A PLASTICITY MODEL FOR 1D SITE RESPONSE ANALYSIS ACCOUNTING FOR LIQUEFACTION-INDUCED GROUND MOVEMENTS

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ABSTRACT

A plasticity model is presented for the non-linear ground response analysis of layered sites. The model is a one-dimensional version of that recently proposed by Tasiopoulou and Gerolymos (2016) for sand behavior, designated as TA-GER sand model. Critical state compatibility for monotonic and cyclic loading, anisotropic plastic flow rule and Bouc-Wen motivated hardening law are among the key-features of the developed 1D model, offering considerable flexibility in representing complex patterns of cyclic behavior such as stiffness decay and increase in strength due to build-up of pore-water pressure. The focus is on the presentation of a calibration procedure targeting the optimum model performance in both drained and undrained load conditions, based on simultaneously matching the response in terms of: (a) the cyclic resistance ratio curves as per the NCEER/NSF methodology, and (b) widely-used experimental shear modulus and damping ratio curves available in the literature.

Keywords: Liquefaction; Cyclic resistance ratio; Excess pore water pressure; Plasticity model; Cyclic loading; Drained and undrained load conditions

1. INTRODUCTION

The one dimensional version of the TA-GER sand model (Tasiopoulou and Gerolymos, 2016) presented in a recently published paper by Anthi et al (2017) for the site response analysis of layered soil deposits under drained load conditions, is reformulated to facilitate the development of a unified calibration procedure for both drained and undrained loading. The calibration is based on best matching the response in terms of (a) the liquefaction resistance (CRR) curve as a function of the relative density and the equivalent number of uniform cycles required to trigger liquefaction, and (b) experimentally derived shear modulus and damping ratio curves commonly used in practice. Regarding the first part of the calibration procedure, the correlation for the reference cyclic resistance ratio \( CRR_{M=7.5, \sigma^\prime=1 \text{atm}} \) from SPT data by Idriss and Boulanger (2008, 2010) was combined with (a) the empirical formula of Seed and Idriss (1982) that relates the earthquake magnitude to the equivalent number of uniform cycles of the seismic motion and (b) the magnitude scaling factor (MSF), as proposed by several researchers (Youd et al., 2001), that associates the reference cyclic resistance ratio with the actual one. The model parameters are divided into two loosely coupled sets, controlling the behaviour in drained and undrained loading conditions, respectively. This way, the response in terms of liquefaction resistance according to a specific engineering design relationship (e.g. Andrus and Stokoe, 1997) can be combined with constitutive stress-strain loops that are consistent with any desired experimental pair of \( G-\gamma, \xi-\gamma \) curves (e.g. Ishibashi and Zhang, 1993). In what follows, a brief introduction to the key model parameters involved in the calibration procedure is given, followed by indicative results demonstrating the capability of the calibrated model in reproducing published data.

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2. BRIEF MODEL DESCRIPTION

Tasiopoulou and Gerolymos (2016) developed a new plasticity-based model for sand behavior formulated in the 6-dimensional stress space. The p-q version of this model was implemented by Anthi et al. (2017) into an explicit finite-difference based algorithm for analyzing the wave propagation in layered soil deposits. The incremental stress-strain form of this model is given by:

\[
\begin{bmatrix}
    dp \\
    dq
\end{bmatrix} = \eta \begin{bmatrix}
    \frac{-K'M_d}{-KM_d + 3G} \zeta_a^n \\
    \frac{-3KM_d}{-KM_d + 3G} \zeta_a^n \\
    \frac{3Gd}{-KM_d + 3G} \zeta_a^n \\
    \frac{-9G^2}{-KM_d + 3G} \zeta_a^n
\end{bmatrix}
\begin{bmatrix}
    de_p \\
    de_q
\end{bmatrix}
\]

in which \(K\) and \(G\) are the elastic (small strain) bulk and shear modulus, respectively, \(d\) is the ratio of the plastic volumetric strain increment \(de_p\) over the plastic deviatoric strain increment \(de_q\) and is based on Rowe’s dilatancy theory as it depends on the distance of the current stress ratio \(q/p\) from the phase transformation line. \(M_p\) and \(M_s\) are the phase transformation and failure stress ratio representing the ultimate strength. Parameter \(\zeta_a\) (Tasiopoulou and Gerolymos, 2016) is a hysteretic dimensionless quantity that provides the loading and unloading rule and is a function of the Bouc–Wen parameter \(\zeta\) (Gerolymos and Gazetas, 2005, Gerolymos et al., 2007), while the exponent \(n\) controls the rate of transition from the elastic state to the perfectly plastic one. Finally, \(\eta\) is inserted as a multiplier of the hardening elastoplastic matrix expressing the dissipated hysteretic energy. The following formulation for the shear modulus \(G\) is adopted:

\[
G = 1532 \rho_a D_r^{0.6464} \left( \frac{p}{\rho_a} \right)^m
\]

where \(\rho_a\) is the atmospheric pressure, \(D_r\) is the relative density and \(p\) is the effective (confinement) pressure. Exponent \(m\) is expressed as a function of the excess pore water pressure ratio \(r_u\) according to:

\[
m = m_{\text{peak}} + (m_0 - m_{\text{peak}}) e^{-r_u}
\]

The initial value of \(m\) is \(m_0\) for zero \(r_u\) (drained conditions), and it tends asymptotically to \(m_{\text{peak}}\) when \(r_u\) approaches unity. Exponent \(n\) can be expressed as the sum of an undrained \(n_u\) and a drained loading \(n_d\) component as:

\[
n = n_u e^{-A \frac{D_r - D_{r0}}{1 - D_{r0}}} + n_d \left( 1 - e^{-A \frac{D_r - D_{r0}}{1 - D_{r0}}} \right)
\]

in which \(D_{r0}\) is the initial value of the relative density, and \(A\) a large number (e.g. \(A \approx 100\)). \(n_u\) is a function of the cumulative deviatoric strain increment according to the following relationship:

\[
n_u = n_f + \left[ n_{\text{peak}} + (n_0 - n_{\text{peak}}) e^{-30 \sum de_q - n_f} \right] e^{-g \sum de_q}
\]

where \(n_0\) is the initial value of \(n\), \(n_{\text{peak}}\) is a potentially reached peak value of \(n\), \(n_f\) is the final value of \(n\) and \(\sum de_q\) stands for the cumulative deviatoric strain increment. \(n_d\) is a constant and is practically activated when the relative density evolves (drained condition). \(g\) is expressed as a function of the relative density \(D_r\), according to:

\[
g = g_1 + g_2 D_r
\]
where \( g_1 \) and \( g_2 \) are constants. The bounding stress ratio has been slightly revised from its original version (Tasiopoulou and Gerolymos, 2017):

\[
M_s = M_{cs} + \left[ M_{sp} + (M_{pt0} - M_{sp}) e^{-2c \sum_{eq} - M_{cs}} \right] e^{-2c \sum_{eq}}
\]

(7)

where \( M_{pt0} \) is an initial value, \( M_{cs} \) is the critical state value and \( M_{sp} \) is a maximum value that can be potentially reached depending on the model parameter \( c \). The latter is given by:

\[
c = 6 + 4I_{r0}
\]

(8)
in which \( I_{r0} \) represents the initial Bolton’s relative density index. In the same context, the phase transformation stress ratio has been revised as follows:

\[
M_{pt} = M_{cs} + (M_{pt0} - M_{cs}) e^{-6c \sum_{eq}}
\]

(9)

with its initial value \( M_{pt0} \) according to the following relationship:

\[
M_{pt0} = \left( M_{speak} - \frac{0.9I_r}{3 + 0.3I_r} \right) \zeta_a
\]

(10)

Parameter \( R_d \) was extended to account not only for drained but also for undrained loading as a function of \( \Delta D_r \) and \( r_u \):

\[
R_d = e^{-a_1 r_u} e^{-a_2 (D_r - D_{r0})}
\]

(11)

Finally, the dissipated hysteretic energy parameter \( \eta \), which was initially proposed by Gerolymos and Gazetas (2005) and Drosos et al. (2012):

\[
\eta = \frac{s_1}{s_1 + \mu s_2}
\]

(12)

where \( \mu \) is a reference ductility defined in terms of the shear strain and \( s_1, s_2 \) are functions of the effective confinement pressure \( \rho \):

\[
s_1 = B \rho^k
\]

(13)

\[
s_2 = C \rho^m
\]

(14)

3. CALIBRATION

An integrated calibration methodology is developed for both drained and undrained load conditions, utilizing published experimental data and case-history based empirical correlations. The model parameters are distinguished into two weakly-coupled sets, each controlling the response in one of the two fundamental load conditions (drained or undrained).

3.1 Undrained conditions

The clean – sand based liquefaction resistance \( CRR \) curves portrayed in Figure 1 (Idriss & Boulanger, 2008, 2010) apply only to magnitude 7.5 earthquakes. To adjust the aforementioned reference curves
to magnitudes smaller or larger than 7.5, Seed and Idriss (1982) and several investigators afterwards introduced correction factors termed “magnitude scaling factors” (MSFs), (Youd et al., 2001). These factors are used to scale up or down the reference curves on CRR versus \((N_1)_{60cs}\), \(q_{ck}\) or \(V_s\) plots. The factor of safety against liquefaction is then defined according to:

\[
FS = \left( \frac{CRR_{7.5}}{CSR} \right)^{MSF}
\]  

Where CSR is the calculated cyclic stress ratio generated by the earthquake shaking, and CRR\(_{7.5}\) is the cyclic resistance ratio for magnitude 7.5 earthquakes. CRR\(_{7.5}\) is determined from Figure 1 for SPT data, and can also be calculated by using CPT data or \(V_s\) data. The most widely used magnitude scaling factors are plotted in Figure 2 while those proposed by Idriss (1995) and Andrus and Stokoe (1997) are also provided in table form (Table 1). The latter two are considered as a lower-bound and upper-bound estimation of the liquefaction resistance, respectively, according to the recommendations by the 1998 NCEER/NSF workshop participants. They are thus formed the basis of the developed calibration procedure.

![Figure 1. Correlations for cyclic resistance ratio (CRR) from SPT data (Idriss and Boulanger, 2010, Seed et al. (1984), Youd et al. (2001), Ziotopoulou and Boulanger (2013))](image)


<table>
<thead>
<tr>
<th>Magnitude Scaling Factor</th>
<th>5.5</th>
<th>6.0</th>
<th>6.5</th>
<th>7.0</th>
<th>7.5</th>
<th>8.0</th>
<th>8.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Idriss (1995)</td>
<td>2.2</td>
<td>1.76</td>
<td>1.44</td>
<td>1.19</td>
<td>1.00</td>
<td>0.84</td>
<td>0.72</td>
</tr>
<tr>
<td>Andrus and Stokoe (1997)</td>
<td>2.8</td>
<td>2.1</td>
<td>1.6</td>
<td>1.25</td>
<td>1.00</td>
<td>0.8</td>
<td>0.65</td>
</tr>
</tbody>
</table>
Adopting the Idriss and Boulanger (2008, 2010) correlation of the reference cyclic resistance ratio ($CRR_{M=7.5, \sigma'=1\text{atm}}$) with the corrected SPT number ($N_{60cs}$):

$$CRR_{M=7.5, \sigma'=1\text{atm}} = \exp \left( \frac{(N_{1})_{60cs}}{14.1} + \left[ \frac{(N_{1})_{60cs}}{126} \right]^2 - \left[ \frac{(N_{1})_{60cs}}{23.6} \right]^3 + \left[ \frac{(N_{1})_{60cs}}{25.4} \right]^4 - 2.8 \right)$$

(16)


and assuming that $(N_{1})_{60}$ is related to the relative density according to (Idriss and Boulanger, 2008):

$$(N_{1})_{60} = 46D_{r}^2$$

(17)

The liquefaction resistance curve can be obtained as a function of the relative density. Then, by using the following curve fitting function (Seed and Idriss, 1982):

$$N = 0.0034 M_{w}^{4.18}$$

(18)

that associates the earthquake magnitude $M_{w}$ with the equivalent number of uniform cycles of the seismic motion and multiplying the reference cyclic resistance ratio ($CRR_{M=7.5, \sigma'=1\text{atm}}$) by the magnitude scaling factor (MSF), the liquefaction resistance curve can be explicitly determined as a function of both the relative density and the number of uniform cycles. The liquefaction resistance curves derived via the Idriss (1995) and Andrus and Stokoe (1997) magnification scaling factors are reproduced by the calibrated model with the help of a MATLAB optimization algorithm. The comparison between computed and suggested values for the cyclic resistance ratio is given in Figures 3 and 4 for two different values of the relative density. The relevant CRR values recommended by other researchers are also informatively provided in the same figures. The onset of liquefaction for the computed CRR curves is deliberately assumed when the excess pore water pressure ratio $r_{u}$ exceeds 0.98. It is observed that the steepest curve (Andrus and Stokoe, 1997) is better reproduced by the calibrated model for high values of the relative density, contrary to the less steep curve (Idriss, 1995) which is matched for lower values of the relative density. The best fit to the Idriss (1995) based CRR
curves is given by the following expression:

\[
MSF = \frac{10^{2.08}}{M_w^{2.36}}
\]  

(19)

while the computed curve that best matches the CRR curve according to the Andrus and Stokoe (1997) magnification scaling factor is given by:

\[
MSF = 1.044 \left( \frac{M_w}{7.5} \right)^{-3.16}
\]  

(20)

Figure 3. Cyclic resistance ratio as a function of the equivalent number of uniform cycles for a relative density of \(D_r = 60\%\). Comparison is given between the predictions of the calibrated model and the curve based on the Andrus and Stokoe (1997) magnification scaling factor (MSF). The derived CRR values in accordance with the recommendations by other researchers regarding the MSF are also presented.

Figure 4. Cyclic resistance ratio as a function of the equivalent number of uniform cycles for a relative density of \(D_r = 40\%\). Comparison is given between the predictions of the calibrated model and the curve based on the Idriss (1995) magnification scaling factor (MSF). The derived CRR values in accordance with the recommendations by other researchers regarding the MSF are also presented.

The performance of the model in terms of the reference cyclic resistance ratio as a function of the corrected SPT value is depicted in Figure 5 in comparison with the design curve by Idriss and Boulanger (2004, 2008).
3.2 Drained conditions

The calibration of the model for drained loading was based on matching some established experimental shear modulus and damping ratio curves from the literature. Three published families of $G : \gamma$, $\zeta : \gamma$ curves have been utilized: (a) the Ishibashi & Zhang (1993), (b) the Vucetic & Dobry (1991), and (c) the pressure ($\sigma_0$)-dependent curves of Darendeli et al. (2001). Starting from the Ishibashi & Zhang (1993) curves and assuming a plasticity index $PI = 0$ for sand, comparison with the predictions of the model for four levels of the mean effective stress $\sigma'_{m}$ (=50, 100, 200, 500 kPa) is presented in Figure 6. The agreement between computed and experimental results is quite satisfactory with some discrepancies observed for small confinement pressure levels. Similarly, Figures 7 and 8 show comparison between the computed $G : \gamma$, $\zeta : \gamma$ curves with those suggested by Vucetic & Dobry (1991) and Darendeli et al. (2001).

![Figure 5](image5.png)

Figure 5. Computed versus suggested by Idriss and Boulanger (2004, 2008) reference cyclic resistance ratio against the corrected SPT number. The calibration of the model parameters for the computed response has been based on the magnification scaling factors by Idriss (1995) and Andrus & Stokoe (1997).

![Figure 6](image6.png)

Figure 6. Approximation of the Ishibashi & Zhang (1993) shear modulus and damping curves for sand ($PI=0$) for selected effective (confinement) pressure levels. Published data is depicted with markers; model results with continuous lines.
3.3 Calibrated Parameters

The constitutive model parameters are grouped into two weakly-coupled sets; those activated for drained load conditions and those when the response is undrained. For intermediate load conditions an appropriate interpolation should be established between the values of the “drained” and the “undrained” parameters with respect to both the relative density and the excess pore water pressure ratio. However, such an interpolation is beyond the scope of this paper. The results of the calibration regarding the two aforementioned extreme load conditions are summarized in Table 2.

Figure 7. Approximation of the Vucetic and Dobry (1991) shear modulus and damping curves for sand ($PI=0$) for selected effective (confinement) pressure $p=80$ KPa. Published data is depicted with markers; model results with continuous lines.

Figure 8. Approximation of the Darendeli et al. (2001) shear modulus and damping curves for sand ($PI=0$) for selected effective (confinement) pressure levels. Published data is depicted with markers; model results with continuous lines.

4. PERFORMANCE AT STRESS-STRAIN LEVEL

Despite the capability of the model to reproduce the phenomenological response of the soil in terms of the cyclic resistance ratio and shear modulus—damping ratio curve, its satisfactory performance at the stress-strain level is not guaranteed. On the contrary, the successful performance at both response levels is often a very difficult task. To demonstrate that the calibrated model yields physically meaningful results at the stress-strain level, consistent with the observed behaviour of sand, indicative model predictions are shown in Figures 9 and 10. The results correspond to the calibrated parameters for the Andrus and Stokoe (1997) magnification scaling factor and Vucetic and Dobry (1991) shear modulus-damping ratio curves. The example under investigation is that of a sand with relative density...
of $D_r = 70\%$. Two numerical cyclic simple shear tests are conducted: (1) The first one is a stress-controlled undrained test of constant amplitude $\tau_{\text{max}} = 20 \text{ kPa}$ with an initial mean effective stress of $p = 66.6 \text{ kPa}$, and (2) the second one is a strain-controlled drained test of constant amplitude $\gamma = 1\%$ and mean effective stress of $p = 80 \text{ kPa}$.

Table 2. Values of the calibrated model parameters corresponding to results presented in Figures 3-8.

<table>
<thead>
<tr>
<th>Calibrated Model Parameters</th>
<th>B</th>
<th>k</th>
<th>C</th>
<th>m</th>
<th>n_d</th>
<th>a_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drained conditions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vucetic &amp; Dobry</td>
<td>5.3</td>
<td>0</td>
<td>1.06</td>
<td>0</td>
<td>*</td>
<td>10</td>
</tr>
<tr>
<td>Darendeli et al.</td>
<td>36.13</td>
<td>-0.3</td>
<td>1.68</td>
<td>-0.06</td>
<td>0.70</td>
<td>10</td>
</tr>
<tr>
<td>Ishibashi &amp; Zhang</td>
<td>19.16</td>
<td>-0.2</td>
<td>1.14</td>
<td>0</td>
<td>0.74</td>
<td>10</td>
</tr>
<tr>
<td>Undrained conditions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Idriss</td>
<td>25</td>
<td>0.2</td>
<td>400</td>
<td>424</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Andrus &amp; Stokoe</td>
<td>42</td>
<td>0.2</td>
<td>484</td>
<td>433</td>
<td>0.4</td>
<td>0.7</td>
</tr>
</tbody>
</table>

*0.0022 p + 0.7, $p=$effective pressure

5. CONCLUSIONS

A methodology was presented for calibrating key parameters of a recently developed constitutive model for sand, designated as TA-GER model (Tasiopoulou and Gerolymos, 2016). The model was slightly modified from its initial version to allow for flexibility in reproducing the response in both drained and undrained load conditions without re-adjusting the values of the parameters. The calibration was based on matching the response in terms of the liquefaction resistance ratio and experimental shear modulus and damping ratio curves available in the literature. It was shown that the calibrated model reproduced the recommended design curves with sufficient engineering accuracy while at the same time maintained its capability to yield physically meaningful results at the stress-strain level consistent with the observed response.

Figure 9. Computed undrained cyclic simple shear response of a sand specimen with $D_r=70\%$, $p_{\text{initial}}=66.6 \text{ kPa}$ and maximum shear stress $\tau=20 \text{ kPa}$. The response is illustrated in the form of applied shear stress versus mean effective stress evolution and shear stress-strain loops. The calibration of the model has been based on the Andrus and Stokoe (1997) magnification scaling factor.
Figure 10. Computed drained cyclic simple shear response of a sand specimen with $Dr=70\%$, $p_{initial}=80$ KPa and maximum imposed shear strain $\gamma=1\%$. The response is illustrated in the form of stress-strain hysteretic loops and volumetric strain evolution with shear strain. The calibration of the model has been based on the Vucetic and Dobry (1991) shear modulus and damping ratio curves.

6. REFERENCES


