Lebanese American University, Byblos, Lebanon, gkaram@lau.edu.lb
undamaged columns still remained in the colonnade of the Jupiter temple at Baalbek in 1751 (Wood, 1757). Only 6 damaged columns survived the November 25th, 1759 Bekaa valley earthquake (Ms ~7) (Ambraseys and Barazangi, 1989; Ambraseys and Jackson, 1998; Lewis, 1999). These large roman columns, probably the largest in the world, have a 2.2m diameter and more than 20m in height. They are constituted by five distinct blocks: a base block, three cylindrical drums of decreasing height on top of each other constituting the shaft, and one capital block. The 6 columns are connected at the top by three layers of lintel blocks: architrave, frieze and cornice. The vertical elements of the columns are connected to each other with three short metallic dowel bars used by the Romans to center the blocks and insure perfect alignment and verticality. Given the size and mass at play, molten lead was poured after positioning each block in lieu of an empolium to stabilize the short dowel bars through a small inclined channel carved at the top of each column block.

The colonnade sits on an elevated foundation made of large masonry blocks. Locally quarried strong yellowish limestone was used for all blocks. Figure 1 shows a photograph of the colonnade.

1.2 Investigation of the Rocking Response

Following early work by the authors (Tabbara and Karam, 2010) it was found necessary to understand intimately the different mechanisms that contribute to the seismic stability of large columns with variable size drums such as those of Baalbek. Reduced scale models of the Baalbek columns were manufactured and their response to harmonic loading was investigated on a 1D shaking table to determine stability under rocking and to categorize their resulting failure modes. A commercial DEM software, UDEC (Itasca, 2014), was also used to model in 2D the response to harmonic loading numerically and to establish correspondence between the reduced models and the full size colonnade.

2. EXPERIMENTAL PROGRAM

2.1 Materials and Equipment

Multi drum models of scales 1/20 and 1/40 of the Baalbek columns were prepared from a local limestone with properties similar to the original limestone. The full height of the column is 20m and it is constituted of five distinct blocks of different sizes. The scaled down columns of 1m and 0.5m respectively were made similarly of five distinct blocks maintaining geometric proportions. The dowel bar connection between blocks was not modeled. A solid single piece monolithic column 0.5m in height was also made. The dimensions of the 1m multi-block model column are shown in Figure 1. The relevant physical properties of the limestone used for the models were established through laboratory testing of sample cubes. The density was measured to be 2.72 g/cm³, the water content was measured to be 0.03% and the limestone to limestone friction angle was measured to be 25.68 degree resulting in a friction coefficient of 0.48. The three different models were subjected to harmonic excitations of varying amplitudes and periods on a 1D horizontal shaking table designed and manufactured at the Lebanese American University (LAU) Laboratories by the investigators. The number of cycles per minute or forth and back stroke of the table was varied from zero to 400 while the stroke amplitude of the shaking table was varied from 0.2 to 7.0 cm allowing the application of sinusoidal harmonic horizontal movements of varying frequency and amplitude depending on the combination of cycles per minute and stroke. The resulting periods, T, of the harmonic excitation signal varied between 0.1s and 2.0s corresponding to frequencies from 0.5 to 10Hz. The acceleration, a, was not allowed to exceed 0.48g to avoid sliding and to limit the investigation to rocking stability only.

At the beginning of each test the column models were assembled and centered over a limestone plate fixed to the table. The table was covered with a thick slab of polyurethane foam with a central cutout to allow the column blocks to move, wobble, rock, fall and tumble freely without getting damaged. The 1m and 0.5m multi-block model columns as well as the solid 0.5m column were subjected to varying combinations of frequency and acceleration for up to 25 cycles noting the rocking response of the column and its collapse or survival.
It was noted in previous numerical investigations that the accelerations required to topple or overturn a multi-drum column by rocking under harmonic excitations show some dependency on the number of loading cycles when the number of cycles is less than 5 but tend to stabilize in value when a large number of cycles is applied (Psycharis et al., 2000; Dimitri et al., 2011). The lowest values of critical acceleration are observed at the larger number of cycles. Hence up to 25 cycles were applied to measure a stable conservative value for the critical parameters of the harmonic excitation. All tests were filmed by a digital camera placed perpendicularly to the shaking table movement axis.

2.2 Testing Program and Observations

The boundaries between the stable and unstable domains were investigated for the model columns in the period-acceleration space by varying the number of cycles per minute and stroke of the shaking table. Results were noted and tabulated. The limitations of the shaking table set the limits of the period-acceleration domains that could be covered. Figures 2 to 5 show photographic snapshots of the rocking and overturning of the model columns. The solid block or monolithic column can only fail by overturning as one block, while multi-drum or multi-block columns can fail by different modes depending on how many of the blocks topple or overturn. The mode of collapse and the process leading to it for each series of tests was noted. For the solid 0.5m monolithic column the following was observed:

(a) At large periods above roughly 0.75s overturning took place without rocking or after one or two impacts from the inception of rocking.
(b) At lower periods multiple rocking and/or wobbling were observed prior to overturning, which may occur in the out of the rocking plane direction.
(c) At very short periods of around 0.25s stable rocking and wobbling were observed without reaching critical accelerations for overturning.

The 0.5m and 1.0m multi-drum columns exhibited similar behaviors but at shifted intervals in the harmonic excitation periods. In general three modes were observed for multi-drum columns:

(a) At low accelerations and high periods multi-drum columns tended to behave as a monolithic column overturning in one piece without rocking. No disorder was observed at the end of the 25 cycles excitation periods where the column remained stable at the limit of the stability boundary.
(b) At intermediate periods and low accelerations multi-drum columns tended to rock as one piece with or without wobbling accompanied by some inter block rocking. Failure took place by overturning in one piece. At the limit of the stability boundary columns that did not overturn showed structural disorder at the end of the 25 cycles excitation periods with inter-block movement.

(c) At high accelerations and small periods of less than 0.3s, i.e. high frequencies, multi-drum columns tended to enter into a multi block rocking mode where the top two or three blocks entered into independent rocking leading to structural disorder and inter block movement. Failure took place by one or more of the top drums overturning alone or overturning and causing the remaining pieces to follow suit through impacts. Accelerations greater than 0.1g were required to activate these modes.

3. NUMERICAL MODELING

Computational methods are used when closed form solutions are intractable or when time stepping is required to update system properties and geometry such as in dynamic problems involving time histories of loading and large displacements. In parallel with the experimental investigation presented in the previous paragraph a commercial DEM software, UDEC, was used to model the response and to develop correspondence between the reduced models and the full size colonnade. A numerical model was developed for 0.5m and 1.0m solid columns and for 0.5m, 1.0m. Blocks were modeled as rigid with non-penetrating contact with density = 2720 kg/m$^3$ and the joint material was assumed to follow a Mohr Coulomb model with a friction angle = 26°, a dilation angle = 0°, cohesion = 0 and tension cutoff= 0. Damping was not considered in the model.
Two types of loads were applied to the model: (i) the self-weight of the blocks and (ii) a harmonic base velocity with a prescribed amplitude and period. All signals were applied for up to 25 cycles, and for a given period the acceleration was incremented until failure of the column was observed within the 25 cycles of application. The period, velocity amplitude and corresponding acceleration amplitude were noted for all the critical combinations. The mode of collapse was noted and the displacement history of the top of the column was also plotted alongside the harmonic excitation signal in order to visualize
Figure 4. 1m multi-drum column (a) Stable disorder at end of 25 cycles $T = 0.15s; a = 1.74m/s^2$
(b) Stable three drums rocking $T = 0.16s; a = 1.49m/s^2$

Figure 5. 1m multi-drum column (a) Toppling of capital $T = 0.17s; a = 3.92m/s^2$
(b) Overturning of three blocks $T = 0.18s; a = 3.6m/s^2$
(c) Overturning of whole column $T = 0.77s; a = 0.8m/s^2$

the response of the column. The results of the numerical simulations are reported in detail elsewhere (Tabbara and Karam, manuscript in preparation). Table 1 presents a sample of the results for the 1m multi-drum column.
### Table 1. Critical combinations for 1m multi-drum column UDEC simulation

<table>
<thead>
<tr>
<th>T (s)</th>
<th>Velocity Amplitude (m/s)</th>
<th>Acceleration Amplitude (g)</th>
<th>Mode of Collapse</th>
<th>Baalbek Column, 1 m, 25 cycles, Displacement history (base and top)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.144</td>
<td>0.4612</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.130</td>
<td>0.2775</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.120</td>
<td>0.1921</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.093</td>
<td>0.1191</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.090</td>
<td>0.0961</td>
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4. DISCUSSION OF THE RESULTS

The results of the shaking table experiments are summarized in Figures 6 to 8 where the outcome of each shaking test is plotted at the corresponding acceleration and period of the shaking table harmonic movement. Each data point represents a single test. When the column did not collapse at the end of 25 cycles the experiment is represented by a green circle. When the experiment resulted in overturning of the whole column or parts of it is represented by a red cross. At critical combinations of harmonic signal period and amplitude the experiment could result alternatively in survival or collapse by overturning, showing different results for every run. This is due to the high sensitivity of the experimental setup to the slightest variations in initial conditions such as alignment of drums, minor eccentricities and minor material and geometric asymmetries among others. However, a separation boundary or band is clearly observed between stable and unstable domains for the 0.5m monolithic column and the 0.5m and 1m multi-drum column models. The separation boundary between the stable and unstable domains seems to follow a general hyperbolic trend for all sizes and types of columns investigated with critical accelerations inversely proportional to critical periods. Figures 6 to 8 also show the boundary between the stable and unstable domains in the acceleration-period space as compiled from the UDEC simulations where the monolithic and multi-block columns used in the experiments were represented with 2D blocks. The UDEC boundary for the 0.5m monolithic column...
is bi-modal showing a flat horizontal line that follows the theoretical minimal required acceleration for overturning: \( a = \alpha g \), where \( \alpha \) is the critical stability angle given by \( \alpha = \cot^{-1}(H/B) \) with \( H \) = height of the column or block and \( B \) = width of the column or block (Housner, 1963; Shenton, 1996; Zhang and Makris, 2001). For Baalbek \( \alpha = 0.10956 \). Below a threshold value, the UDEC boundary rises sharply with decreasing periods.

Figure 6. Experimental results and UDEC model for 0.5m monolithic block column

Figure 7. Experimental results and UDEC model for 0.5m multi-drum column
For the multi-drum 0.5m and 1.0m columns the UDEC compiled boundary follows a more or less horizontal line that rises sharply in a hyperbolic fashion below a threshold period. This threshold period is different for each size. The experimentally determined boundary between the stable and unstable domains falls below the UDEC compiled boundary; this probably due to the 3D nature of the experimental setup, the cylindrical shape of the columns, the multiple sources of eccentricities in the setup and the roughness and imperfections of the contact surfaces between the column drums and with the column base block.

Multi-drum columns under harmonic excitation periods that correspond to the horizontal part of the stability boundary, behave similarly to monolithic ones showing block overturning after very few cycles. At intermediate periods closing on the threshold wobbling is also observed. As described by Stefanou et al. (2011), wobbling happens when the motion is not limited in the vertical plane of rocking, but can take place out-of-plane, in the three dimensional space. The cylindrical column can rolls on its edge under small inclination angles and, contrary to rocking, the pole of rotation does not change abruptly from one to the other corner of its base as in rocking, but rather it follows a smooth translation along that edge.

At very short periods of the order of 0.1s to 0.3s the required accelerations for overturning grow hyperbolically and may exceed those required to overturn smaller columns with shape factors $\alpha_2$, $\alpha_3$, $\alpha_4$ greater than $\alpha_1 = \alpha = 0.10956$. These shape factors $(\alpha_2, \alpha_3, \alpha_4)$ correspond to those of columns made up of the four top drums, then the three top drums, then the two top drums respectively. At these accelerations the rocking of the top parts of the multi-drum columns can be activated by the harmonic excitation signal applied and multiple rocking modes with overturning or toppling of one or more of the upper drums are observed. Experimental and numerical observations are in agreement as to the activation of these different collapse modes at very small periods as seen in Figures 3 to 5 and the first entry in Table 1.

Figure 9 plots the critical shape factors for the rocking of top drums in the 1m multi-drum column showing how collapse modes can change as the critical accelerations rise above the rocking activation limit of the top drums.

In his seminal contribution Housner (1963) proposed a simple 2D equation for the overturning of a rigid block under a half-cycle sinusoidal pulse that relates the critical acceleration, a, to the period of the signal, T, and the size and shape factor of the block. The conditions assumed for the derivation of that equation were over-constrained so Makris and Roussos (2000) proposed a correction to Housner’s...
equation that retains essentially the same properties. Housner’s equation and the simplified form of Makris and Roussos’ equation are plotted in Figure 9 for comparison purposes. It is seen that both equations follow the same trend but fall much higher than the experimental and the numerical data. This observation is probably due to the fact that both equations were derived for a single pulse whereas the numerical and experimental data presented is for multiple cycle applications up to 25 cycle. Further analysis of the applicability of these two equations as upper bounds for the stability domain is presented elsewhere (Tabbara and Karam, 2018).

![Figure 9. Comparison of all results for 1m column](image)

5. CONCLUSIONS

The experimental investigation of the stability under harmonic excitations of scaled single columns of the Baalbek colonnade showed that the general rocking response of the multi-drum column is similar to that of the monolithic column of the same size but for small excitation periods, i.e. high frequencies, where the accelerations reached are sufficient to activate multi-block rocking and various alternative collapse modes. The 1/20 model behaved qualitatively the same as the geometrically similar 1/40 model but with a shift in the time domain. Overall at the equal size of 0.5m the monolithic column is more stable than the multi-drum column. And for the multi-drum columns of 0.5m and 1.0m the larger one is more stable.

The numerical UDEC simulations identified the same type of response and collapse modes observed experimentally but for wobbling which involves 3D modeling. The numerical stability boundaries agree well with the experimental results and represent some sort of an upper bound.

The difference between numerical and experimental data is due to unavoidable errors and eccentricities, imperfections in the models especially at the level of contact surfaces, and 3D effects. Comparison of the results with the 2D equations of Housner (1963) and Makris and Roussos (2000) for the overturning of a rigid block under a half-cycle sinusoidal pulse shows general agreement in trend but requires further work to establish their usefulness as upper bounds.

The work presented here is part of a much larger ongoing research effort to establish stability curves for the actual colonnade and identify the critical energy content characteristics of earthquakes that may cause collapse of the structure.
6. ACKNOWLEDGMENTS

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7. REFERENCES


