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NEW DYNAMIC DECOUPLING CRITERIA FOR SECONDARY SYSTEMS

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ABSTRACT

This article aims to propose new criteria for dynamic decoupling between primary and secondary systems, supplementing existing criteria in the French nuclear industry. The criteria developed are based on frequency and response decoupling of both systems and can be directly used by engineers to complement existing criteria. They provide information on the types of interactions between the systems (inertial or kinematic interactions) and on the types of modeling to be adopted (lumped masses or complete model).

Keywords: Decoupling criteria; equipment-structure interaction; primary and secondary systems; mass and frequency ratios; nuclear industry;

1. INTRODUCTION

For various practical reasons in dynamic equipment-structure interaction studies, it is generally relevant to perform the analysis of the primary system (the building) and the secondary system (materials and equipment) separately. This is equivalent to assuming that the two systems are decoupled, in the sense that the response of one is not able to significantly influence the response of the other. Although the consideration of coupling tends to mitigate the response of the primary system – which remains conservative because if there is coincidence of frequencies, the secondary system behaves like a dynamic damper and mobilizes, through its proper vibration, a significant part of the total vibratory energy – this is not the case for the secondary system, for which the response is often greatly amplified by the primary system. Therefore, a decoupled analysis of the primary system can be rationally justified if the decoupling results in no significant error in the response calculation of the equipment.

In the French nuclear industry (see ASN guide 2006, CEA 2008 or ETC-C 2012), criteria are available for the mass ratio between the secondary structure and the primary structure, as well as the ratio of fundamental vibration frequencies, which specify in which cases the coupling must be taken into account. The rule, illustrated on Figure 1, is:

- to neglect the coupling if the mass ratio is less than 1%;
- to take it into account if the mass ratio is greater than 10%;
- to take it into account, for mass ratios ranging between 1 and 10%, if the frequency ratio is between 0.8 and 1.25.

These criteria, which seem at first sight very easy to use, actually raise many questions, especially when mass and frequency ratios are close to values for which, the consideration of the coupling is necessary. Indeed, if we understand that this coupling zone reflects an interaction between the primary and the secondary structures, it does not provide any precision on the nature of this interaction

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(kinematic and/or inertial interaction), and on the representation of the secondary system in the dynamic model (modeling as lumped mass, or complete model, and so on). In addition, the percentage of error on the frequency of the primary structure, when the coupling is not required according to the criteria of nuclear industry, is very different from those proposed by the American code ASCE 4-98 (2000). In fact, the coupling boundaries appear to have been drawn in an empirical and arbitrary way.

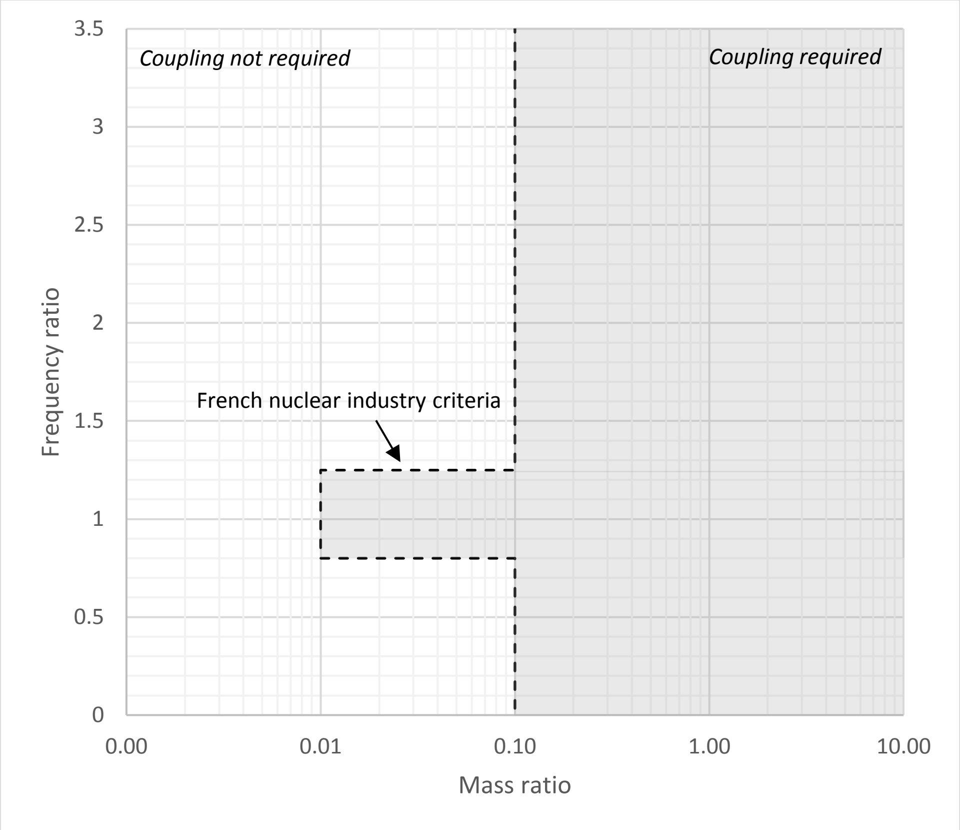

Figure 1. Decoupling criteria applied in the nuclear industry.

The above remarks motivate a redefinition of the decoupling criteria for dynamic equipment-structure interaction in order to provide engineers with “advanced” and rigorously justified decoupling criteria, which will enable them to tackle this problem with precision and rationality. The redefinition of decoupling criteria for dynamic equipment-structure interaction constitutes the main objective of this paper.

2. SYSTEMS WITH TWO DEGREES OF FREEDOM

Before embarking in the analysis of the decoupling criteria, it is necessary to introduce the equations of motion of a two-mass-two-spring system, with two degrees-of-freedom. This system is illustrated by the model type C on Figure 2. In a practical system, the spring k_p and the mass m_p constitute the primary system, and spring k_s and mass m_s the secondary system. The free vibration of this undamped system is represented by the following equations (Pecker 2013):

$$\begin{bmatrix} m_p & 0 \\ 0 & m_s \end{bmatrix} \begin{Bmatrix} \ddot{u_p} \\ \ddot{u_s} \end{Bmatrix} + \begin{bmatrix} k_p + k_s & -k_s \\ -k_s & k_s \end{bmatrix} \begin{Bmatrix} u_p \\ u_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{1}$$

which may be rewritten as a system of two ordinary differential equations as follows:

$$m_p \ddot{u}_p + k_p u_p + k_s (u_p - u_s) = 0 \tag{2.a}$$

and

$$m_s\ddot{u}_s + k_s(u_s - u_p) = 0 \tag{2.b}$$

By substituting into equations (2), $u_p = U_p \sin\omega_n t$ and $u_s = U_s \sin\omega_n t$, with ω_n the frequency of the coupled system, we obtain:

$$(-m_p\omega_n^2 + k_p)U_p + k_sU_p - k_sU_s = 0 \tag{3.a}$$

and

$$-m_s\omega_n^2U_s + k_sU_s - k_sU_p = 0 \tag{3.b}$$

From equations (3), we obtain:

$$\frac{-m_p\omega_n^2 + k_p + k_s}{k_s} = \frac{k_s}{-m_s\omega_n^2 + k_s}$$

Simplifying this we have:

$$\omega_n^4 - \left(\frac{k_p}{m_p} + \frac{k_s}{m_p} + \frac{k_s}{m_s}\right)\omega_n^2 + \frac{k_p}{m_p}\frac{k_s}{m_s} = 0 \tag{4}$$

The frequency of the primary system is $\omega_p = \sqrt{k_p/m_p}$, and the frequency of the secondary system is $\omega_s = \sqrt{k_s/m_s}$. Equation (4) can be written in the following form:

$$\omega_n^4 - \left(\omega_p^2 + \omega_s^2\frac{m_s}{m_p} + \omega_s^2\right)\omega_n^2 + \omega_p^2\omega_s^2 = 0 \tag{5}$$

Moreover, it can be shown that the displacement vector can be written in the following form:

$$\begin{Bmatrix} u_p \\ u_s \end{Bmatrix} = \begin{Bmatrix} 1 - \omega_n^2 m_s/k_s \\ 1 \end{Bmatrix} \tag{6}$$

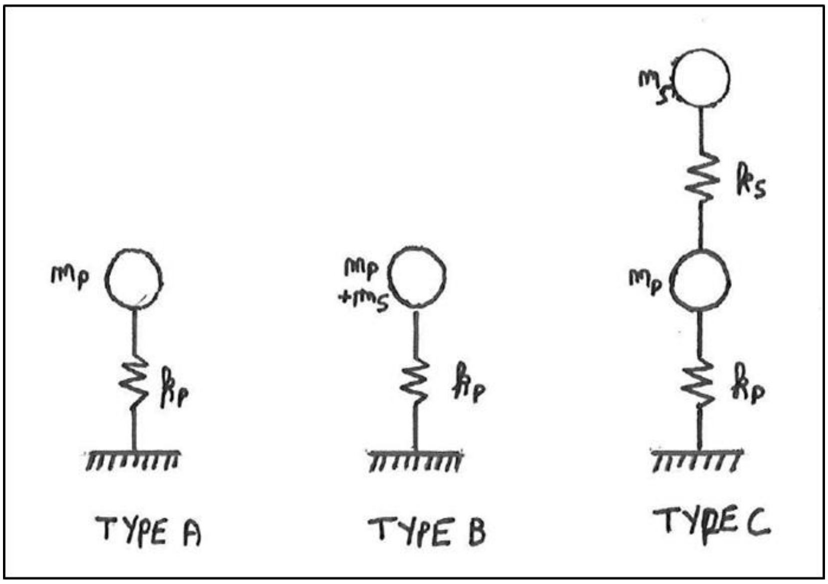

Figure 2. Models for analyzing decoupling criteria.

3. DECOUPLING CRITERIA IN FREQUENCY

We have pointed out in paragraph 2 that the coupled frequencies of a two degrees-of-freedom system (model C on Figure 2) is noted ω_n. If we write the ratio $R_f = \omega_n/\omega_p$, the frequency ratio $r_f = \omega_s/\omega_p$, the mass ratio $r_m = m_s/m_p$, equation (5) can be written in the following form:

$$R_f^4 - \left(1 + r_f^2 + r_m r_f^2\right) R_f^2 + r_f^2 = 0 \tag{7}$$

Equation (7) yields two positive roots for R_f, and hence ω_n, the frequencies of the coupled system. If decoupling has to be achieved, one of these frequencies should be close to the frequency of the primary structure ω_p (model A in Figure 2), the corresponding R_f close to 1, say $R_f^2 = 1 + \varepsilon$, where ε is small. The other mode is not likely to contribute significantly to the response. Substituting $(1 + \varepsilon)$ for R_f^2 in equation (7), we get (Gupta 1992):

$$r_f^2 = \frac{\varepsilon(1+\varepsilon)}{r_m(1+\varepsilon)+\varepsilon} \tag{8}$$

Equation (8) fixes a relationship between r_f and r_m for any change in frequency between model A and model C. For example, for a 10% change in frequency, $R_f = (1 \pm 0.1) = 1.1\,,0.9$, $R_f^2 = 1.21\,,0.81$, $\varepsilon = 0.21\,,-0.19$. The value of ε is positive for $r_f < 1$, and negative for $r_f > 1$.

The r_f versus r_m curves for 5%, 10% and 15% changes in the value of ω_p are shown on Figure 3. The region on the left side of any curve will assume an error on the frequency of the primary structure, if the system is uncoupled, less than that used for the particular curve. As one would expect, to limit the error, r_m should be small and r_f away from unity. Moreover, nuclear industry criteria seem to assume a percentage of error less than 5%. These curves are similar to those obtained by Hadjian and Ellison (1984).

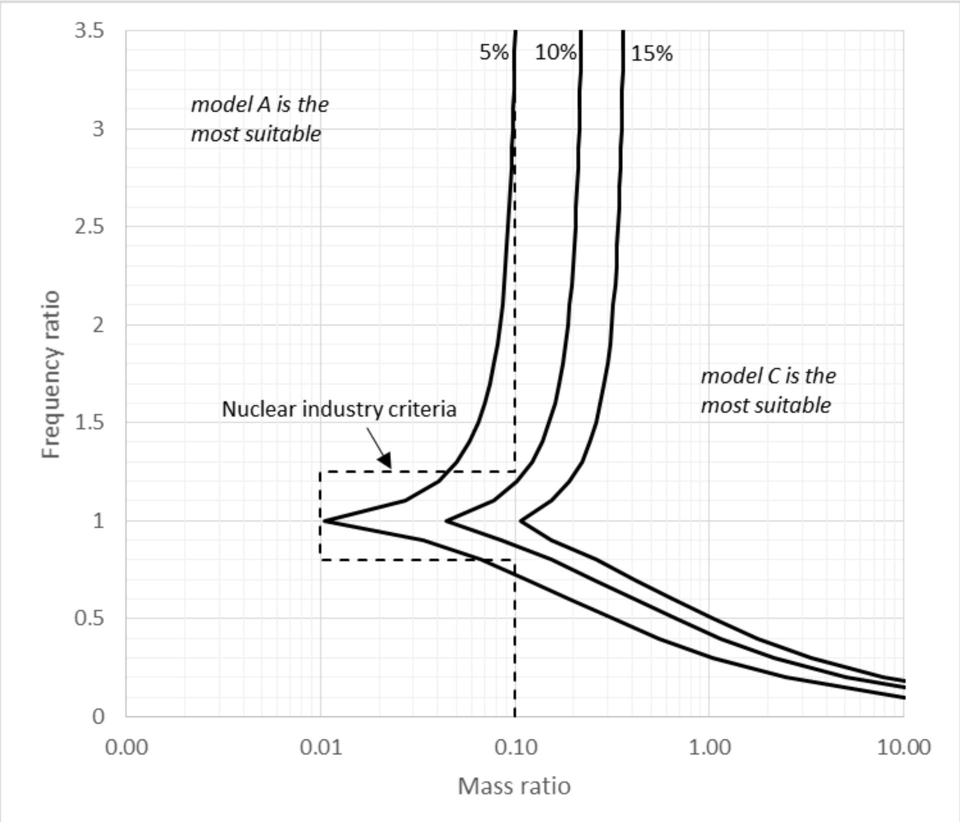

Figure 3. Curves for error of 5%, 10% and 15% in the natural frequency if the system is uncoupled.

Let's go back to equation (4). By introducing $r_m = m_s/m_p$ into it, we write:

$$\omega_n^4 - \left(\frac{k_p(m_p+m_s)}{(m_p+m_s)m_p} + \frac{r_m k_s}{m_s} + \frac{k_s}{m_s}\right)\omega_n^2 + \frac{k_p}{m_p+m_s}\frac{(m_p+m_s)}{m_p}\frac{k_s}{m_s} = 0 \quad (9)$$

If we assume that the kinematic interaction due to the presence of the secondary system is negligible, the frequency of the system is given by the model B in Figure 2, which is $\omega_{n/B} = \sqrt{k_p/(m_p + m_s)}$. Then, the equation (9) takes the following form:

$$\omega_n^4 - (1 + r_m)\left(\omega_{n/B}^2 + \omega_s^2\right)\omega_n^2 + (1 + r_m)\omega_{n/B}^2\, \omega_s^2 = 0 \quad (10)$$

If we write $R_f = \omega_n/\omega_{n/B}$ and $r_f = \omega_s/\omega_{n/B}$, we obtain:

$$R_f^4 - (1 + r_m)\left(1 + r_f^2\right)R_f^2 + (1 + r_m)r_f^2 = 0 \quad (11)$$

If kinematic interaction is negligible, one of the two frequencies ω_n should be close to $\omega_{n/B}$, the corresponding R_f close to 1, say $R_f^2 = 1 + \varepsilon$, where ε is small. The other mode is not likely to contribute significantly to the response. Substituting $(1 + \varepsilon)$ for R_f^2 in equation (11), we get:

$$r_f^2 = (1 + \varepsilon) - r_1\left(\frac{1}{\varepsilon} + 1 + r_f^2\right) \quad (12)$$

Similarly to equation (8), equation (12) fixes a relationship between r_f and r_m for any change in frequency between model B and model C. The r_f versus r_m curves for 5%, 10% and 15% changes in the value of ω_p are shown on Figure 4. The region on the left side of any curve will assume an error on the primary structure, if kinematic interaction is neglected, less than that used for the particular curve.

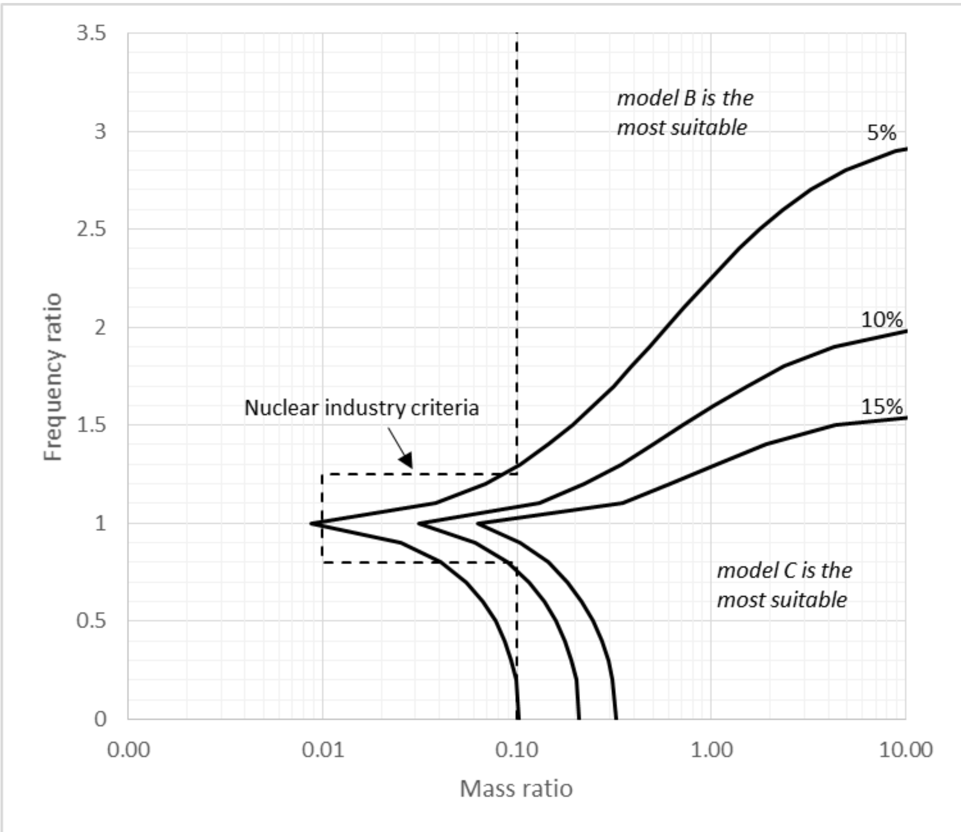

Figure 4. Curves for error of 5%, 10% and 15% in the natural frequency if kinematic interaction is neglected

Let us compile, on Figure 5, the curves from Figure 3 and Figure 4 for a 5% change in frequencies.

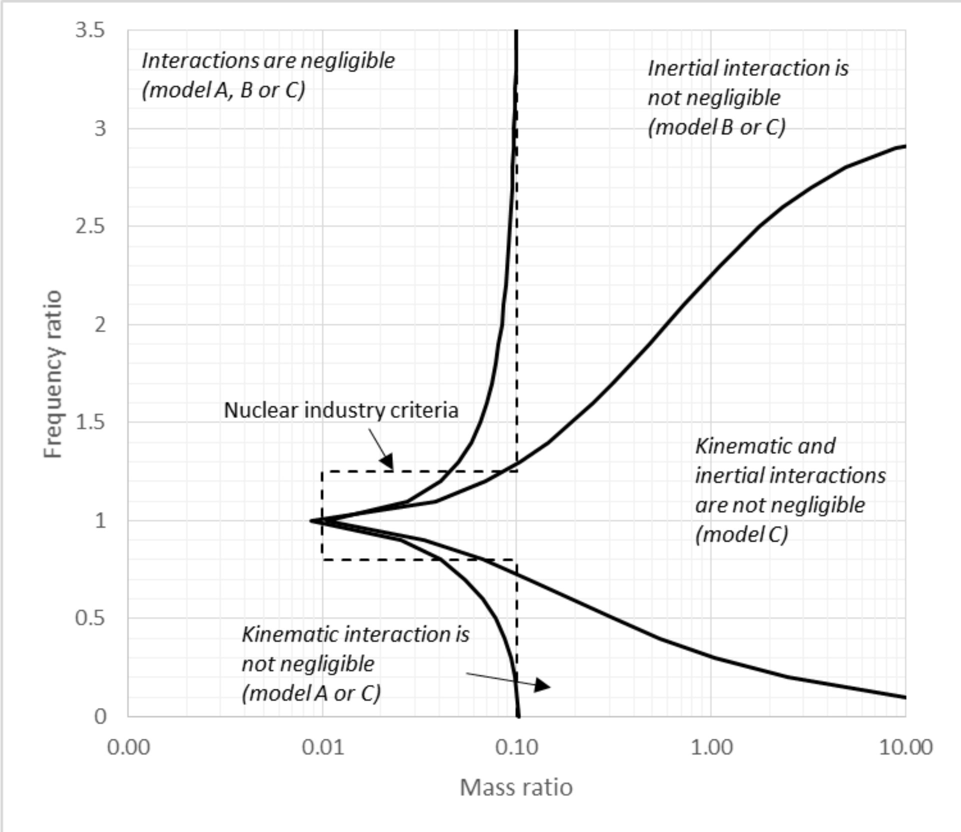

Figure 5. Decoupling criteria for frequency changes in the space of frequency and mass ratios

The curves divide the graph into four regions, corresponding to four forms of interaction between primary and secondary systems:

- a region where interactions can be neglected without modifying by more than 5% the frequencies of the primary structure. In this region, all the models are suitable (model A, B or C) ;
- a region where the inertial interaction, which corresponds to the effects of inertia forces associated with the movement resulting from the consideration of masses of the secondary system, is not negligible. The model which best represents this interaction is model B, but model C is also adequate ;
- a region where the kinematic interaction, which corresponds to the modifications of the movement due to the rigidity of the secondary system, but considering that its mass is zero, is negligible. This behavior can be represented by models A or C ;
- a region where both kinematic and inertia interactions are not negligible. In this case, model C is the only applicable model.

These curves are similar to those presented in code ASCE 4-98 (2000), except that for this code, curves are only drawn for a 10% change in frequencies.

To conclude this paragraph, it appears that a necessary condition for a minimal error in the response is a minimal change in the frequency of vibration. In practice, most codes, like ASCE 4-98, consider this condition to be sufficient, which is open to criticism.

3. DECOUPLING CRITERIA IN RESPONSE

The uncoupled primary system has a maximum relative displacement equal to the spectral displacement S_D at the frequency ω_p. By using equations (6), the displacement vector of the coupled system is (Gupta and Tembulkar, 1984):

$$\begin{Bmatrix} u_p \\ u_s \end{Bmatrix} = \begin{Bmatrix} k_s - \omega_n^2 m_s \\ k_s \end{Bmatrix} = m_s \omega_p^2 \begin{Bmatrix} r_f^2 - R_f^2 \\ r_f^2 \end{Bmatrix} \tag{12}$$

The relative displacement of the primary mass in the coupled system can be shown to be:

$$u_p = \left[1 + \frac{r_m}{\left(1 - R_f^2/r_f^2\right)^2}\right]^{-1} \left(1 + \frac{r_m}{1 - R_f^2/r_f^2}\right) S_D \tag{13}$$

If we assume that the spectral displacement S_D does not change significantly between the frequency of the uncoupled primary system and the coupled system, the response ratio R_R becomes:

$$R_R = \left(1 + \frac{r_m}{1 - R_f^2/r_f^2}\right) \Big/ \left[1 + \frac{r_m}{\left(1 - R_f^2/r_f^2\right)^2}\right] \tag{14}$$

The two-degrees-of-freedom coupled system considered here will have two modes, and hence two values of R_f for any set of r_m, r_f values. Gupta (1992) shows that for both of these modes, the response ratio R_R, is always less than unity, *i.e.*, the response of the primary mass in the coupled analysis in any one of the two modes is always less than that in the uncoupled analysis. Figure 6 shows r_f versus r_m curves for 5%, 10% and 15% reduction in the primary system response ($R_R = 0.95, 0.90, 0.85$) in the mode for which R_f is closest to unity. The acceptable domain (which would limit the reduction) is below the bottom curve and above the top curve for any given reduction.

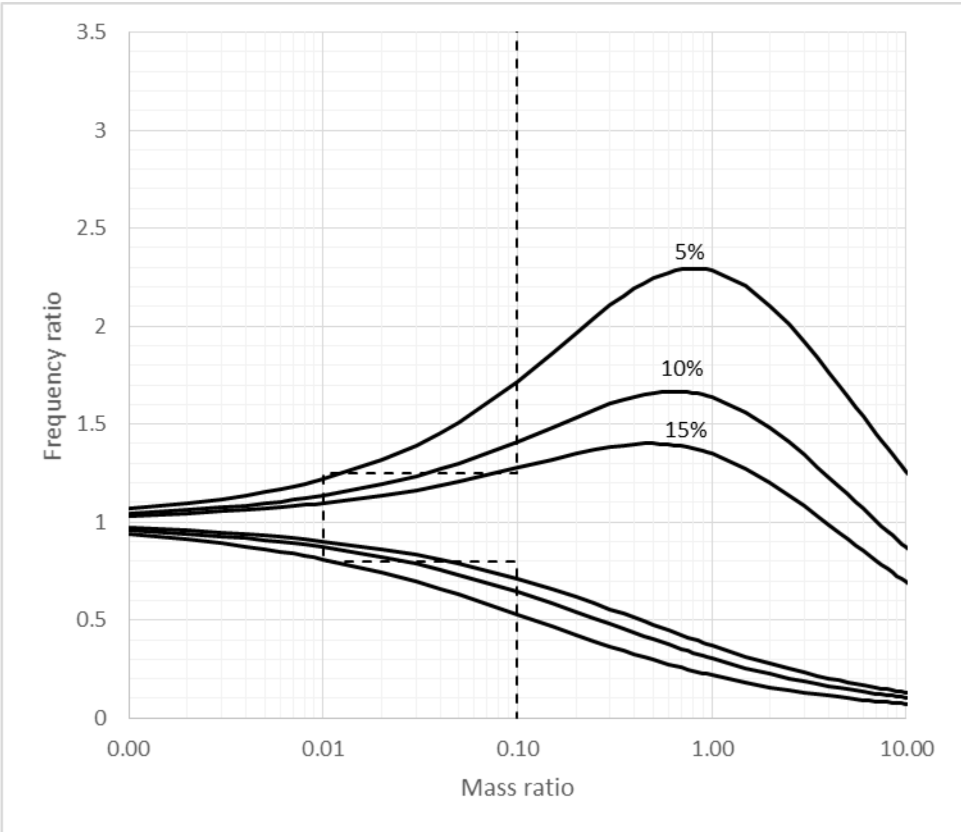

Figure 6. Curves for error of 5%, 10% and 15% in response values if the system is uncoupled

We see that all the curves are asymptotic to the $r_f = 1$ line. This is caused by a resonance between the two systems and a dynamic amplification factor equal to the infinite when the frequency ratio tends to 1. In reality, amplification of the response is limited by damping but our analysis is based on an undamped system.

When the frequency ratio is greater than 1, the response is only affected by the mass of the secondary system, even if this mass if much higher than the mass of the primary system. However, such a system can not be considered as decoupled due to the restriction on frequency change. In addition, the change in frequency which would accompany such large secondary masses, would invalidate the assumption that the spectral displacement S_D did not change.

We observe that for small frequency ratios, the mass of the secondary system no longer controls the response of the structure. This is due to the fact that the stiffness of the secondary system becomes more and more flexible, and the mass of the secondary system no longer moves relative to the primary system.

4. PROPOSAL FOR NEW CRITERIA

Figure 7 shows r_f versus r_m curves for a 5% change in frequencies and response values. Compared to the frequency criteria, the criteria in response widen the region where kinematic and inertia interactions are not negligible, for mass ratios lower than 1.

Although the curves of the criteria in response tend to 1, it is reasonable to assume that the decoupling criteria are achieved for mass ratios less than 0.01.

The proposal for new dynamic decoupling criteria of secondary systems is illustrated on Figure 8.

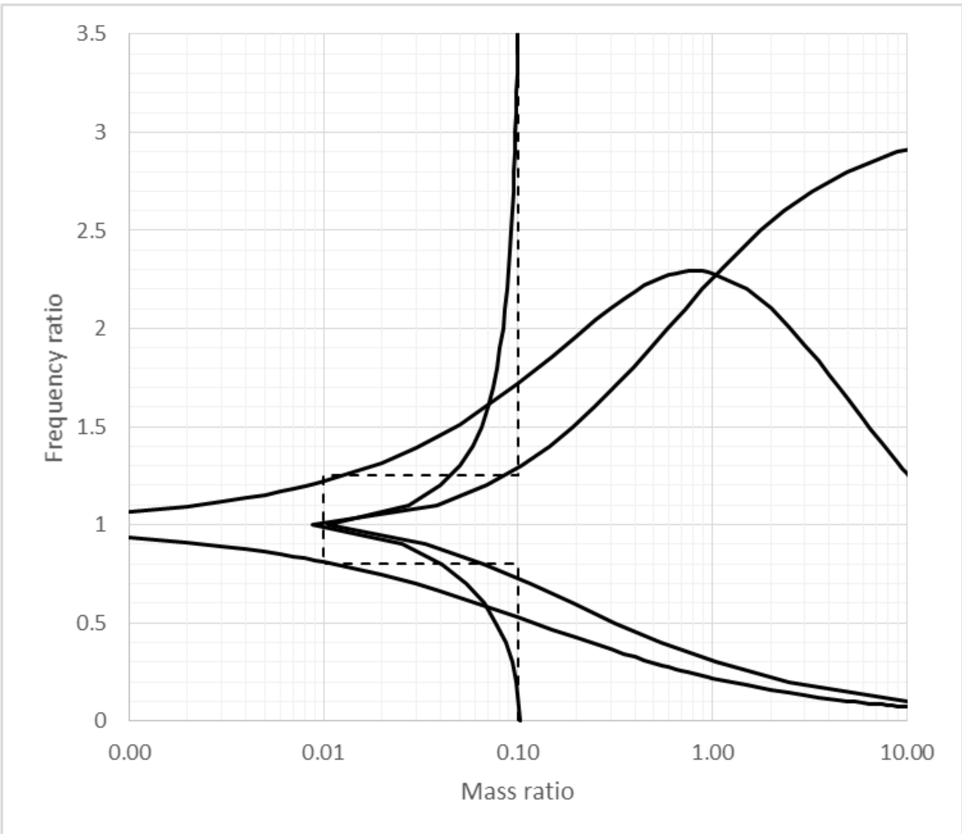

Figure 7. Decoupling criteria for a 5% change in response and frequencies of the primary system

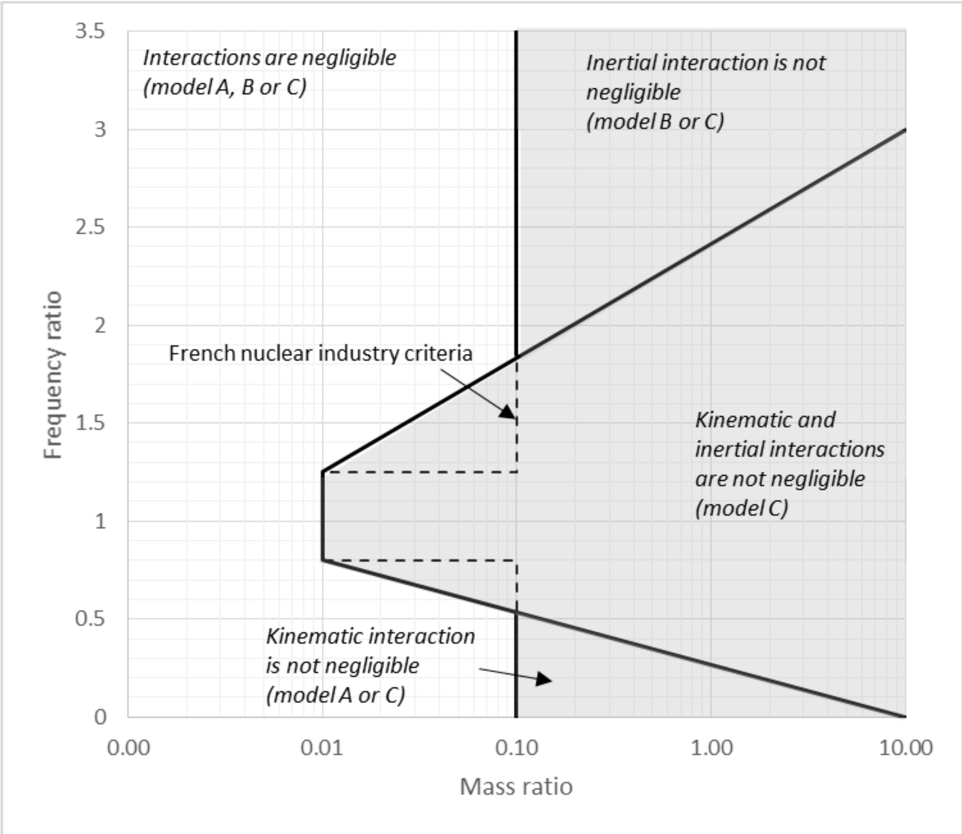

Figure 8. New dynamic decoupling criteria for use in seismic analysis.

It can be seen that, for mass ratios between 0.01 and 0.10, the new criteria indicate that coupling is necessary in areas where the old criteria did not take into account coupling.

The graph is divided into four regions. Each region provides information about the representation of the secondary system in the calculation model of the primary system:

- in the left region, where interactions between the systems are negligible, all the representations of the secondary system are allowed (model A, B or C). Indeed, in this region, whatever the model chosen, the percentage of error in the frequency and the response of the primary system is less than 5%. However, it is recommended to adopt a modeling of the secondary system in the form of lumped masses (model B), in order to obtain the correct mass in the static analysis;
- in the upper right zone, the inertial interaction with the secondary system cannot be neglected. The mass of the secondary system must be taken into account in order to not induce errors of more than 5% in the response and in the frequency of the primary system. It is sufficient to represent the secondary system by lumped masses (model B) although a complete modeling is also possible (model C);
- in the lower right region, the kinematic interaction is dominant and cancels the mass of the secondary system. The mass of the secondary system must not be included in the dynamic analysis (model A), but it is important to take this mass into account for the static analysis. A complete model (model C) is also suitable to restore this kinematic interaction;
- finally in the central region on the right, the model C is the only modeling which is able to restore both the kinematic and inertial interactions with the secondary system.

The values associated with the graph of Figure 8, allowing to build the zones of interactions, are given in the table 1.

Table 1. Values associated with new dynamic decoupling criteria graph.

Point	Mass ratio	Frequency ratio
1	10	3
2	0.01	1.25
3	0.01	0.8
4	10	0
5	0.1	1.85
6	0.1	3.5
7	0.1	0.54
8	0.1	0

5. CONCLUSIONS

The new decoupling criteria developed in this paper can be used by engineers in all their projects of dynamic calculation, in complement of the existing criteria in the nuclear industry.

Future developments will assess the influence of damping on these criteria.

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