IMPEDANCE FUNCTIONS OF ADJACENT EMBEDDED STRIP FOUNDATIONS

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ABSTRACT

The interaction between two adjacent strip foundations embedded in a soil layer on rigid substratum is studied. The response of the foundations under vertical, horizontal and rocking loading is investigated by the application of the finite element method. The accuracy of the finite element model is verified by a comparison study with the rigorous results from the boundary element method concerning a single strip foundation. Impedance functions are presented for typical values of the distance ratio S/L, where S is the distance between the foundations and L is the width of each foundation, over a wide range of frequencies. The condition under which the interaction between adjacent embedded strip foundations should be taken into consideration is further discussed.

Keywords: Soil-foundation-structure interaction, embedded strip foundations, Finite Element Method.

1. INTRODUCTION

Fixed based structure is an assumption that can be considered appropriate only in very limited cases of a hard substratum site. Consequently, the soil-structure interaction (SSI) phenomenon must be taken into account during dynamic structural design analysis. Furthermore, in the cases of closely spaced structures or structures on relatively close supports, the cross-interaction through the soil between adjacent foundations might also be important for the solution of some practical problems (Betti 1997). Therefore, in order to evaluate their dynamic characteristics, it is necessary not only to investigate the classical interaction between the foundation and the soil medium, but also the interaction between the adjacent foundations through the soil (Wang et al. 2015). The presence of a foundation may affect its adjacent ones in a way such that each foundation which diffracts the incident wave field can be regarded as a disturbance producing a secondary wave field affecting its neighboring (Tham et al. 1998). This phenomenon is known as dynamic cross interaction (DCI) problem (Wang et al. 2015), structure-soil-structure interaction (SSSI) (Bharadwaj and Ahmad 1992) or foundation-soil-foundation interaction (FSFI) (Karabalis and Mohammadi 1998).

The present paper investigates the dynamic behavior of an isolated, rigid, massless strip foundation embedded in a soil layer resting on rigid substratum as well as the dynamic cross interaction between two strip foundations with the aforementioned characteristics. Many researchers investigated the dynamic characteristics of arbitrary shaped embedded foundations. Karabalis and Beskos (1986) used the full-space Green’s functions in order to investigate the dynamic response of an arbitrary shaped embedded rigid foundation. The vertical and horizontal response of arbitrarily shaped embedded foundations was calculated by Gazetas et al. 1985 and Gazetas et al. 1987. Rajapakse and Shah (1988) studied the dynamic response of rigid strip foundations of arbitrary geometry embedded in a homogeneous elastic half-space. Hatzikonstantinou et al. 1989 estimated the static and dynamic stiffness coefficients while Fotopoulou et al. 1989 estimated the radiation damping coefficients of arbitrarily shaped rigid foundations embedded in an elastic homogeneous halfspace.

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Wang and Rajapakse (1991) studied the dynamic response of an arbitrary shaped rigid strip foundation embedded in an orthotropic elastic soil under time-harmonic vertical, horizontal and moment loadings. Esmaeilzadeh Seylabi et al. (2016) calculated the impedance functions of rigid massless soil-structure interfaces that are embedded in arbitrarily heterogeneous half-spaces.

The soil-structure interaction of an embedded strip foundation was investigated also by many researchers. In particular, the boundary integral equation scheme was used by Kobori et al. (1982) and by Nakai et al. (1984) to study the dynamic behavior of embedded strip foundations. Israil and Ahmad (1989) investigated in detail the dynamic response of surface and embedded rigid strip foundation in viscoelastic soils under vertical excitation. Ahmad and Bharadwaj (1991) investigated in detail the influence of key mechanical and geometrical parameters on the dynamic horizontal stiffness of rigid strip foundations embedded in layered soil. Bharadwaj and Ahmad (1992) investigated in detail the influence of key mechanical and geometrical parameters on the rocking impedance of rigid strip foundations embedded in layered soil.

Although there are many studies referring to the dynamic behavior of a single, massless, rigid embedded foundation, very few deal with the dynamic cross interaction problem. It is worth mentioning that Rajapakse and Shah (1988) investigated the effect on impedance due to the presence of an adjacent embedment for various distances between foundations and embedment ratios. Furthermore, Betti (1997) studied the dynamic cross-interaction between two identical rigid square foundations embedded in a homogeneous viscoelastic half-space by using the boundary element method.

The main target of the present study is the estimation of the impedance matrix referring to a system of two rigid, massless, strip foundations embedded in a soil layer resting on a rigid substratum. The influence of the distance between the two foundations on the impedance values is further investigated.

2. PROBLEM AND MODEL DESCRIPTION

The problem under study is depicted in Figure 1. Two rigid, massless, strip foundations are embedded to a depth, D in a soil layer which is resting on a rigid substratum. The lateral welding between the soil and each foundation is effective over the entire depth of embedment. Therefore, the two foundations are perfectly bonded to the surrounding soil. Perfect bond is defined to be the type of attachment between the foundation and the soil which is characterized by the complete continuity of displacements in the area of contact. This type of attachment is also referred to as welded contact (Hryniewicz 1981). The soil comprises of a layer of thickness, H which is homogeneous, elastic, isotropic and rests on a rigid substratum.

Figure 1. Adjacent, embedded strip foundations bonded to the surrounding soil.
The two foundations have the same length (L), the same depth (D) and their separation distance is denoted by S. Each foundation is excited by a vertical force (V), a horizontal force (H) and a moment (M). Each external load varies harmonically in time. Subscript 1 refers to the left and subscript 2 refers to the right foundation. The letter w refers to the vertical displacement, u to the horizontal displacement and φ to the rocking angle. The positive directions of external loads and the corresponding positive displacements as well are depicted in Figure 1.

The relationship between the external forces and the corresponding displacements of the two embedded foundations is expressed by the following equation.

\[
\begin{bmatrix}
V_1 \\
H_1 \\
M_1 \\
V_2 \\
H_2 \\
M_2 \\
\end{bmatrix} = 
\begin{bmatrix}
K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\
K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\
K_{33} & K_{34} & K_{35} & K_{36} & K_{37} \\
K_{44} & K_{45} & K_{46} & K_{47} & K_{48} \\
K_{55} & K_{56} & K_{57} & K_{58} & K_{59} \\
K_{66} & & & & & \\
\end{bmatrix} 
\begin{bmatrix}
w_1 \\
u_1 \\
φ_1 \\
w_2 \\
u_2 \\
φ_2 \\
\end{bmatrix}
\]  
(1)

Each foundation has three degrees of freedom. Therefore, matrix \([K]\) is a 6x6 matrix. Furthermore, matrix \([K]\) is characterized as the impedance matrix. Each matrix term is frequency-dependent and complex. In a simplified model, the real parts represent the springs which are related to the restraining action of the supporting soil or the true stiffness, whereas the imaginary parts represent the dashpots or the out-of-phase components which are related to the effect of energy dissipation by radiation (Bharadwaj, Ahmad 1992, Karabalis, Mohammadi 1998). The impedance matrix is symmetrical.

During the analyses, it was observed that the impedance matrix is simplified in the following form:

\[
\begin{bmatrix}
V_1 \\
H_1 \\
M_1 \\
V_2 \\
H_2 \\
M_2 \\
\end{bmatrix} = 
\begin{bmatrix}
K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\
K_{22} & K_{23} & -K_{15} & K_{24} & K_{25} & K_{26} \\
K_{33} & -K_{16} & K_{34} & K_{35} & K_{36} & K_{37} \\
K_{44} & -K_{17} & -K_{12} & K_{45} & K_{46} & K_{47} \\
K_{55} & K_{56} & K_{57} & K_{58} & K_{59} & K_{60} \\
\end{bmatrix} 
\begin{bmatrix}
w_1 \\
u_1 \\
φ_1 \\
w_2 \\
u_2 \\
φ_2 \\
\end{bmatrix}
\]  
(2)

Therefore, the calculation of the complete impedance matrix can be achieved by imposing the appropriate loading (vertical force, horizontal force, moment) in only one of the two adjacent foundations. In particular, the following displacements are considered to be necessary: (i) vertical loading – complete displacement field of the two foundations, (ii) horizontal loading – horizontal displacements and rocking angles of both foundations and (iii) moment loading - rocking angles of both foundations. The same notion also applies for the case of two adjacent surface strip foundations resting on half-space (Terzi 2017).

The shear modulus (G) of the soil equals to 132MPa, the Poisson’s ratio (v) equals to 0.4 and the density (ρ) equals to 1.75t/m³. In the case of a strip foundation, referring to a track, dam or building foundation with a high ratio of length to width, plane strain conditions is reasonable to be assumed (Wang et al. 2015). Therefore, the finite element model is created in 2D space by using 9-node quadrilateral 2-D solid elements (ADINA 2011).

The ratio between the foundation’s depth and half-width is considered to be unity (D/B=1), whereas the ratio between the soil’s stratum depth and foundation’s half-width is considered to be two (H/B=2).

The wave propagation problem that takes place in a model constructed by the use of the finite element method, cannot be simulated perfectly due to the restriction imposed by the model length and the finite elements size (Smith 1974). In particular, the model must be large enough in order to reduce any reflected waves at its boundaries (Roesset, Ettouney 1977) and the finite elements’ size must be as small as possible in order to accurately describe a wavelength. Therefore, in order to avoid any wave reflections, quiet boundaries suggested by Lysmer and Kuhlemeyer (1969) were used. Furthermore, the minimum finite element size for each frequency was equal to λₚ / 10, where λₚ is the wavelength of Rayleigh waves.
3. FINITE ELEMENT RESULTS

3.1 Validation study

In order to validate the finite element model, the impedance functions of an isolated, rigid, massless strip foundation embedded to a depth D of a soil layer resting on rigid substratum, were compared with the results of the study of Huh (1986). The researcher uses the boundary element method and calculates the impedance functions of an embedded foundation for various ratios of D/B, whereas the H/B remains equal to two (H/B=2). The calculated by the use of the finite element method, impedance matrix was transformed into the flexibility matrix by $[F]=[K]^{-1}$. The range of frequency $0<\alpha_0<2$ is considered in the numerical study since most forced vibrations of machine foundations are within this range (Gazetas 1983). In Figure 2, the comparison charts are depicted.

![Comparison charts](image)

Figure 2. Validation study results for the case of an isolated, rigid, massless strip foundation embedded to a soil layer resting on a rigid substratum: a) vertical, b) horizontal, c) coupled horizontal-rocking and d) rocking degree of freedom.

The charts refer to the vertical, the horizontal, the coupled horizontal-rocking and the rocking degree of freedom. The horizontal axis of the charts represents the normalized frequency $\alpha_0$, $\alpha_0 = (\omega \cdot B) / V_s$, where $V_s$ is the shear wave velocity and the vertical axis is normalized by the parameters $\pi$, $G$ and $B$. The vertical loading of the strip foundation does not produce any horizontal or rocking movement. Therefore, no coupling terms exists for the vertical degree of freedom. The finite element method results are denoted with the red color, whereas the boundary element method results are denoted with the blue color. No information is given by Huh (1986) regarding the coupled horizontal-rocking degree of freedom. It can be seen that the results of the present study are in very good agreement with the results of the aforementioned researcher. Consequently, the validation study can be considered successful.

3.2 Coupling of two adjacent foundations study

The main core of the present study refers to the estimation of the dynamic cross interaction between two adjacent, rigid, massless strip foundations embedded to a soil layer over a rigid substratum. Therefore, the main question that must be answered is the following: In what degree an external loading imposed on a foundation affects the dynamic behavior of its adjacent one? In order to answer
the above question, the necessary terms of the impedance matrix, according to equation (2), were estimated for the following S/L ratios: 0.125, 0.5, 3 and 5.

Based on the analyses results, it was observed that the displacement field of each foundation is attributed not only on its external loading but on the adjacent foundation’s loading as well. Therefore, not only coupling terms between the horizontal and rocking movement exist but coupling terms also appear between the adjacent foundations even for the vertical degree of freedom. In particular, Figure 3 depicts the two foundations displacement fields at a random time frame for the case of S/L=0.5.

![Figure 3](image1.png)

Figure 3. Foundations response at a random time frame for the case of S/L=0.5: a) vertical, b) horizontal and c) moment loading.

The foundations initial position is depicted by the blue color, whereas the deformed position is depicted by the cyan color. It can be observed that even the vertical excitation causes a rocking and horizontal movement not only the foundation itself but on the adjacent foundation as well. As expected, the smaller the ratio of S/L is, the bigger the effects of the foundation-soil-foundation interaction phenomenon are.

The foundation-soil-foundation interaction is expressed by the impedance matrix, which relates the external loading of a foundation to the foundations displacement fields. The following figure, Figure 4, depicts the real parts of the necessary terms for the definition of the impedance matrix, according to equation (2). The horizontal axis of the charts represents the normalized frequency $\alpha_0$ and the vertical axis is normalized by the parameters $\pi$, G and B.

Some of the most important observations regarding the real parts are the following:

**Vertical excitation**

- In the case of S/L=0.125, the term F11 receives smaller values than in all other cases, which approximate the behavior of a single foundation.
- In the case of S/L=0.125, the terms F12 and F13 receive greater values than in all the other cases, which approximate the behavior of a single foundation. Therefore, the vertical-horizontal and vertical-rocking coupling for the loaded foundation is stronger in small values of ratio S/L.
- In the case of S/L=0.125, the term F14 receives greater values in the low-middle frequency
range than in all other cases. Whereas, in the high frequency range the vertical-vertical coupling between the adjacent foundations is strong.

- In the cases of S/L=0.125 and 0.5, the term F15 receives greater values in the low frequency range than in the other two cases (S/L=3 and 5). Whereas, in the middle-high frequency range the vertical-horizontal coupling of the unloaded foundation is strong.
- In the case of S/L=0.125, the term F16 receives greater values in the low-middle frequency range than in all other cases. Whereas, in the high frequency range the vertical-rocking coupling of the unloaded foundation is strong.

In the cases of S/L=0.125 and 0.5, the term F22 receives lower values in the low frequency range than the other two cases. Whereas, in the high frequency range all the cases of S/L approximate the behavior of a single foundation.

- In the case of S/L=0.125, the term F23 receives smaller values in the high frequency range than the other cases, which approximate the behavior of a single foundation.
- In the cases of S/L=0.125 and 0.5, the terms F25 and F26 receives greater values in the low frequency range than in the other two cases (S/L=3 and 5). Whereas, in the middle-high frequency range the horizontal-horizontal and horizontal-rocking coupling between the

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**Figure 4.** Real parts of the necessary terms for the definition of the impedance matrix.

**Horizontal excitation**

- In the cases of S/L=0.125 and 0.5, the term F22 receives lower values in the low frequency range than the other two cases. Whereas, in the high frequency range all the cases of S/L approximate the behavior of a single foundation.
- In the case of S/L=0.125, the term F23 receives smaller values in the high frequency range than the other cases, which approximate the behavior of a single foundation.
- In the cases of S/L=0.125 and 0.5, the terms F25 and F26 receives greater values in the low frequency range than in the other two cases (S/L=3 and 5). Whereas, in the middle-high frequency range the horizontal-horizontal and horizontal-rocking coupling between the
adjacent foundations is strong.

Moment excitation

- In the case of \( S/L = 0.125 \), the term \( F_{33} \) receives lower values than the case of a single foundation. In the cases of \( S/L = 0.5, 3 \) and \( 5 \), the term \( F_{33} \) receives lower values than the case of a single foundation but higher values than the case of \( S/L = 0.125 \). Therefore, a greater distance between the adjacent foundations is needed in order to approximate the rocking behavior of a single foundation.
- In the cases of \( S/L = 0.125 \) and \( 0.5 \), the term \( F_{36} \) receives greater values in the low frequency range than in the other two cases (\( S/L = 3 \) and \( 5 \)). Whereas, in the middle-high frequency range the rocking-rocking coupling of the unloaded foundation is strong.

The following figure, Figure 5, depicts the imaginary parts of the necessary terms for the definition of the impedance matrix, according to equation (2).

Some of the most important observations regarding the imaginary parts are the following:

Vertical excitation

- In all cases of \( S/L \), the term \( F_{11} \) approximates the behavior of a single foundation.
• In the low frequency range, the terms F12 and F13 approximate the behavior of a single foundation. Whereas, in the high frequency range the vertical-horizontal and vertical-rocking coupling of the loaded foundation cannot be ignored especially for the cases of S/L=0.125 and 0.5.

• In the low-middle frequency range, the terms F14, F15 and F16 receive almost zero values. Therefore, no interaction can be considered between the adjacent foundations. Whereas, in the high frequency range, the vertical-vertical, vertical-horizontal and vertical-rocking coupling cannot be ignored.

Horizontal excitation
• In the low frequency range, the term F22 approximates the behavior of a single foundation for all the cases of S/L. Whereas, in the high frequency range small differences are observed for the cases of S/L=0.125 and 0.5.

• In the low-middle frequency range, the term F23 approximates the behavior of a single foundation for all the cases of S/L. Whereas, in the high frequency range, the term F23 receives smaller values for the case of S/L=0.125.

• In the low frequency range, the terms F25 and F26 receive zero values. Therefore, no interaction can be considered between the adjacent foundations. Whereas, in the middle-high frequency range the horizontal-horizontal and horizontal-rocking coupling of the unloaded foundation cannot be ignored.

Moment excitation
• In the low-middle frequency range, the term F33 approximates the behavior of a single foundation for all cases of S/L. Whereas, in the high frequency range, the term F33 receives the smallest values for the case of S/L=0.125 and lower than the case of a single foundation for all the other cases.

• In the low-middle frequency range, the term F36 receives zero values. Therefore, the rocking-rocking coupling can be ignored. The same does not apply in the high frequency range.

In the following figure, Figure 6, the necessary terms for the definition of the impedance matrix are depicted in the complex number space. In particular, the horizontal axis represents the normalized real part while the vertical axis represents the normalized imaginary part of each impedance term. Each point in every chart follows a frequency step, \( \omega_0 \), of 0.1, starting from 0.10 and ending at 2.

Some of the most important observations regarding representation of impedance terms on complex number space are the following:

Vertical excitation
• For all cases of S/L, except the S/L=0.125, the term F11 approximates the behavior of a single foundation. Therefore, the presence of the unloaded foundation cannot be ignored for a very small separation distance (S/L=0.125).

• The vertical-horizontal coupling of the loaded foundation, which is expressed by the term F12 cannot be ignored for the cases of small separation distances between the two adjacent foundations (S/L=0.125 and 0.5).

• For all cases of S/L, except the S/L=0.125, the vertical-rocking coupling, which is expressed by the term F13 approximates the behavior of a single foundation. Therefore, the presence of the unloaded foundation cannot be ignored for a very small separation distance (S/L=0.125).

• The vertical-vertical, vertical-horizontal and vertical-rocking coupling between the two adjacent foundations, which are expressed by the corresponding terms of F14, F15 and F16 cannot be ignored for all the cases of S/L. However, the larger the separation distance is the smaller the values of the impedance terms are.

Horizontal excitation
• For the large values of separation distances (S/L=3 and 5), the term F22 approximates the behavior of a single foundation. Therefore, the presence of the unloaded foundation cannot be ignored for a very small separation distances (S/L=0.125 and 0.5).

• For all cases of S/L, except the S/L=0.125, the term F23 approximates the behavior of a single foundation. Therefore, the presence of the unloaded foundation cannot be ignored for a very small separation distance (S/L=0.125).

• For all cases of S/L, the term F25, which represents the horizontal-horizontal coupling
between the two foundations cannot be ignored.

- For all cases of S/L, the term F26, which represents the horizontal-rocking coupling of the unloaded foundation, receives smaller values in comparison to the other horizontal degrees of freedom.

**Moment excitation**

- For the case of S/L=0.125, the term F33 receives the smallest values, whereas for all the other cases of S/L, it receives greater values than the S/L=0.125 case but smaller than the single foundation case. Therefore, the presence of the unloaded foundation cannot be ignored for very small separation distances.
- For all cases of S/L, the term F36, which represents the rocking-rocking coupling between the two foundations cannot be ignored.

Furthermore, in the case of a single foundation and of low values of S/L, the lines are smooth like spirals, whereas for large values of S/L, the lines are polygons.

![Figure 6. Representation of impedance terms on complex number space.](image-url)
4. CONCLUSIONS

The through-the-soil coupling of two rigid, massless strip foundations embedded in a soil layer resting on rigid substratum has been studied by the use of the finite element method. The accuracy of the model has been verified by the comparison studies referring to an isolated strip foundation. The foundation-soil-foundation is expressed by the impedance matrix. For low values of S/L, the coupled response cannot be ignored even for the vertical excitation. The charts of real and imaginary parts can be used in a simplified model of springs and dashpots.

5. REFERENCES


