SEISMIC RESPONSE OF YIELDING FRAMES COUPLED WITH RESTRAINED ROCKING WALLS

Mehrdad AGHAGHOLIZADEH¹, Nicos MAKRIS²

ABSTRACT

This paper investigates the inelastic response of a yielding structure coupled with a vertically restrained rocking wall. The paper first derives the nonlinear equations of motion of a yielding oscillator coupled with a vertically restrained rocking wall and the dependability of the one-degree of freedom idealization is validated against the nonlinear time-history response analysis of the 9-story SAC moment-resisting steel frame that is coupled with a stepping, vertically restrained rocking wall. While, the coupling of weak building frames with rocking walls is an efficient strategy that controls inelastic deformations by enforcing a uniform interstory-drift distribution, therefore, avoiding mid-story failures, the paper shows that even for medium-rise buildings the effect of vertical tendons on the inelastic structural response is marginal, with the exception of increasing the vertical reactions at the pivoting points of the rocking wall. Accordingly, the paper, concludes that for medium- to high-rise buildings vertical tendons in rocking walls are not beneficial.

Keywords: Inelastic structures; Rocking wall; Recentering; Seismic protection; Earthquake engineering

1. INTRODUCTION

In an effort to eliminate the appreciable seismic damage in moment-resisting frames that occasionally resulted to a weak-story failure, the concept of a rigid core system gained appreciable ground (Paulay 1969, Fintel 1975, Emori and Schnobrich 1978, Bertero 1980). When the core walls in tall buildings are fixed-based, the ductility capacity of the base of the core wall may be limited given the significant axial loads; while, the ductility demands are appreciable under long-duration pulse motions (Paulay 1986, Zhang and Wang 2000). Furthermore, the base of the core wall may suffer from cyclic degradation under prolonged shaking which usually results to permanent inelastic deformations. Such inelastic response may result to permanent drifts and lead to large repair costs; therefore, the entire design becomes unsustainable.

During the last three decades, there has been a growing effort to direct the attention of engineers to the unique advantages associated with allowing major vertical structural elements (piers in bridges or shear wall in buildings) to uplift in an effort to intentionally mobilize a lower "failure" mechanism. In this way failures associated with cyclic degradation are essentially avoided; while, permanent displacements remain small due to the inherent recentering tendency of the rocking mechanism. For instance, as early as the PRESS Program (Priestley 1991, Priestley 1996), the jointed shear wall system was allowed to lift-off and rock (Nakaki et al. 1999, Priestley et al. 1999). About the same time Kurama et al. (1999, 2002) examined the lateral load behavior of unbonded segmented post-tensioned precast walls; while, Mander and Cheng (1997) introduced the damage avoidance design (DAD) in which the free-standing piers of a bridge frame are only vertically restrained through their center line and are allowed to rock atop the pile-cap and bellow the pier-cap beam without inducing any damage. Following these studies, Holden et al. (2003) presented experimental studies on the cyclic loading of a precast, partially prestressed system that incorporated post-tensioned unbonded tendons; while Ajrab et al. (2004) presented a performance-based design methodology for the frame-building-rocking-wall

¹PhD Candidate, University of Central Florida, Orlando, USA, mehrdad@knights.ucf.edu
²Professor of Civil Engineering, University of Central Florida, USA and University of Patras, Greece, nicos.makris@ucf.edu
system with various prestressed tendon configurations and energy dissipation devices. In their proposed methodology Ajrab et al. (2004) adopt an "equivalent-static" lateral force procedure, and the study concludes that the proposed performance-based, capacity-demand method predicts larger displacements than those obtained from time-history analysis.

In the aforementioned studies, central post tensioned steel tendons inside the rocking wall or bridge-pier are provided to increase the lateral resistance of the entire structure. The force-deformation curve of the vertically restrained solitary rocking wall reported in these studies has invariably a positive post-uplifting stiffness, indicating that the axial stiffness of the steel tendon is large enough to the extent that the post-uplift stiffness of the rocking wall is positive. By introducing such a stiff tendon that reverses the negative stiffness of the solitary rocking wall, one creates a strong system; nevertheless, at present it is not well understood to what extent a stiff vertical tendon that offers a positive lateral stiffness enhances the seismic stability of the overall structure or it merely contributes to accentuate the crushing of the pivoting points of the rocking wall due to the increased vertical load. Part of the motivation of this study is to build upon the previously referenced work and examine the role of vertical restrainers in the seismic response of moment-resisting frames coupled with rocking walls.

The motivation for coupling of a moment-resisting frame with a strong rocking wall is to primarily enforce a uniform distribution of interstory drifts; therefore, the first mode of the frame becomes dominant as was first indicated in the seminal paper by Alavi and Krawinler (2004). Further analytical evidence to the first-mode dominated response is offered in Qu et al. (2012) and also shown in a recent paper by Aghagholizadeh and Makris (2018). These results together with additional evidence by other investigators were recently evaluated in Grigorian (2015) and it was concluded that a moment resisting frame coupled with a rocking wall can be categorized as a single-degree-of-freedom (SDOF) system. Accordingly, in this study we adopt the SDOF idealization shown in Figure 1.

2. DYNAMICS OF A YIELDING OSCILLATOR COUPLED WITH A VERTICALLY RESTRAINED STEPPING ROCKING WALL

With reference to Figure (1), this study examines the dynamic response of a yielding single-degree-of-freedom (SDOF) structure, with mass, \( m \), pre-yielding stiffness, \( k_1 \), post yielding stiffness, \( k_2 \), and strength, \( Q \), that is coupled with a free-standing stepping rocking wall of size, \( R = \sqrt{b^2 + h^2} \), slenderness, \( \tan \alpha = b/h \), mass, \( m_w \) and moment of inertia about the pivoting (stepping) points \( O \) and \( O' \), \( I = 4/3 m_w R^2 \), that is vertically restrained with an elastic tendon with axial stiffness \( EA \) which can be prestressed with a prestressing force \( P_o \). In the interest of simplicity, it is assumed that the arm with length, \( L \), that couples the motion is articulated at the center of mass of the rocking wall at a height, \( h \), from its foundation as shown in Figure 1.

During rocking motion, the center of mass of the rocking wall uplifts by \( v \); therefore, the initially horizontal coupling arm rotates by an angle \( \psi \). Accordingly, the horizontal translation of the center of mass of the rotating wall, \( x \), is related to the horizontal displacement of the SDOF oscillator, \( u \), via the expression, \( \cos \psi = 1 - (u - x)/L \); whereas, \( \sin \psi = v/L \). From the identity, \( \cos^2 \psi + \sin^2 \psi = 1 \), one concludes that the horizontal displacement, \( u \) of the SDOF oscillator is related to the horizontal displacement \( x \) of the center of mass of the rotating wall via the expression:

\[
\frac{u}{L} = 1 + \frac{x}{L} \sqrt{1 - \frac{v^2}{L^2}}
\]

In this paper, the coupling arm is assumed to be long enough so that \( v^2/L^2 \) is much smaller that unity \( (v^2/L^2 << 1) \); and in this case \( u = x \). A recent study by Makris and Aghagholizadeh (2017) on the response of an elastic oscillator coupled with a rocking wall showed that the effect due to a shorter coupling arm is negligible.

The system under consideration is a single-degree-of-freedom system where the lateral translation of the mass, \( u \) is related to the rotation of the stepping rocking wall \( \theta \) via the expression:
Figure 1. Yielding single-degree-of-freedom oscillator coupled with a vertically restrained stepping rocking wall.

\[ u = \pm R \left[ \sin \alpha - \sin(\alpha \Theta) \right] \]
\[ i \& e = R \theta \cos(\alpha \Theta) \]
\[ \theta \& = R \left[ \theta \cos(\alpha \Theta) \pm \theta \sin(\alpha \Theta) \right] \]

In equations (2) to (4), wherever there is a double sign (e.g. ±) the top sign is for \( \Theta > 0 \) and the bottom sign is for \( \Theta < 0 \). Dynamic equilibrium of the mass \( m_s \) gives:

\[ m_s (\theta \& + \theta \& \pm) = -F_s - c \theta \& + T \]

where \( F_s \) is the force the develops in the nonlinear spring and is described by the Bouc-Wen model (Bouc 1967, Wen 1975, Wen 1976 and Baber and Wen 1981)

\[ F_s(t) = ak_1u(t) + (1-a)k_2u_z(t) \]

in which, \( a = k_2/k_1 \) is the post-to-pre yielding stiffness ratio and \(-1 \leq z(t) \leq 1\) is a dimensionless internal variable described by:

\[ \dot{\theta}(t) = \frac{1}{u_0} \left[ |\dot{\theta}(t)| - \beta |\dot{\theta}(t)| - \gamma |\dot{\theta}(t)| \right] \]

In equation (7), constants \( \beta, \gamma \) and \( n \) are model parameters. Furthermore, in equation (5), \( T \) is the axial force (positive = tensile) that develops in the coupling arm.

During rocking motion of the vertically restrained wall, the tendon is elongated by (Vassiliou and Makris 2015)
\[ e = \sqrt{2} R \sin \alpha \sqrt{1 - \cos \theta} \]  \hspace{1cm} (8)

In addition to the elongation, \( e \), given by equation (8), the analysis accounts for an initial elongation due to a possible initial post-tensioning force, \( P_0 \).

Accordingly, during rocking motion, the restoring moment on the rocking wall from the tendon alone is \((Vassiliou and Makris 2015)\)

\[ M_r = -R \sin \alpha \sin \left( \frac{1}{2} EA \tan \alpha + \frac{P_0}{\sqrt{2 \sqrt{1 - \cos \theta}}} \right) \]  \hspace{1cm} (9)

With reference to Figure 1 (bottom), as the elasticity of the tendon increases it offsets the negative stiffness originating from rocking. The value of the axial stiffness of the tendon that is needed to introduce positive stiffness is \((Vassiliou and Makris 2015)\)

\[ \frac{EA}{m_w g} = 2 \frac{1}{\tan^2 \alpha} \]  \hspace{1cm} (10)

For instance, for a slenderness value, \( \tan \alpha = 1/6 \), a rigid-plastic behavior is reached when \( \frac{EA}{m_w g} = 72 \).

**Case 1: \( \theta > 0 \)**

For positive rotations \( (\theta > 0) \), dynamic equilibrium of the vertically restrained stepping rocking wall with mass \( m_w \) shown in Figure (1), gives:

\[ I \ddot{\theta} = -TR \cos(\alpha - \theta) - m_w g R \sin(\alpha - \theta) - m_w \ddot{u} R \cos(\alpha - \theta) - R \sin \alpha \sin \left( \frac{1}{2} EA \tan \alpha + \frac{P_0}{\sqrt{2 \sqrt{1 - \cos \theta}}} \right) \]  \hspace{1cm} (11)

where \( P_0 \) is the initial post-tensioning force and \( EA \) is the axial stiffness of the elastic tendon. The axial force \( T \) appearing in equation (12) is replaced with the help of equations (5) and (6), whereas for a rectangular stepping wall, \( I = 4/3 m_w R^2 \). Accordingly, equation (12) assumes the form:

\[ \frac{4}{3} m_w R^2 \ddot{\theta} = \left[ m_s (\dddot{u} + \dddot{\theta}) + a_k u(t) + (1-a) k \dot{u}, z(t) + c \dddot{\theta} R \cos(\alpha - \theta) \right] \]  \hspace{1cm} (12)

\[ = -m_w R \ddot{u} \cos(\alpha - \theta) + g \sin(\alpha - \theta) - R \sin \alpha \sin \left( \frac{1}{2} EA \tan \alpha + \frac{P_0}{\sqrt{2 \sqrt{1 - \cos \theta}}} \right) \]  \hspace{1cm} (13)

Upon dividing with \( m_w R \) equation (13) gives:

\[ \frac{4}{3} R \ddot{\theta} = \left[ \sigma (\dddot{u} + \dddot{\theta}) + a_k \frac{m_s}{m_w} u(t) + (1-a) \frac{k}{m_w} \dot{u}, z(t) \right] \]  \hspace{1cm} (14)

\[ = - \dddot{\theta} \cos(\alpha - \theta) - g \sin(\alpha - \theta) - \sin \alpha \sin \left( \frac{1}{2} \frac{EA \tan \alpha + \frac{P_0}{m_w \sqrt{2 \sqrt{1 - \cos \theta}}} \right) \]  \hspace{1cm} (14)

in which \( \sigma = m_s / m_w \) is the mass ratio parameter.

Substitution of the expressions of the relative displacement, velocity and acceleration given by equations (2) to (4) for positive rotations, and after dividing with \( R \) equation (14) is expressed only in terms of the variable, \( \theta(t) \).
The Bouc-Wen model described by equations (6) and (7) is a phenomenological model of hysteresis originally proposed by Bouc 1967 and subsequently generalized by Wen 1976 and Baber and Wen 1981. It is a versatile model that can capture various details of the nonlinear force-displacement loop. Subsequent studies on the modeling of yielding systems by Constantinou and Adnane 1981, 1987 concluded that when certain constraints are imposed on the parameters $\beta$ and $\gamma$ ($\beta+\gamma=1$), the model

$$\ddot{x} = \frac{1}{\sigma + 1} g \tan(a) \left(1 + \frac{P_s}{m_s g}\right)$$ (17)
reduces to a viscoplastic element with well-defined physical characteristics. The Bouc-Wen model essentially builds on the bilinear idealization shown in the bottom-left of Figure 1.

For the five-parameter system shown with the bilinear idealization. \((k_1=\text{pre-yielding stiffness, } k_2=\text{post-yielding stiffness, } u_y=\text{yield displacement, } Q=\text{strength and } F_y=\text{yielding force})\), only three parameters are needed to fully describe the bilinear behavior (see for instance Makris and Kamps 2013). In this work, the authors select the pre-yielding stiffness \(k_1 = m_s \omega_1^2\), the post-yielding stiffness \(k_2 = a k_1\) and the strength of the structure \(Q\). With reference to Figure (1) (bottom-left), \(F_y = k_1 u_y = Q + k_2 u_y\). Accordingly, \(u_y = Q/(k_1 - k_2)\) and \(F_y = k_1 Q/(k_1 - k_2)\). The parameters \(\beta, \gamma\) and \(n\) appearing in equation (7) are established from past studies on the parameter identification of yielding concrete structures and assume the values: \(\beta = 0.95, \gamma = 0.05\) and \(n = 2\) (Kunnath et al. 1997, Goda et al. 2009). With the parameters \(\beta = 0.95, \gamma = 0.05\) and \(n = 2\) being established, the peak inelastic displacement, \(u_{\text{max}}\) of the SDOF system shown in Figure (1) is a function of the following parameters:

\[
u_{\text{max}} = f(\omega_1, \frac{Q}{m_s}, a, \xi, p, \tan \alpha, \sigma, g, EA, P_s, \text{parameters of excitation})
\]

(18)

In this study, it is assumed that upon yielding, the structure maintains a mild, positive, post-yielding stiffness \(k_2 = 0.05 k_1\), therefore \(a = 0.05\). Furthermore, it is assumed that the pre-yielding damping ratio, \(\xi = c/(2 m_s \omega_1) = 0.03\) and the authors focus on rocking walls with slenderness, \(\tan \alpha = 1/6\).

4. NORMAL FORCE AT THE PIVOTING CORNERS

By increasing the axial stiffness, \(EA\), of the vertical tendon one increases the lateral stiffness of the entire structural system; nevertheless, at present it is not clear to what extent a stiffer vertical tendon improves the seismic performance of the overall structure, or it merely contributes to accentuate the vertical reaction force at the pivoting points. With reference to Figure 2, a rotation of the wall = \(\theta\) creates an elongation to the tendon = \(e\), given by equation (8). In addition to gravity and inertia forces, the vertical reaction at the pivot corner, \(N\), balances the vertical forces from the tendon

\[
F_v = \frac{EA}{2h} e \cos \phi + P_s \cos \phi
\]

(19)

Using that \(e \cos \phi = b \sin \theta\) and \(\cos \phi = (1/\sqrt{2}) \sqrt{1 + \cos \theta}\), equation (19) assumes the form

\[
F_v = \frac{1}{2} EA \tan \alpha \sin \theta + \frac{P_s}{\sqrt{2}} \sqrt{1 + \cos \theta}
\]

(20)

During rocking motion, the vertical reaction at the pivoting corners, \(N\), balances the weight of the wall, the inertia forces and the vertical force, \(F_v\), from the tendon gives by equation (20)

\[
N(t) = m_s \ddot{\nu} + m_s g + EA \left( \frac{1}{2} \tan \alpha \sin \theta + \frac{P_s}{EA} \frac{\sqrt{1 + \cos \theta}}{\sqrt{2}} \right)
\]

(21)

where \(\ddot{\nu}\) is the vertical acceleration of the center of mass of the wall. For instance, for a positive rotation \(\theta > 0\), the vertical uplift of the center of mass of the wall is given by (22) and successive differentiation gives,

\[
\nu = R[\cos(\alpha - \theta) - \cos \alpha]
\]

(22)

\[
\ddot{\nu} = R \sin(\alpha - \theta)
\]

(23)

\[
\dddot{\nu} = R \left[ \sin(\alpha - \theta) - \dot{\theta} \cos(\alpha - \theta) \right]
\]

(24)
By virtue of equation (24), the normalized to the weight of the wall vertical reaction of the pivoting point is given by

$$\frac{N(t)}{m_w g} = 1 + \frac{R}{g} \left[ \phi \sin(\alpha - \theta) - \phi \cos(\alpha - \theta) \right] + \frac{1}{2 m_w g} \tan \alpha \sin \theta + \frac{1}{\sqrt{2} m_w g} \sqrt{1 + \cos \theta}$$

(25)

Figure 2. Free-body diagram of a rocking wall with an elastic tendon passing through its center-line.

Figure 4 plots displacement, $u(t)$, rotation $\theta(t)$ and vertical reaction at the pivot points, $N(t)$ time histories for a structure having $T_1 = 1.5 \text{ sec}$, $Q/m_s = 0.12g$ which is coupled with a rocking wall with $\omega_1/p = 10$ ($p = 0.778 \text{ rad/sec}$), $\tan \alpha = 1/6$ and $\sigma = m_s/m_w = 10$, now the vertical tendon is prestressed with $P_o = 0.5m_w g$ and subjected to the Newhall/360 ground motion recorded during 1994 Northridge, California earthquake (left) and the Takarazuka/000 ground motion recorded during the 1995, Kobe, Japan earthquake (right). The dashed line is when there is no wall, the heavy dark line is where there is a rocking wall without tendon; whereas the thinner solid lines show the response when a tendon is prestressed with ($P_o = 0.5m_w g$). Figure 3 shows that whereas a stiff tendon ($\frac{E_A}{m_w g} = 200$) increases the vertical reaction at the pivot points by more than 50% its effect in reducing peak inelastic deformations is marginal.

While equations (15) and (16) only describe the dynamics of the SDOF idealization shown in Figure 1, they are of engineering value since they show the relative contribution of the various parameters of the problem. For instance, consider a moment frame-rocking wall system with mass ratio, $\sigma = m_s/m_w = 10$, when the rocking wall with slenderness, $\tan \alpha = 1/6$, restrained with a stiff vertical tendon (say $E_A/m_w g = 200$) and subjected to a ground motion with an acceleration amplitude of $u_g = 0.5g$. The right-hand side of equations (15) and (16) show that the term associated with the input ground acceleration, $(\sigma + 1) \frac{u_g}{g} \cos(\alpha - \theta)$, is of the order of 5; whereas, the term associated with the contribution of the tendon is $\frac{1}{2} \sin \alpha \tan \alpha \frac{E_A}{m_w g} \sin \theta \approx 2.74 \theta$. Given that for most cases of interest $\theta_{\text{max}}$ is less than $\alpha/10 \approx (\tan \alpha)/10$ (see Figures 4 and 5 of this paper), the contribution of the tendons at peak wall rotation $= \theta_{\text{max}}$ is of the order of 2.74 ($\tan \alpha)/10 \approx 0.05$—that is two order of magnitude smaller than the term associated with the input ground acceleration. This explains the marginal contribution of the vertical tendons even if they are stiff.
Figure 3. Time-history analysis of a nonlinear SDOF oscillator coupled with a vertically restrained stepping rocking wall with preyielding period, $T_1 = 1.5 \text{ sec}$, normalized strength $Q/m_s = 0.12g$, wall size ratio, $\omega_1/p = 10$, structure-to-wall mass ratio, $\sigma = 10$ when subjected to the 1994 Newhall/360 ground motion (left) and the 1995 Takarazuke/000, Japan ground motion (right). Even stiff tendons ($EA/m_w g = 200$) have a marginal effect on the response, except of drastically increasing the vertical reaction (more than 50\%) at the pivot points.

Tendons are prestressed with, $P_0/m_w g = 0.5$.

5. VALIDATION OF THE SDOF – IDEALIZATION

In view of the small differences between the peak response of a yielding structure coupled with a stepping rocking wall (either free-standing or vertically restrained) and the nonlinear response of the solitary yielding structure (other than the reduction of permant displacements – see Figures 3, 4 and 5), the dependability of the single-degree-of-freedom idealization shown in Figure 1 is examined against the results obtained with the open-source code OpenSees (McKenna et al. 2000) when analyzing the nine-story moment resisting steel structure designed for the SAC Phase II Project (2000). This structure that is well-known to the literature (Gupta and Krawinkler 1999, Chopra and Goel 2002) was designed to meet the seismic code (pre-Northridge Earthquake) and represents typical medium-rise buildings designed for the greater area of Los Angeles, California.

This moment-resisting, steel building is 40.82 m tall with 9-stories above ground level and a basement. The bays are 9.15 m wide, with five bays in north-south (N-S) and east-west (E-W) directions. Floor-to-floor height of each story is 3.96 m, except for the basement and first floor which are 3.65 m and 5.49 m respectively as shown in Figure 5 (left). Columns splices are on the 1st, 3rd, 5th and 7th floors and located 1.83 m above the beam-column joint. The column bases are modeled as pinned connection and it is assumed that the surrounding soil and concrete foundation walls are restraining the structure in horizontal direction at the ground level.
The columns are 345 MPa wide-flange steel sections and the floor beams are composed of 248 MPa wide-flanges steel sections. All beam column connections of the frames are rigid except for the corner columns which are pinned in order to avoid bi-axial bending of the members. In this study, the exterior frame in N-S direction is chosen for the 2-D validation of our planar analysis.

Figure 4 (top-right) plots the computed push-over curve (base shear vs roof displacement) of the 9-story moment resisting steel building without rocking wall, which is compared with the push-over curve presented in past investigations (Gupta and Krawinkler 1999, Chopra and Goel 2002). The resulting pre-yielding period of the building is $T_1 = 2.27 \text{ sec}$, while its normalized strength is $Q/m_s = 0.17g$. The remaining two subplots in Figure 4 (right) plot the base-shear versus the mid-height displacement of the 9-story building without rocking wall together with the corresponding force-displacement loops computed with Matlab of the SDOF inelastic model shown in Figure 1 when excited with the 1994 Newhall/360, Northridge (c) and the 1995 Takarazuka/000, Kobe (d) ground motions.

Figure 5 compares response histories computed with OpenSees at mid-height of the 9-story SAC steel building with the solutions obtained with MATLAB for the SDOF idealization shown in Figure 1. The top plots are when the rocking wall is not restrained (No tendon), the center plots are when the rocking wall is restrained with a stiff tendon with $EA/m_w g = 200$ without being prestressed ($P_o = 0$); while, the bottom plots are when the tendon with $EA/m_w g = 200$ is prestressed with $P_o = m_w g$. The left plots are when the structure is subjected to the Newhall/360 ground motion recorded during the 1994 Northridge, California earthquake whereas the right plots are when the structure is subjected to the 1995 Takarazuka/000 ground motion recorded during the 1995 Kobe, Japan earthquake. The comparison of the OpenSees and Matlab solutions are in good agreement—in particular for the peak-response values and supports the use of the SDOF idealization introduced in Figure 1.

6. EARTHQUAKE SPECTRA OF A YIELDING OSCILLATOR COUPLED WITH A ROCKING WALL

Following the verification of the single-degree of freedom idealization by comparing its response with that of the 9-story steel SAC building computed with OpenSees, the equations of motion (15) and (16) are used to generate inelastic response spectra.
Figure 5. Comparison of the displacement time histories at mid-height of the 9-story steel building shown in Figure 4, computed with OpenSees with the displacement time-histories of the SDOF idealization shown in Figure 1, when excited with the 1994 Newhall/360, Northridge, California (left) and the 1995 Takarazuka/000, Kobe, Japan (right) ground motions.

Figure 6 plots displacement spectra of a yielding SDOF oscillator coupled with a vertically prestressed, stepping rocking wall when excited by the Newhall/360 ground motion recorded during the 1994 Northridge, California earthquake. The 1st and 3rd column of the plots (from left) are for a structure with a yielding strength \( Q/m_s = 0.15g \); whereas, the 2nd and 4th plots (from left) are for a weaker structure, \( Q/m_s = 0.08g \). The first and most important observation is that the effect of vertical tendons even when they are stiff (\( EA/m_w g = 200 \)) and highly prestressed (\( P_o = m_w g \)) is marginal. In contrast, the weight of the rocking wall has more noticeable effects with the heavier wall (\( \sigma = 5 \)) being more effective in some regions of the spectra.

Figure 6. Displacement spectra of a yielding SDOF oscillator coupled with a vertically restrained stepping rocking wall with slenderness \( \tan \alpha = 1/6 \), for two valued of strength, \( Q/m_s = 0.15g \) (1st and 3rd column from left) and \( Q/m_s = 0.08g \) (2nd and 4th column from left) with mass ratios, \( \sigma = 5, 10 \) and \( \infty \) (no wall); several values of tendon stiffness (\( EA/m_w g = 0, 40, 72 \) and 200) with (\( P_o = m_w g \)) and without (\( P_o = 0 \)) pre-tensioning when subjected to the Newhall/360 ground motion recorded during the 1994, Northridge California earthquake.
7. CONCLUSIONS

This paper investigates the dynamic response of a yielding SDOF oscillator coupled with a vertically restrained, stepping rocking wall. The full nonlinear equations of motion were derived, and the dependability of the one-degree-of-freedom idealization is validated against the nonlinear time-history response analysis of the 9-story SAC steel building. The equations of motion of the SDOF idealization show explicitly that the contribution of vertical tendons, even when they are stiff, is two orders of magnitude less than the inertia forces on the moment frame-rocking wall system. This paper offers a comprehensive parametric analysis which reaches the following conclusions.

The participation of the stepping rocking wall suppresses peak inelastic displacements with the heavier wall being in most cases more effective. In contrast, the effect of the vertical tendons even when they are stiff ($\frac{EA}{m_{wall}} = 200$) and highly prestressed ($P_o = m_w g$) is marginal. Given than the vertical tendons increase the vertical reactions at the pivoting corners by more than 50%, the paper concludes that for medium- to high-rise buildings, vertical tendons in rocking walls are not recommended.

The SDOF idealization presented in this paper compares satisfactory with finite-element analysis of a 9-story steel SAC building coupled with a stepping rocking wall; therefore, the SDOF idealization can be used with confidence for preliminary analysis and design.

8. REFERENCES


