SEISMIC RESPONSE CONTROL USING ELASTOPLASTIC TUNED MASS DAMPER

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ABSTRACT

Tuned Mass dampers (TMDs) are used to suppress the vibratory response of structures due to dynamic loads particularly wind and seismic excitations. It is well established that efficiency of tuned mass damper (TMD) can be increased with proper tuning of its parameters i.e. frequency of TMD and damping ratio. A typical TMD comprises of mass, stiffness and damper elements, wherein major energy dissipation occurs due to damping mechanism. One possible way to enhance energy dissipating capacity of TMD is by including the hysteresis behaviour i.e. by making the TMD elastoplastic. In the present work, the effect of inclusion of plastic behaviour in TMD on its efficiency is studied. The main system is considered as elastic single degree of freedom (SDOF) and TMD is considered as another SDOF with elastoplastic behaviour. The effect of yield level of TMD on the response of main system under harmonic base excitations and seismic ground motions is studied. The governing equations of motion are solved numerically using Newmark beta method. The numerical study is carried out to assess the optimum parameters of elastoplastic TMD, which may be different than optimum elastic TMD. It is seen that once plastic behaviour of TMD is included, then its performance is better if damping of TMD is same as that of main system.

Keywords: Tuned mass damper; Elastoplastic; Optimum parameters; Yield level; Seismic response control

1. INTRODUCTION

Earthquake and wind loads are the extreme dynamic loads which a structure encounters and suffers excessive vibrations and damage due to them. To ensure acceptable dynamic response of structures, vibration controlling devices like tuned mass damper (TMD) are used. TMDs have been deployed in many buildings, bridges and other structures. Some of the examples include John Hancock tower, Boston; Petronas Twin Towers, Malaysia; Millennium Bridge, London; Taipei 101, Taipei and Hotel Burj Al Arab, Dubai (Soto and Adeli 2013). A TMD is attached to the main system as a secondary mass with properly tuned spring stiffness and a viscous damper which dissipates the input energy by enhancing the damping of system. Researches have shown that the effectiveness of TMD can be improved by proper tuning of its frequency and damping ratio. Studies are also reported on enhancing TMD’s effectiveness by adding certain additional features in TMD. In the literature two such approaches are seen, one is inclusion of friction in the TMD (Inaudi and Kelley 1995, Gewei and Basu 2010, Pisal and Jangid 2016) and other is inclusion of hysteresis behaviour i.e. plasticity in the TMD (Jagadish et al. 1979, Abe 1994, Jaiswal 2008, Parulekar 2009).

In earthquake resistant design philosophy, it is expected that at design earthquake loads, structure and TMD system will undergo yielding and suffer repeated inelastic deformation that may cause damage and certain level of plastic deformation in the system. Thus, the elastoplastic behaviour of TMD is quite logical for earthquake approach since damage is acceptable in seismic design. In the early works of some of the authors on the plastic behaviour of TMD (Jagadish et al. 1979, Jaiswal 2008), an insight into the dynamic behaviour of structure is provided, but due to the complexity of large number

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of parameters influencing the performance of elastoplastic TMD, comprehensive results are not yet available. This paper sheds new light into the seismic response of elastic main system with elastoplastic TMD through the development of a systematic parametric study aimed at exploiting the influence of optimum parameters governing the performance of elastoplastic TMD. In this context, combinations like elastic system with elastic TMD and elastic system with elastoplastic TMD are studied for a discrete system i.e. two degree of freedom lumped mass system. The response of the system is obtained numerically using Newmark beta method. The effect of TMD yielding is studied on the response of elastic main system for harmonic base excitation and few recorded earthquake ground motions.

2. SYSTEM WITH TMD

A main system (SDOF) with TMD subjected to base excitation \( \ddot{x}_g(t) \) is shown in Figure 1, where, \( m, k, c \) are mass, stiffness, and viscous damping coefficient of the main system and \( m_d, k_d, c_d \) are corresponding parameters of TMD.

![Mathematical model of structure with TMD considered in present study](image)

Figure 1. Mathematical model of structure with TMD considered in present study

The equations of motion for elastic main system with elastic TMD subjected to base excitation is given by:

\[
\begin{bmatrix}
m & 0 \\
0 & m_d
\end{bmatrix}\begin{bmatrix}
\ddot{x} \\
\ddot{x}_d
\end{bmatrix} + \begin{bmatrix}
c + c_d & -c_d \\
-c_d & c_d
\end{bmatrix}\begin{bmatrix}
\dot{x} \\
\dot{x}_d
\end{bmatrix} + \begin{bmatrix}
k + k_d & -k_d \\
-k_d & k_d
\end{bmatrix}\begin{bmatrix}
x \\
x_d
\end{bmatrix} = -\begin{bmatrix}
m \\
m_d
\end{bmatrix}\begin{bmatrix}
\ddot{x}_g \\
\ddot{\dot{x}}_d
\end{bmatrix}
\] (1)

Similarly, the equations of motion of elastic main system with elastoplastic TMD subjected to base excitation is given by:

\[
\begin{bmatrix}
m & 0 \\
0 & m_d
\end{bmatrix}\begin{bmatrix}
\ddot{x} \\
\ddot{x}_d
\end{bmatrix} + \begin{bmatrix}
c + c_d & -c_d \\
-c_d & c_d
\end{bmatrix}\begin{bmatrix}
\dot{x} \\
\dot{x}_d
\end{bmatrix} + \begin{bmatrix}
k & 0 \\
0 & k
\end{bmatrix}\begin{bmatrix}
x \\
x_d
\end{bmatrix} + \begin{bmatrix}
f(x_d, \dot{x}_d) \\
f(x_d, \dot{x}_d)
\end{bmatrix} = -\begin{bmatrix}
m \\
m_d
\end{bmatrix}\begin{bmatrix}
\ddot{x}_g \\
\ddot{x}_d
\end{bmatrix}
\] (2)

where, \( f(x_d, \dot{x}_d) \) is the nonlinear restoring force of damper, whose value depends upon the relative displacement and velocity of TMD in the previous state (Chopra 2004). In the present study, TMD is idealized as a bilinear elastic-perfectly plastic system, characterized by its yield strength level given as:

\[
\text{Yield strength level } f_y^* = \frac{f_y^*}{f_0}
\] (3)
where, $f_0$ is the maximum elastic force required by the system to remain linear and $f_y$ is yield strength of the system. A typical TMD is characterized by its optimum parameters defined in terms of its mass ratio ($\mu$), frequency ratio ($f$) and optimum damping ratio ($\xi$), given as:

$$\mu = \frac{m_d}{m} \quad (4)$$

$$f = \frac{\omega_d}{\omega} \quad (5)$$

$$\xi = \frac{c_d}{2m_\omega \omega_d} \quad (6)$$

where, $\omega$ and $\omega_d$ are the natural frequency of main system and TMD respectively.

For a given elastic system with elastic TMD, many researchers have obtained the optimum parameters. It is noted that the optimum parameters of TMD depend on the nature of loading. For transient loads like earthquake excitations, the optimum TMD damping required is more whereas for steady state harmonic loads, the optimum TMD damping required is comparatively less. In the present work, the optimum parameters of TMD for fixed acceleration harmonic base excitation are taken from Tsai and Lin (1993) and for that of earthquake excitations from Sadek et al. (1997).

3. PARAMETRIC STUDY AND NUMERICAL SOLUTION

The effect of yield strength of elastoplastic TMD is studied on the response of main system subjected to base excitation. Type of base excitation considered are harmonic base acceleration with fixed amplitude. Limited results are also obtained for certain recorded earthquake ground motions. The main system parameters have been selected as representatives of typical mid-rise buildings with time period ($T$) as 1 sec and critical viscous damping ratio ($\eta$) as 2%. The frequency ratio of elastoplastic TMD is considered to be same as that of optimum elastic TMD. For the optimum damping ratio, two cases of TMD damping are considered. In the first case, elastoplastic TMD with damping equal to optimum damping of elastic TMD (TMD-OD) is considered. In the second case, damping ratio of TMD is kept same as that of main system (TMD).

Numerical solution of the equations with elastoplastic TMD is obtained using Newmark beta method, a numerical integration technique. Computer programs in MATLAB are developed to solve the linear (Equation 1) and nonlinear equations (Equation 2) using linear acceleration approach of Newmark beta method with parameters $\alpha = 1/2$ and $\beta = 1/6$.

4. RESULTS OF HARMONIC BASE EXCITATION

A SDOF main system with TMD is subjected to harmonic base excitation $\ddot{x}_g(t) = A \sin \lambda t$, where $A$ is the amplitude of motion and $\lambda$ is the excitation frequency. The maximum amplitude of displacement in the steady state is recorded and using this maximum amplitude, frequency response curve (FRC) is obtained for various excitation frequency. One of the classical way to assess the performance of TMD is to compare the FRC of system without TMD and with TMD. In the present case, the frequency response curve is obtained for excitation frequency ratio $\lambda/\omega = 0.5$ to 1.5. The results are presented in comparative form, wherein the response of main system without TMD, with elastic TMD and with elastoplastic TMD are compared.
4.1 TMD with Optimum Damping (TMD-OD)

Figure 2 provides the FRC of main system \((T = 1 \text{ sec and } \eta = 2\%)\). The elastoplastic TMD considered here has mass ratio as 10\%, and its frequency and damping ratio are taken same as that of optimum elastic TMD. From the FRC, it can be seen that the response of the main system reduces with the addition of elastic TMD-OD near resonance range \((\lambda/\omega = 1.0)\). However, for the low excitation frequency range, i.e. \(\lambda/\omega < 0.85\), the response of main system is higher due to deployment of TMD-OD. As compared to the elastic TMD-OD, elastoplastic TMD-OD is less effective near the resonance range, but it gives more reduction in low excitation frequency range. For the high frequencies i.e. \(\lambda/\omega > 1.2\), there is no appreciable change in the response due to elastic or elastoplastic TMD-OD.

![Figure 2. Effect of yield strength of TMD-OD on FRC of main system](image)

4.2 TMD with Damping Same as Main System (TMD)

Figure 3 provides the FRC of main system with TMD, with the selected values from section 4.1 except that the damping ratio of TMD is kept same as the main system damping i.e. \(\xi = \eta = 2\\%\). The FRC of the main system in the Figure 3 shows asymmetry since TMD has damping same as that of main system. With the deployment of elastoplastic TMD, the response of main system reduces considerably near resonance range, while increases at frequencies away from resonance range. As the yield strength of TMD is reduced, the response of main system also reduces up to a certain yield level for all the excitation frequencies. The maximum response reduction can be seen is achieved at a yield strength level \(f_\gamma = 0.2\) of TMD. As the yield strength of TMD is further reduced, the response of main system starts increasing at a range of excitation frequencies near resonance. This indicates that there may exist an optimum value of \(f_\gamma\).

![Figure 3. Effect of yield strength of TMD on FRC of main system](image)
The above results are shown for a particular mass ratio of TMD, with frequency of elastoplastic TMD kept same as optimum frequency of elastic TMD. In order to search the optimum parameters of elastoplastic TMD, another parametric study is carried out to assess the effect of variation of mass ratio and frequency of TMD on its performance and optimum yield strength level.

4.2.1 TMDs with Different Mass Ratio

TMD with higher mass ratio are more robust for seismic applications, since their performance is not sensitive to variation in earthquake frequency content and design parameters (Hoang et al. 2007, Angelis et al. 2012). In order to achieve higher mass ratio, some researchers have used the existing part of structure as TMD without disturbing the functionality, structural and architectural functions (Jaiswal 2004, Chey et al. 2008, Makino et al. 2008, Johnson et al. 2012). In the present case, a range of mass ratios of TMD from low to heavy is considered (i.e. 1, 5, 10 and 25%), to study its effect on the performance of elastoplastic TMD. Figure 4 shows the FRC of main system with TMD of different mass ratio. From the given figures, it can be observed that as the mass ratio of TMD is increased, the yield strength of TMD at which maximum response reduction can be achieved, reduces. For mass ratios of 1% and 5%, the maximum response reduction of main system is achieved at $f_{y} = 0.8$ and 0.4 respectively (Figure 4a and 4b). For heavy mass ratio TMD i.e. greater than 10% (Figure 4c and 4d), maximum response reduction is seen at $f_{y} = 0.2$ of TMD.

![Figure 4. FRC of main system with TMD of different mass ratio](image-url)
4.2.2 TMDs with Variation in Frequency

The pertinent question considered in this case is, whether elastoplastic TMD also gives maximum reduction at the same frequency as that of elastic optimum TMD. In order to assess this, frequency of TMD is deviated (mistuned) on either side of elastic optimum frequency in the range of ±10% and ±20%. Figure 5 shows the FRC of main system with different frequency (mistune) of TMD. The parameters are used for 10% mass ratio of TMD.

![Figure 5](image_url)

**Figure 5. Effect of mistuning of TMD on FRC of main system**
The maximum response at $\lambda/\omega = 1$ is reduced effectively by the addition of the elastoplastic TMD. It may also be noted that this response reduction is available over a wide range of parameters. From Figure 5, it can be seen that even if the frequency ratio is varied on either side of elastic optimum frequency in the range of $\pm 10\%$ and $\pm 20\%$ and the yield ratio varied from 0.6 to 0.2, there is reduction in the main system response. The region over which reduction is obtained shifts with change in the frequency of TMD. For frequency of TMD less than optimum frequency, response reduces at low excitation frequencies, while for optimum frequency and frequencies greater than optimum, response of main system reduces near resonance range and at higher excitation frequencies.

4.3 Comparison of Elastic TMD-OD and Elastoplastic TMD

Figure 6 shows comparison of FRC of main system with elastic TMD-OD and elastoplastic TMD having damping value same as that of main system at optimum yield level $f_y = 0.2$. The properties of the TMD are taken for a mass ratio of 10% with optimum frequency. From the FRC, it can be seen that appreciable response reduction can be obtained with the addition of elastic TMD-OD as well as elastoplastic TMD, however the region at which response reduction is obtained varies. Elastoplastic TMD gives maximum response reduction compared to elastic TMD-OD near resonance range ($\lambda/\omega = 1.0$), with a slight increase in response around $\lambda/\omega = 1.1$ to 1.3. At low excitation frequencies i.e. $\lambda/\omega < 0.80$, the response reduction obtained by elastic TMD-OD and elastoplastic TMD is same. Thus, one gets a choice to design either an elastic TMD-OD or an elastoplastic TMD with damping ratio same as that of main system. Specific care should be taken for the design of elastic TMD-OD, since at design earthquake load if elastic TMD-OD yields, the response of main system increases, rendering the need of TMD installation as insignificant.

5. RESULTS OF EARTHQUAKE EXCITATION

Three ground motion time histories are used to study the effect of elastoplastic TMD. The details of ground motions are given in Table 1. The effect of yield strength level of TMD on the maximum displacement of the main system is given in Table 2, 3 and 4. The main system ($T = 1$ sec, $\eta = 2\%$) and TMD of different mass ratios (1, 5, 10 and 25%) is considered with optimum frequency ratio and damping ratio same as that of main system. Figure 7 shows the response of main system subjected to Taft earthquake time history. From the inspection of results, it is observed that the maximum displacement of the main system decreases with reduction in yield strength of TMD, up to a certain yield level only. The optimum yield strength level and the extent of response reduction varies for different earthquakes and for different mass ratios of TMD.
Table 1. Details of earthquake ground motions used

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Imperial valley</th>
<th>Kern County</th>
<th>Loma Prieta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recorded Station</td>
<td>El-Centro</td>
<td>Taft Lincoln School Tunnel</td>
<td>Oakland Outer Harbor Wharf</td>
</tr>
<tr>
<td>Year</td>
<td>1940</td>
<td>1952</td>
<td>1989</td>
</tr>
<tr>
<td>Peak Ground Acceleration (PGA)</td>
<td>0.21g</td>
<td>0.18g</td>
<td>0.28g</td>
</tr>
<tr>
<td>Duration</td>
<td>53.46 sec</td>
<td>54.38 sec</td>
<td>40 sec</td>
</tr>
</tbody>
</table>

Table 2. Effect of yield strength of TMD on maximum displacement (in m) of main system subjected to El-Centro earthquake

<table>
<thead>
<tr>
<th>Mass ratio</th>
<th>$\mu = 1%$</th>
<th>$\mu = 5%$</th>
<th>$\mu = 10%$</th>
<th>$\mu = 25%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without TMD</td>
<td>0.0715</td>
<td>0.0715</td>
<td>0.0715</td>
<td>0.0715</td>
</tr>
<tr>
<td>Elastic TMD</td>
<td>0.0761</td>
<td>0.0894</td>
<td>0.1050</td>
<td>0.0718</td>
</tr>
<tr>
<td>TMD $f_\gamma = 0.8$</td>
<td>0.0757</td>
<td>0.0863</td>
<td>0.1026</td>
<td>0.0667</td>
</tr>
<tr>
<td>TMD $f_\gamma = 0.6$</td>
<td>0.0751</td>
<td>0.0843</td>
<td>0.1024</td>
<td>0.0592</td>
</tr>
<tr>
<td>TMD $f_\gamma = 0.4$</td>
<td>0.0724</td>
<td>0.0829</td>
<td>0.0988</td>
<td>0.0571</td>
</tr>
<tr>
<td>TMD $f_\gamma = 0.2$</td>
<td><strong>0.0706</strong></td>
<td>0.0760</td>
<td>0.0828</td>
<td>0.0689</td>
</tr>
</tbody>
</table>

Table 3. Effect of yield strength of TMD on maximum displacement (in m) of main system subjected to Taft earthquake

<table>
<thead>
<tr>
<th>Mass ratio</th>
<th>$\mu = 1%$</th>
<th>$\mu = 5%$</th>
<th>$\mu = 10%$</th>
<th>$\mu = 25%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without TMD</td>
<td>0.0530</td>
<td>0.0530</td>
<td>0.0530</td>
<td>0.0530</td>
</tr>
<tr>
<td>Elastic TMD</td>
<td><strong>0.0424</strong></td>
<td>0.0598</td>
<td>0.0578</td>
<td>0.0619</td>
</tr>
<tr>
<td>TMD $f_\gamma = 0.8$</td>
<td>0.0424</td>
<td>0.0525</td>
<td>0.0559</td>
<td>0.0604</td>
</tr>
<tr>
<td>TMD $f_\gamma = 0.6$</td>
<td>0.0424</td>
<td>0.0461</td>
<td>0.0530</td>
<td>0.0599</td>
</tr>
<tr>
<td>TMD $f_\gamma = 0.4$</td>
<td>0.0424</td>
<td><strong>0.0418</strong></td>
<td>0.0462</td>
<td>0.0553</td>
</tr>
<tr>
<td>TMD $f_\gamma = 0.2$</td>
<td>0.0440</td>
<td>0.0418</td>
<td><strong>0.0413</strong></td>
<td><strong>0.0404</strong></td>
</tr>
</tbody>
</table>

Table 4. Effect of yield strength of TMD on maximum displacement (in m) of main system subjected to Loma Prieta earthquake

<table>
<thead>
<tr>
<th>Mass ratio</th>
<th>$\mu = 1%$</th>
<th>$\mu = 5%$</th>
<th>$\mu = 10%$</th>
<th>$\mu = 25%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without TMD</td>
<td>0.1145</td>
<td>0.1145</td>
<td>0.1145</td>
<td>0.1145</td>
</tr>
<tr>
<td>Elastic TMD</td>
<td><strong>0.1089</strong></td>
<td>0.0970</td>
<td>0.1045</td>
<td>0.1155</td>
</tr>
<tr>
<td>TMD $f_\gamma = 0.8$</td>
<td>0.1093</td>
<td><strong>0.0980</strong></td>
<td>0.1002</td>
<td>0.1133</td>
</tr>
<tr>
<td>TMD $f_\gamma = 0.6$</td>
<td>0.1093</td>
<td>0.0980</td>
<td><strong>0.0948</strong></td>
<td>0.1113</td>
</tr>
<tr>
<td>TMD $f_\gamma = 0.4$</td>
<td>0.1093</td>
<td>0.0980</td>
<td>0.0948</td>
<td><strong>0.0999</strong></td>
</tr>
<tr>
<td>TMD $f_\gamma = 0.2$</td>
<td>0.1093</td>
<td>0.1003</td>
<td>0.0948</td>
<td>0.0999</td>
</tr>
</tbody>
</table>
6. CONCLUSIONS

Effectiveness of TMD is due to its ability to absorb energy. Linear TMD relies on viscous damper for energy absorption. Classically, it is known that for elastic TMD there is an optimum damping at which energy absorption is maximum. In the past, efforts have been made to supplement energy absorption either through hysteresis or by adding friction. In the present study, hysteresis behaviour is added to TMD by making the TMD elastoplastic. Similar to optimum damping of linear TMD, it is noted that there is an optimum yield level of TMD at which its performance is best. Most interestingly, it is noted that if viscous damping itself is of linear optimum level, then addition of hysteresis does not lead to any efficiency in TMD’s energy absorption. In cases where TMD damping is same as that of main system, addition of hysteresis is quite effective. In such cases, an optimum yield strength level of TMD exists. Results of this study indicate that higher damping can be achieved through elastoplastic design of TMD. Such elastoplastic TMD is as effective as elastic TMD with higher damping (Figure 6). If hysteresis is included, then, the design of such TMD is more realistic and economical, since the cost of adding additional damping devices is not required.

7. REFERENCES


