QUANTIFYING THE EFFECT OF BEATING INFERRED FROM RECORDED RESPONSES OF TALL BUILDINGS

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ABSTRACT

The beating phenomenon observed in recorded earthquake responses of a tall building in Japan and of two others buildings in the U.S. are examined in this paper. The objective of the paper is to discuss the significance of beating and to estimate what percentage of total shaking energy impacting a building is contributed by beating when it occurs. Beating is a periodic vibrational behavior caused by distinctive coupling between translational and torsional modes that typically have close frequencies. Beating is prominent in the prolonged resonant responses of lightly damped structures. Resonances from site effects may also enhance beating. Spectral analyses and system identification techniques are used herein to quantify the periods and amplitudes of the beating from strong-motion recordings of the three buildings. Quantification of beating is a first step towards determining remedial actions to improve building resilience to this phenomenon. It is shown by the analysis presented in this paper that the ratio of additional vibrational energy of a building exhibiting beating with respect to a postulated zero beating status can be as much as 105% depending on the building and the strong shaking record. Hence, beating should be considered during design and analyses process. Alternatively, remedies maybe implemented for existing buildings.

Keywords: beating, frequencies, damping, vibrational energy, tall buildings

1. INTRODUCTION

From a cursory survey of several textbooks on structural dynamics it is noted that beating is not discussed. However, as more long-duration earthquake response records are being acquired from buildings instrumented by modern digital instruments it is becoming increasingly evident that the beating phenomenon is common. Investigating the degree to which beating may impact the instantaneous and long-term shaking performances of buildings during large or small earthquakes is the main motivation for this study. Ultimately, we aim to demonstrate that beating can be significant in the response of buildings and other structures. Herein, we develop a simple, rational procedure to qualitatively estimate the percentage of additional energy due to beating as compared to a status postulated to have no beating.

Beating is a periodic, resonating and prolonged vibrational behavior caused by distinctive close coupling of translational and torsional modes of a lightly damped structure (Boroschek and Mahin, 1991, Çelebi, 1994, 2006, 2007, 2014). Thus, repetitively stored potential energy during the coupled translational and torsional deformations turns into repetitive vibrational energy, causing the ensuing prolonged motions. The energy periodically flows back and forth between closely coupled modes and mostly with regular periodicity. The coupled motions reinforce and weaken each other. Figure 1 demonstrates how two simple harmonic signals with constant frequencies (0.45 and 0.57 Hz) and with and without light damping ratio of 1 %, interfere to produce beating. The “beat frequency” \((f_b)\) - as it is generally referred to in acoustical physics - is denoted by the absolute value of the differences in frequencies \((f_1-f_2)\) that cause the phenomenon [https://en.wikipedia.org/wiki/Beat (acoustics)]. The “beating period” \((T_b)\) is twice the inverse of beat frequency \((T_b=2/f_b)\) as shown in Figure 1. Throughout this paper, the beating period will be computed by the following equation (see also Boroschek and Mahin, 1991):

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\[ T_b = 1/F_b = 2/\text{abs}(f_1-f_2) = 2T_1T_2/\text{abs}(T_1-T_2) \]  

(1)

Of course, it is understood that the real life response of a structure is not this simple. The amplitudes of reinforcing signals may vary drastically with time, and there may be contributions from other modes that further complicate the resulting response and visual identification of the beating period.

In several case studies it was noted that resonance caused by site effects also contributes to beating (e.g. Çelebi et al., 2014). Thus, for any structure, it becomes necessary to determine site frequencies to infer whether there might be resonant coupling between the site and the structure at close frequencies. (e.g. for Case 3 presented later in the paper, site transfer function is computed using shear wave velocity (Vs) versus depth profiles of the site of the building and a nearby free-field station [Çelebi et al., 2014]). Thus, engineering implications of the beatings are:

(a) Prolonged cyclic shaking due to beating takes its toll on the structural system – particularly if the structure is brittle and old (e.g. historical buildings [Çelebi and Rinaldis, 2006]),

(b) Repetitive shaking accentuates fatigue and low-cycle fatigue, and

(c) Beating cycles can cease functionality of a building or cause serious discomfort to occupants (as it did in Case 3 [Çelebi et al., 2014]).

The purpose of this paper is to review previous relevant studies and to present new results from further analyses of three buildings whose responses consistently exhibit beating during earthquakes. Such studies can help guide design and construction methods to mitigate negative impact caused by beating and, as a result, improve their resiliency. Throughout the paper, spectral analyses methods described by Bendat and Piersol (1980) and system identification procedures described by Ljung (1987) are used. System identification allows computation of modal damping values in addition to the modal frequencies. It was already shown in Figure 1.

2. PROCESS TO QUANTIFY BEATING

I use the strong shaking duration as being between 5-95% of the sum of square of a motion parameter (e.g. acceleration, velocity) of a record as postulated by Trifunac and Brady (1975). However, such computations do include the additional shaking (both at the ground level [or 1st floor] and at the roof of a building) that was not captured with older generation of recorders.

Needless to say, the older recorders based on threshold exceedance of accelerations were considerably shorter than what is continuously recorded with modern recorders with real-time digitizers. The older recorders did acquire structural response records that include translational, torsional, rocking but not beating effects, if present. Hence, in this paper we establish an approach to quantify the effect of beating to the total energy of shaking as defined by the longer recorded response records. Such quantification will shed light on the significance of such effects. In this paper we focus on long-duration response records from modern digital instrument because earlier instruments often did not record the prolonged response of building to earthquakes in which beating occurs.

A measure of vibrational energy is obtained using the cumulative sum of the squared values of a time-
history record, either velocity \(v(t)\) ; hence \(e_v = \sum v(t)^2\), or acceleration \((a(t)) ; e_a = \sum a(t)^2\) ^{eq^{}} , similar to measure developed for ground motions (for example, Trifunac and Brady, 1975; Bommer, J.J., Martinez-Pereira, A., 1996, 1999). Several alternative parameters used to compute vibrational energy are summarized by Bommer and Martinez-Pereira (1999) that include \(e_v\) or \(e_a\) but also Arias Intensity \(A_I = (\pi/2g)\sum a(t)^2\) attributed to Husid (1969)).

Of course, these developments were made only for ground motions – not structural response motion. So, one of the aims of this paper is to explore how significant additional strong shaking energy is for structural response shaking – no matter what the cause (beating effect, resonance, and other factors). Hence, the need for at least a pair of records – one on ground level (free-field or ground level of a building) and the other from the roof or highest instrumented floor of the building.

In Figure 2, we show two velocity time histories, one at the ground level (in this case 1st floor) and the other at the roof of a 20-story building. These velocity time histories were derived from the acceleration record from the 20-story Atwood building in Anchorage, AK, for the Iniskin, AK, earthquake (M7.1, January 14, 2016; www.strongmotioncenter.org, last visited August 28, 2017). These data, used here as an illustrative example, are part of Case 1 discussed later in the paper.

Figure 2. (a) defining ssd from the plot of ground related 1st fl NS velocity time-history and its normalized \(e_v = \sum v(t)^2\), (b) assessing \(as_1, as_2, A_1, B_1\) and \(B_2\) as defined from the roof NS velocity time-history and its normalized \(e_v = \sum v(i)^2\).

In Figure 2a, we define:
- \(L_1 = \) start time of shaking energy build-up above 5% of total time-history at ground (1st level).
- \(L_2 = \) end time of shaking energy build-up at 95% of total time-history at ground (1st level).
- \(Ssd = \) strong shaking duration = \(L_2 - L_1\).

In Figure 2b, we define:
- \(as_1 = \) additional shaking duration when 95% is reached
- \(as_2 = \) additional shaking duration when 100% is reached.
- \(A_1 = \) percentage of strong shaking with respect to total
- \(B_1 = \) percentage of shaking beyond strong shaking but up to 95%
- \(B_2 = \) percentage of shaking beyond strong shaking but up to 100% (not needed but useful to compute averages).

In this case, \(L_1 = 70s, L_2 = 149s, ssd = L_2 - L_1 = 79s, as_1 = 41s, as_2 = 181s\).

As seen in Figure 2a and b, \(e_v = B_1/A_1 = B_1/(0.9-B_1) = 30.7\%\) or \(e_v = B_2/A_1 = (B_1+0.05)/(0.90-B_1) = 37.9\%\). Therefore, it is sufficient to get a percentage of \(B_1\) to estimate either way. The best is to take an average \(e_{ave} = 34.3\%\).

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All pertinent parameters for the NS direction are included as Case 1 in the summary Table 4 at the end of the paper.
2. 1 Case 1: Atwood Building, Anchorage, Alaska

The Atwood Building, designed according to 1979 Uniform Building Code (UBC) and constructed in 1980 is a 20-story, 264 ft (80.5 m) tall steel moment-resisting framed (MRF) structure with only one level of basement. The building is 130ft x130ft (39.6m x39.6 m) in plan with a 48ft x 48ft (14.6m x 14.6m) center steel shear-walled core, and. The building foundation is without any piles and consists of a 5ft (1.52m) thick reinforced concrete mat below the core and a 4-ft 6-in. (1.37m) thick reinforced concrete perimeter mat interconnected with grade beams. A picture of the building, its three-dimensional schematic showing major dimensions, locations of accelerometers within the building and its associated downhole array is shown in Figure 3.

Figure 3. (left) A photograph of the Atwood Building (Anchorage, AK). (right) Three-dimensional schematic of building showing the general dimensions and locations of accelerometers deployed within the structure as well as the associated downhole array. The superstructure array of this building’s monitoring scheme is designed to capture (rocking) SSI effects in addition to the traditional translational and torsional responses.

To date, more than two dozen earthquakes have been recorded by the building and downhole array (www.strongmotioncenter.org, last visited April 14, 2016). A comprehensive study of 3 earthquakes prior to August 2005 (Çelebi, 2006), as well as the recent M7.1 January 24, 2016 Iniskin (AK) earthquake, all show that there is close coupling between the translational and torsional modes. In general, the critical damping percentages were <5% (Çelebi, 2006, Çelebi and others, 2016). As noted in a previous study (Çelebi, 2006), beating was clearly exhibited, and beating periods were reliably computed according to Equation 1. In this study we examine only data from the M7.1 January 24, 2016, Iniskin (AK) earthquake.

Figure 4 shows acceleration and displacement time-histories at the roof in (left) NS and (center) EW directions. Beating is clearly displayed in the time-history plots. In particular, the amplified displacements better display the beating periods. For example, for NS, between ~150-190 seconds, a beating cycle observation leads to ~ 40 s beating period. Figure 4 (right) shows amplitude spectra of roof accelerations in the NS (CH30 or CH31), EW (CH32) and torsional (CH30-CH31). The significant frequencies displayed expose how close the translational and torsional frequencies are (NS [0.48 Hz], EW [0.40 Hz] and torsional [~0.48Hz]). These frequencies are also confirmed by system identification analyses as summarized in Table 1. When translational f=0.47Hz or 0.48 Hz and torsional f=0.40Hz, the beating period (T_b) is estimated using Equation 1 as T_b~ 2*(1/f1)*(1/f2)/(abs(1/f1)-(1/f2)) ~ 25 – 28.6 seconds. There is variation in the figures as well.
Figure 4. At the roof, (left) NS and (center) EW accelerations (top frame) and displacements (middle and lower frames). For both NS and EW, the bottom frames are amplified plots of displacements to better display beating in the response of the building from 120 seconds into the record. (right) Amplitude spectra computed from acceleration data.

Table 1. Iniskin earthquake: NS, EW and torsional modal frequencies and damping percentages computed by system identification.

<table>
<thead>
<tr>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>f(Hz)</td>
<td>ξ (%)</td>
</tr>
<tr>
<td>NS1</td>
<td>0.47</td>
<td>1.47</td>
</tr>
<tr>
<td>EW</td>
<td>0.40</td>
<td>1.34</td>
</tr>
<tr>
<td>TOR</td>
<td>0.47</td>
<td>0.12</td>
</tr>
</tbody>
</table>

While only NS direction computations are shown in Figure 2, Figure 5 demonstrates a general plot of normalized $\varepsilon_n$ for all pertinent data from the roof, 1st floor and the free-field (Çelebi, 2006) associated with this building. The figure shows the general parameters that were identified previously. Note that $\varepsilon_n$ corresponding to locations 3-8 (covering 1st floor, and free field and for both NS and EW) are very similar while for the roof locations, they are different.

Figure 5. Normalized sum of velocity squared for nine time histories (ground related for both NS and EW: free-field downhole and surface, and 1st floor of the building [similar to that of the basement], and structural related: roof [NS, EW and torsional]). The figure displays definitions adopted in this paper (L1, L2, ssd, as1, as2, A1, B1, B2). A mean of normalized ground related curves is also depicted.

2.2 Case 2: Santa Clara County (CA) Office Building

The most detailed study of the causes and resulting behavior of a building influenced by beating is by Boroschek and Mahin (1991) of the responses of the 13-story Santa Clara County Office Building (SCCOB) in San Jose, California during three California earthquakes (www.strongmotioncenter.org, last visited, July 27, 2017). Figure 6 shows a general schematic of the SCCOB and a map showing the location of SCCOB and the epicentral locations of the 24 April 1984 (MHE) Morgan Hill (Ms=6.1), the 31 March 1986 (MLE) Mt. Lewis (Ms=5.5) and the 17 October 1989 (LPE) Loma Prieta (Ms=7.1) earthquakes. Figure 6 also shows the instrumentation scheme of the building installed and managed by California Strong Motion Instrumentation Program (CSMIP) of the California Geological Survey.
Built in 1975 according to UBC 1970 provisions, the 56-m tall SCCOB is a moment-resisting steel-framed building on a mat foundation on an alluvial site. There are six column lines in each direction of the approximately 51 m x 51 m (in plan) building (www.strongmotioncenter.org, last visited, July 27, 2017). The depth to bedrock at the site is estimated to be between 270m-500m (Çelebi, 1994, 1998). Site transfer functions were computed (Çelebi, 1994, 1998) for two estimated depths to bedrock (270m and 500 m) for which shear wave velocities (Vs) assigned to each layer based on available geotechnical reports (Earth Sciences, 1971). From these transfer functions it was concluded that the site is capable of generating resonating surface waves at low frequencies (<1 Hz) that are close to the resonant frequencies of the building (Çelebi, 1994, 1998). Frankel and Vidale (1992) studied the basin effect of the Santa Clara Valley and concluded that the basin is capable of generating motions with longer periods between 2-5 seconds.

Lin and Papageorgiou (1989) also studied SCCOB response data from the Morgan Hill earthquake and concluded that strong beating-like phenomena occur in buildings with identifiable closely coupled torsional and translational modal characteristics. However, while they stressed the cause of beating to be due to close coupling of [clarify: structural response modes, or the interplay of structure and site modes], they did not elaborate on the effect of low damping, as reiterated by Çelebi (1994, 1998), and that beating is further accentuated due to both low damping and site resonance (1994, 1998). The possible influence of site resonance in prolonging the response and, in particular, contributing to the beating responses of a structure was not considered in previous studies.

Only the LPE is chosen to further characterize the structural response. Figure 7a shows acceleration responses at the roof level, as well as the difference of parallel accelerations at the roof level (nominal torsional accelerations). Figure 7b shows the amplitude spectra of the NS and EW translational accelerations clearly peaking at the translational frequency (period) [0.45 Hz (2.22 sec)]. Figure 7c shows amplitude spectra of the nominal torsional accelerations (calculated from parallel records in the NS and EW directions respectively) that peak at 0.57 Hz (1.69 sec) and 0.45 Hz (2.22 sec). Figures 7d and e show the coherence, phase angle and cross-spectra of the unidirectional response and confirm the translational frequency at 0.45 Hz. At this frequency, the motions are coherent and are in phase, clearly indicating that they are related. Figure 7f, on the other hand, confirms the torsional frequency to be at 0.57 Hz, as identified from the nominal torsional accelerations. Unity coherence and zero phase angle at 0.57 Hz means that the closely-coupled translational-torsional frequencies are related to one another. Peak accelerations at the roof and basement levels, the frequencies (and periods) and damping percentages extracted from the records by system identification and spectral analysis techniques are

Figure 6. Left: General location map showing the building site and the epicenters of three earthquakes. Right: Distribution of building instrumentation. Arrows indicate the location and orientation of each accelerometer channel. Note: The building figure is not to scale. No free-field stations are associated with this site.
presented in Table 2 for the fundamental translational and torsional modes (Çelebi, Phan and Marshall, 1993, Çelebi, 1994, 1998). The small differences between the periods determined from the three earthquake response data are noted. The proximity of these two frequencies causes the observed coupling and beating.

Figure 7. (a) Acceleration time-histories of the roof, (b and c) amplitude spectra, (d, e and f) coherence (solid lines) and phase angle (dotted lines) plots (with cross-spectra, $S_{xy}$ normalized to 1 superimposed as dashed lines) to distinguish translational and torsional frequencies.

Table 2. Accelerations, Structural and Site Frequencies as Determined from Records obtained during the M7.1, LPE-Loma Prieta earthquake. Also included are the beating periods computed from identified periods and ambient test periods.

<table>
<thead>
<tr>
<th>M7.1 Loma Prieta Earthquake (LPE) 17 October 1989</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Accel. (g) : Roof [NS=0.34, EW=0.34], Base[NS=0.10, EW=0.09]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fundamental Periods/ Frequencies : $T/(f)$ (s/Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translational ($\xi=2.7%$)</td>
</tr>
<tr>
<td>2.22/(0.45)</td>
</tr>
</tbody>
</table>

Figure 8. For Santa Clara county Office Building (SCCOB), (a) defining L1, L2 and ssd from the plot of ground level 1st fl (CH21) EW acceleration time-history and its normalized $e_a= \Sigma a(t)^2$ plot, (b) identifying A1, B1 and B2 as defined from the roof NS velocity time-history and its normalized $e_a= \Sigma a(t)^2$ plot.

The close-coupling of the torsional and translational frequencies at low damping percentages (lightly damped system) clearly explains that for SCCOB, the translational and torsional modes reinforce one another during the vibration with only small dissipation. The observed beating period is computed using Equation 1 (Boroscheck and Mahin, 1991). Thus, a beating period for LPE is computed as $T_b=2(2.22)(1.75)/(2.22-1.75)= 16.5$ seconds. The computed beating periods (with an average value 16.9 seconds are listed in Table 2 also.
From Figure 8, we identify the pertinent parameters as: \( L_1 = 3 \text{s}, \ L_2 = 40 \text{s}, \ ssd = L_2 - L_1 = 37 \text{s}, \ as_1 = 41 \text{s}, \ as_2 = 181 \text{s} , \ B_1 \sim 0.45\% , \ B_2 \sim 0.5\% . \) As seen in Figure 2a and b (Normalized \( e_b = \Sigma a(t)^2 \) each for ground floor and roof motions). Hence, \( eb_1 = B_1/A_1 = 0.45/(0.90 - 0.45) = 100\% \) or \( eb_2 = B_2/A_1 = (0.45 + 0.05)/(0.90 - 0.45) = 110\% . \) Therefore, it is sufficient to get a percentage of \( B_1 \) to estimate the effect either way. Hence, \( eb_{ave} \approx 105\% . \)

It is important to note that, following the LPE, 96 dampers were installed in SCCOB (www.strongmotioncenter.org, last accessed July 27, 2017). Since, the LPE, there has been no strong shaking with amplitudes sufficient to assess the effectiveness of the dampers in reducing the beating on this building.

### 2.3 Case 3: 55-Story building in Osaka, Japan

Unprecedented long-duration response records were acquired from a 55-story tall building located 769 km from the epicenter of the mainshock of the M9.0 March 11, 2011, Tohoku, Japan, earthquake (http://earthquake.usgs.gov/earthquakes/equinthenews/2011/usc0001xgp/, last visited July 15, 2011). Vertical sections of the building, a typical plan view and locations of tri-axial accelerometers are shown in Figure 9. In a separate study, Çelebi and others (2014) provide a detailed study of the mainshock as well as numerous aftershock records of the building including discussions of average drift ratios as related to performance. In this paper, we discuss only beatings as observed from only the mainshock records. The fact that the shaking of the building was both strong and prolonged caused the loss of functionality of the building and significant problems for the occupants and visitors (e.g. elevator cables entangled and could not be used for some time).

The building response records echo the consistent long-duration, strong-shaking characteristics of the hundreds of surface and downhole free-field records for both the mainshock and aftershocks that have been publicly released by K-Net and KiK-Net of Japan (http://www.kyoshin.bosai.go.jp/, last visited April 21, 2016). In addition, site transfer functions computed from geotechnical logs of KIKNET for station OSKH02, the free-field station closest to the building (~2.5 km), suggests the possibility of resonance due to soil-structure interaction (SSI), which was not considered during the design/analysis phase of the building. In Çelebi and others (2014), the fundamental frequency of the site is computed to be in the range of 0.13-0.17 Hz.

Figure 10 displays the unprecedented 1000 s-long recorded acceleration time-histories (in both X [219] and Y [319] directions) at 1st floor (as input) and at 52nd floor (as output) as well as the computed 52nd floor accelerations using system identification. To the best knowledge of the authors, such long-duration response records have not previously been obtained, even though there likely have been many buildings that experienced such shaking. The long durations of repetitious cycles in the responses suggest that the building is in resonance. This is also supported by the fact that the site frequency (between 0.13-0.17Hz)
is very close to the fundamental frequencies of the building (~0.15Hz) as exhibited in amplitude spectra in Figure 10. The amplitude spectra and spectral ratios (Figure 11) further confirm these structural frequencies.

Figure 10. System identification applied to mainshock records in the X (229) and Y (319) directions. The first floor accelerations are used as input and the 52nd floor accelerations as output. The computed 52nd floor accelerations match well with the recorded. System identification also facilitates computation of the modal damping ratios. Amplitude spectra peaks at the 52nd floor are very narrow – indicative of low damping.

In addition, the damping percentages extracted from system identification analyses (Table 3) are quite low (1.2 and 1.6 % in X and Y directions, respectively) which herewith is asserted to be one of the main causes for the prolonged shaking, including beating phenomenon particularly in the Y-direction of the building. The computed low damping values are also supported by the very narrow bandwidths of the dominant peaks in the amplitude spectra of both the recorded and computed accelerations (Figure 11).

Figure 11. (left) Amplitude spectra of accelerations at the 52nd floor exhibit the frequencies in principal axes of the building. Torsional frequencies are identified from the difference (N-S) between two parallel accelerations at the 52F1 and 52F2 locations (Figure 2). Third mode (~0.9 Hz) in X-direction is not identified from amplitude spectrum, but is identified by spectral ratios (right) of the amplitude spectra of the 52nd, 38th and 18th floors with respect to the 1st level.

Table 3. Summary of frequencies [periods] determined by spectral analyses and system identification applied to mainshock. Critical damping percentages are identified by system identification only.

<table>
<thead>
<tr>
<th>Orientation</th>
<th>X [229]</th>
<th>Y [319]</th>
<th>Torsional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modes</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Freq. (Hz)</td>
<td>0.152</td>
<td>0.489</td>
<td>0.905</td>
</tr>
<tr>
<td>[T (s)]</td>
<td>[6.58]</td>
<td>[2.06]</td>
<td>[1.11]</td>
</tr>
<tr>
<td>Damping ratio (ξ)</td>
<td>0.012</td>
<td>0.020</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Denoting the identified translational fundamental periods (Table 3) of the building as T1 (6.58 s or 6.9
s) in the X- or Y-directions respectively, and torsional period as \( T_2 \) (4.69 s), then, using equation 1 yields
\[
T_b = \frac{2T_1T_2}{T_1-T_2} = 2 \times \frac{6.58 \times 4.69}{(6.58-4.69)} = 32.66 \text{ s} \text{ or } 2 \times \frac{6.9 \times 4.69}{(6.9-4.69)} = 29.29 \text{ s}. 
\]
These computed beating periods are shorter than the beating periods (between 50-150 s) observed from the response records depicted in Figure 10. The mismatch between observed and computed beating periods is significant but does not diminish that low damping is a cause of the observed long-duration beating vibrations.

Immediate remediation to improve the behavior and future resiliency of the building was accomplished by applying response modification technologies (e.g., adding dampers at select bays and floors) to dissipate vibrational energy and thus decrease the prolonged shaking and suppress the beating of the building during future strong shaking events. As noted in a recent study (Çelebi and others, 2017), during the M7.3 Kumamoto earthquake of April 24, 2016, the records showed that the beating behavior of the building was not suppressed after retrofit with dampers and buckling restrained braces (BRBs) following its behavior during the M9.0 Tohoku earthquake of March 11, 2011.

To quantify the additional vibrational energy due to beating as compared to the postulated non-beating behavior, we show for each pair of 1st floor and 52nd floor responses in either X (229N) direction (Figures 12a and b) or Y(319N) direction. Velocities time histories \( v(t) \) and normalized \( e_v = \sum v(t)^2 \) are plotted in each frame. We will call these Pair X (Figures 12a and b) and Pair Y(319N).

![Figure 12. Defining L1, L2 and ssd from the plot of ground level 1st fl (a) X(229) direction, (c) Y(319) direction. Defining A1, B1 and B2 for 52nd floor (b) X(229) direction. (d) Y(319) direction. Note the modified L2* rather than L2.](image)

Examining Figure 12 shows that for each of the pairs X (229N) (Figures 12a and b) or Y(319N) direction (Figures 12c and d), from 1st floor time-history and normalized ev plots (Figure 12a or Figure 12c), L2 is located at \(~480\) seconds into the record. However, placing L2 into the 52nd floor plots shows that L2 crosses the normalized ev plots at approximately 95%. This would imply zero additional shaking energy at 52nd floor due to beating when compared to that at the 1st floor, and determined from 1st floor normalized ev plot on the 52nd Floor normalized ev plot crosses the normalized ev plots at approximately 95%. This would imply zero additional shaking energy at 52nd floor due to beating when compared to that at the 1st floor. Hence, the need to modify the process. Therefore, we extend a straight line at an
angle that follows the straight sloping part of the normalized ev plot to intersect at L2* and note the time this intersection occurs into the record (~275s). We then place the sloping L2* line at the same time location (~280s) of the 52nd floor normalized ev plot and then identify B1, B2 and A1. For X (229) direction: B1=0.37, B2=0.42, A1=0.53, and therefore B1/A1 = .37/.53=.70, B2/A1=.42/.53=.79, average 74.5%. And for Y (319) direction: B1=0.23, B2=0.28, A1=0.67, and therefore, B1/A1 = .23/.67=.34, B2/A1=.28/.67=.42, average 38%.

3. DISCUSSION AND CONCLUSIONS

Pertinent parameters to quantify and estimate the effect of beating in causing additional vibrational energy to shaking of a building as compared to a postulated status of zero-beating are summarized in Table 4 for three buildings of varying heights, each subjected to strong shaking from different earthquakes.

In this paper, we draw attention to this real physical phenomenon that was observed in a building in Japan and two others in the US. Quantification of the presence of beating can stimulate finding solutions to eliminate it, or to decrease the possible adverse effects that it may cause – as were implemented in the buildings in Cases 2 and 3. For the subject buildings it is shown that additional vibrational energy as a percentage of total vibrational energy \([B1/(B1+A1)]\) can be as much as 25-50% (when expressed in terms of additional vibrational energy with respect to a postulated zero-beating status \([B1/A1]\), the ratio can be as much as 105%).

Table 4. For the three cases, summary of parameters and results of computations of additional energy due to beating

<table>
<thead>
<tr>
<th></th>
<th>L1</th>
<th>L2</th>
<th>ssd = L2-L1</th>
<th>A1 (%)</th>
<th>B1 (%)</th>
<th>B2 (%)</th>
<th>(e_{b1} = B1/A1) %</th>
<th>(e_{b2} = B2/A1) %</th>
<th>(e_{ave}) %</th>
<th>(B1/(B1+A1)) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 (NS)</td>
<td>70</td>
<td>149</td>
<td>79</td>
<td>76.5</td>
<td>23.5</td>
<td>29</td>
<td>30.7</td>
<td>37.9</td>
<td>34.3</td>
<td>26</td>
</tr>
<tr>
<td>Case 2</td>
<td>3</td>
<td>40</td>
<td>37</td>
<td>45</td>
<td>45</td>
<td>50</td>
<td>100</td>
<td>110</td>
<td>105</td>
<td>50</td>
</tr>
<tr>
<td>Case 3 (X)</td>
<td>100</td>
<td>275</td>
<td>175</td>
<td>53</td>
<td>37</td>
<td>42</td>
<td>70</td>
<td>79</td>
<td>74.5</td>
<td>41</td>
</tr>
<tr>
<td>Case 3 (Y)</td>
<td>100</td>
<td>275</td>
<td>175</td>
<td>67</td>
<td>23</td>
<td>28</td>
<td>34</td>
<td>42</td>
<td>38</td>
<td>26</td>
</tr>
</tbody>
</table>

Therefore, during the design and analyses process, the possibility of beating could be considered and appropriate mitigating remedies can be incorporated into the design. For existing buildings, retrofits are employed to minimize or eliminate the effect beating.

Data Source and Disclaimer: Japan data from http://www.kyoshin.bosai.go.jp (last visited April 21, 2016), US data from www.strongmotioncenter.org (last visited May 4, 2016). Any use of trade, firm, or product names is for descriptive purposes only and does not imply endorsement by the U.S. Government.

4. REFERENCES


