BAYESIAN ESTIMATION OF NONLINEAR SOIL MODEL PARAMETERS: THEORY AND FIELD-SCALE VALIDATION AT KIK-NET DOWNHOLE ARRAY SITES

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ABSTRACT

We present an inversion algorithm that can recover constitutive model parameters of dynamic nonlinear soil behavior and their uncertainties from downhole array recordings. The algorithm borrows elements from the structural health monitoring literature, and in particular the nonlinear Bayesian filtering technique. Quantifying the model parameters and their uncertainties is important because results are not ground motion dependent – as would be the case for an inversion algorithm based on an equivalent linear forward site response model, for example – and can be used for predictive analyses of site response to future events. In the following sections, we first present an inversion framework that we developed using constrained unscented Kalman filtering; and verify it using canonical problems of wave propagation through inhomogeneous and inelastic layered soil overlaying rigid bedrock. We then employ the inversion technique to estimate the statistical distribution of the parameters for a multiaxial plasticity model using measured data from downhole arrays in Japan. We lastly show that by using the estimated parameters and a subset of recorded ground motions not used in the inversion, the nonlinear model can successfully predict the soil response for a wide amplitude and frequency range of input excitations. While the presented technique is not model dependent, the inversion results are clearly a function of the constitutive model used to formulate the forward problem. In turn, successful implementation of the technique and its results for future predictions is conditioned on selecting a reliable constitutive law that can capture the site conditions of interest; and an adequate number of ground motion records that can probe the shear strain range of interest.

Keywords: Site response, inversion, bounding surface plasticity, parametric uncertainty, Bayesian filtering

1. INTRODUCTION

One-dimensional (1D) site response inversion has been widely studied during the past. The majority of these studies (e.g., Assimaki et al. 2006), however, were focused on the identification of velocity profiles, material damping, scattering attenuation parameters and layer thickness, and have been used to inform problems of linear viscoelastic site response analyses. When the recorded ground motion is strong enough to trigger nonlinear effects, linear analyses cannot capture the associated strain-dependent soil stiffness (e.g., Kaklamanos et al. 2013; Kaklamanos et al. 2015; Shi and Asimaki 2017). In this case, equivalent linear or nonlinear analyses need to be employed, the majority of which are formulated or calibrated using the so-called normalized modulus reduction curve (that is, the normalized derivative of the stress-strain monotonic curve) as input parameters. Measured site-specific modulus reduction curves are rarely available, however, especially for field scale problems, and engineers by and large resort to empirical equations available in literature (e.g., Darendeli 2001; Zhang et al. 2005). Recent studies (e.g., Kaklamanos et al. 2015) have quantified the performance of equivalent linear and nonlinear models for the assumed modulus reduction curve and have found that their accuracy strongly depends on the selection of the appropriate curves. A few recent studies have focused on inference of modulus reduction and damping curves using downhole array data. Yang et al. (2017) used data from 165 earthquake events at 18 sites of the Kiban-
Kyoshin network (KiK-net) downhole array in Japan to derive empirical curves for shear modulus reduction curves under different confining pressures by comparing the response amplification spectra subjected to strong motions with those subjected to weak motions. Mercado et al. (2017), on the other hand, used a Bayesian inference framework to estimate the optimal set of parameters, including shear wave velocity profiles, damping, and nonlinear soil properties, that produces a close fit to the actual response spectra from a downhole acceleration array. In this study, we describe a technique based on a sequential Bayesian learning approach that can recover both linear and nonlinear parameters of the constitutive soil behavior and their uncertainties using downhole array data. Instead of frequency domain measures used in the two studies above, we use the acceleration time series at the surface and at depth for the estimation task. In the following sections, we first briefly present the technique, followed by two numerical examples that show the approach can successfully capture the physics of the problem in absence of measurement noises and uncertainties. Then, we apply the framework to a set of KiK-net downhole array recordings that have been previously shown to satisfy site and source conditions of 1D wave propagation, to (1) estimate a more refined shear wave velocity and damping profiles at the site, and (2) infer the normalized modulus reduction curve using a total stress multiaxial cyclic plasticity model as a proxy to approximate nonlinear soil behavior at the site. For the dataset used here, we show that training the constitutive model using one strong motion event can yield estimated parameters that can be used to successfully predict the surface response to other strong motion events.

2. PROBLEM DEFINITION

Consider a downhole array with two strong motion instruments recording ground motions at the surface and at depth, and assume that a coarse profile, as well as a set of weak and relatively strong motion recordings are available. Moreover, assume that 1D wave propagation assumptions (i.e., vertical incidence and soil layers with horizontal stratifications) are reasonable approximations of site response at the site. The main objective is to use a probabilistic framework for assimilating data and therefore training a relatively simple soil model to infer a modulus reduction curves. To this end, we consider two phases of (1) using the weak motions with PGA<0.1g for the estimation of the shear wave velocity profile and damping, and (2) using the stronger motions with PGA>0.1g for the estimation of the nonlinear soil model parameters. For modeling nonlinear soil behavior, we use a simple but rigorous total stress plasticity model, which is first introduced by Borja and Amies (1994) and has been extensively tested since after and has consistently shown a good performance (e.g., Borja et al. 1999b; Borja et al. 1999a; Rodriguez-Marek 2000; W. Zhang et al. 2017). This model is based on the J2-bounding surface plasticity theory with a vanishing elastic region and implements a smooth mapping rule between the stress state and its image point on the bounding surface to define the hardening modulus within the bounding surface. For the simple shear test, it can be shown (Borja and Amies 1994) that the relationship between the normalized shear modulus $G/G_{\text{max}}$ and maximum shear strain $\gamma$ can be approximated as

$$\frac{G}{G_{\text{max}}} = 1 - \frac{3}{2y} \tau_{f}^{2r} \left[ \frac{\tau_{f} - \gamma}{\xi} \right]^{m} + H_{0} \right]^{-1} d\xi$$

where $h$ and $m$ are the hardening parameters; $\tau_{f}$ is the simple shear test soil strength, which can be used to set the radius of the bounding surface (i.e., $R = \sqrt{2} \tau_{f}$) and $H_{0}$ is the kinematic hardening parameter of the bounding surface.

While the modulus reduction curve is not only a function of maximum shear strain but also confining pressure, plasticity index, over consolidation ratio, etc., we implicitly account here only for the effects of confining pressure to reduce the number of free parameters, while assuming that the soil strength (or equivalently, the radius of the bounding surface) varies linearly with depth as in:

$$R(z) = \left( a_{R0} + a_{R1} \frac{z}{H} \right) G_{\text{max}}(z)$$

(2)
where $H$ is the total height of the soil deposit between the two sensors in the borehole and at the surface and $G_{\text{max}}(z)$ is the maximum shear modulus at depth $z$. To test the validity of this assumption, we computed the normalized shear modulus reduction curve in a soil deposit with height $H = 200$ m and density of 2000 kg/m$^3$ using the empirical equation 
$$
\frac{G}{G_{\text{max}}} = \frac{1}{1 + \left(\frac{\gamma_r}{\gamma_r^1}\right)^k} \tag{1}
$$
(Zhang et al. 2005). In this equation, $\gamma_r = \gamma_{r1} \left(\frac{\sigma_m}{P_a}\right)^k$; $\gamma_{r1}$ is the reference strain at a mean effective confining pressure $\sigma_m = P_a = 100$ kPa; we set $\gamma_{r1} = 0.03\%$, $\alpha = 1$, and $k = 0.3$. Then, we used Eq. (1) to fit the free parameters of the constitutive model. Figure 1 shows the fitted $G/G_{\text{max}}$ curves at different depths as well as the variation of each parameter with depth. As can be seen for this specific example, linear variation of the bounding surface radius with depth, and constant hardening parameters ($h$) and exponent ($m$) could approximately capture the effects of confining pressure on normalized modulus reduction curves. It should be noted that we will explore the effects of relaxing this assumption in our future studies.

![Figure 1: An illustrative example of soil model parameters’ variation with depth.](image)

3. METHODOLOGY

3.1 One Dimensional Nonlinear Site Response Analysis

For the 1D nonlinear site response analyses presented below, we use the Open System for Earthquake Simulation (OpenSEES) software (Mazzoni et al. 2006) and the implementation of the Borja and Amies (1994) model by Wang and Sitar (2011). For the forward model, we use plane strain quadrilateral elements and slave the degrees of freedom at each depth to move equilaterally in both vertical and horizontal directions. We further assume that the base of the soil column is fixed, which allows us to prescribe the borehole recorded motion as a total motion at the base. To model small-strain damping, we use the stiffness proportional Rayleigh damping matrix, i.e., $C = a_k K$, where $K$ is the elastic stiffness matrix of the soil column. For nonlinear analysis, we define the soil model parameters at each soil layer with shear wave velocity $V_{sl}$ as $h_i = a_k G_{\text{max},i}$, $m$ and $R_i = \left(a_{R0} + a_{R1} \frac{z}{H}\right) G_{\text{max},i}$ where $G_{\text{max},i} = \rho V_{sl}^2$ and it is assumed that for all layers mass density $\rho$ is constant and equal to 2000 kg/m$^3$.

3.2 Bayesian Filtering for Estimation of Soil Model Parameters from Downhole Array Data

We define the predicted soil acceleration response at the $k$-th time window $[t_k^1, t_k^2]$ at the soil surface as $y_k = T \left[\theta_k, u_{g,t_k^1}, t_k^2\right]$ where $T$ is a nonlinear functional mapping soil column properties $\theta_k \in \mathbb{R}^n$ and input acceleration $u_{g,t_k^1,t_k^2}$ to the soil surface response vector $y_k \in \mathbb{R}^n$. Considering the actual measured surface acceleration in the same time window as $y_k^1 \in \mathbb{R}^n$, we want to find a set of soil properties that can approximate the actual soil response. In this study, we assume that the soil properties are time invariant and therefore their evolution during the estimation process can be modeled as a random walk process using a time invariant zero mean Gaussian noise $w \in \mathbb{R}^n$ with a known covariance matrix $Q \in \mathbb{R}^{n \times n}$. On the other hand, we assume that the error due to the misfit between the measured
and predicted soil responses can be represented as a time invariant zero mean Gaussian noise $v \in \mathbb{R}^{n_y}$ with a known covariance matrix $R \in \mathbb{R}^{n_x \times n_y}$. As such,

$$\theta_{k+1} = \theta_k + w, \quad y_k^\Gamma = y_k + v .$$

(3)

On the other hand, in order to avoid non-physical estimations, we consider enforcing $n$ interval constraints such that

$$A_i \leq \theta_i \leq B_i \quad \text{for} \quad i = 1, \ldots, n .$$

(4)

To find the most probable solution, i.e., $\tilde{\theta}^{\text{est}}$ and its covariance matrix $P^{\text{est}}$, we use the Bayesian inference technique based on the constrained unscented Kalman filtering (CUKF). UKF is a prediction-correction approach in which unscented transformation is used for propagation of uncertainties from the model parameters $\theta_k$ into the response function $y_k$. We provide details of the deterministic sampling and enforcing interval constraints in both prediction and correction steps in the next two sections; the estimation process is summarized in Figure 2.

### 3.3 Symmetric Interval Constrained Unscented Transformation

At the $k$-th iteration, i.e., the $k$-th estimation time window, we define the prior estimates of the mean vector and covariance matrix as $\bar{\theta}_k^-$ and $P_k^-$. In the prediction step, for the unconstrained random variables with the Gaussian probability density function (PDF), unscented transformation can be used to deterministically sample the PDF. For the constrained random variables, the enforced constraints must be satisfied for each sampling point, a.k.a. sigma point. Therefore, in this study, we use the symmetric interval constrained unscented transformation (Mandela et al. 2012) with sigma points $\bar{\theta}_k^{(i)}$ and weights $W^{(i)}$ defined as follows.

$$\bar{\theta}_k^{(i)} = \begin{cases} \bar{\theta}_k^-, & i = 0 \\ \bar{\theta}_k^- + \alpha_k s_i, & i = 1, \ldots, 2n \end{cases}, \quad W^{(i)} = \begin{cases} 1 - \frac{n}{n+\kappa} \times \frac{1}{\alpha_k^2}, & i = 0 \\ \frac{1}{2(n+\kappa)} \times \frac{1}{\alpha_k^2}, & i = 1, \ldots, 2n \end{cases}$$

(5)

where $s_i$ is the $i$-th column of the matrix $S = \{s_{ij}\} = \{\sqrt{P_k^-} - /\sqrt{P_k^-}\} \in \mathbb{R}^{n \times 2n}$ and $\alpha_k = \min(X)$; each entry of the matrix $X = \{x_{ij}\}$ is determined as follows:

$$x_{ij} = \begin{cases} \sqrt{n + \kappa}, & s_{ij} = 0 \\ \min \left( \sqrt{n + \kappa} - \frac{A_{ij} - \bar{\theta}_{ik}^-}{s_{ij}}, \frac{B_{ij} - \bar{\theta}_{ik}^-}{s_{ij}} \right), & s_{ij} > 0 \\ \min \left( \sqrt{n + \kappa} - \frac{B_{ij} - \bar{\theta}_{ik}^-}{s_{ij}}, \frac{A_{ij} - \bar{\theta}_{ik}^-}{s_{ij}} \right), & s_{ij} < 0 \end{cases}$$

(6)

It should be noted that $\alpha_k = \sqrt{n + \kappa}$ for the plain (unconstrained) UT; $\kappa$ is a secondary scaling parameter and it is set to zero in this study.

### 3.4 PDF Truncation

In the correction step, it is possible that the posterior (corrected) estimate of the mean vector, i.e., $\bar{\theta}_k^+$, violates the enforced physical constraints. Therefore, it is necessary to adjust $\bar{\theta}_k^+$ and the covariance matrix $P_k^+$ before sliding the estimation window for the next prediction step. In this study, we employ the PDF truncation method – c.f. Simon (2006) for more details of the method – where we truncate the posterior Gaussian PDF at the $n$ enforced physical constraints and compute the mean and covariance of the truncated PDF, i.e., $\tilde{\theta}_k$ and $\tilde{P}_k$. It should be noted that we enforce this truncation sequentially, and since the algorithm is nonlinear, the constrained estimation depends on the order in which the constraints
are applied, unless the constraints are decoupled (Simon and Simon 2010) as is in this study.

<table>
<thead>
<tr>
<th>Initialization ($k = 0$)</th>
<th>( \hat{\boldsymbol{\theta}}_0 = E(\boldsymbol{\theta}_0) )</th>
<th>Initial estimate of the mean vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \boldsymbol{P}_0 = E\left[ (\boldsymbol{\theta}_0 - \hat{\boldsymbol{\theta}}_0) (\boldsymbol{\theta}_0 - \hat{\boldsymbol{\theta}}_0)^T \right] )</td>
<td>( \hat{\boldsymbol{\theta}}^+ )</td>
<td>Initial estimate of the covariance matrix</td>
</tr>
<tr>
<td>( \boldsymbol{Q} = E\left[ \mathbf{w}_k \mathbf{w}_k^T \right] )</td>
<td>( \hat{\boldsymbol{\theta}}^+ )</td>
<td>Estimate of process noise covariance matrix</td>
</tr>
<tr>
<td>( \boldsymbol{R} = E\left[ \mathbf{v}_k \mathbf{v}_k^T \right] )</td>
<td>( \hat{\boldsymbol{\theta}}^+ )</td>
<td>Estimate of measurement noise covariance matrix</td>
</tr>
</tbody>
</table>

For \( k = 1, \ldots, n_k \) do

**Prediction**

\[ \hat{\boldsymbol{\theta}}_k^+ = \hat{\boldsymbol{\theta}}_{k-1} + \hat{\boldsymbol{\theta}}_k \]
\[ \hat{\boldsymbol{P}}_k^+ = \hat{\boldsymbol{P}}_{k-1} + \hat{\boldsymbol{\theta}}_k \]

Generate \( \hat{\boldsymbol{\theta}}_k^{(i)} \) and \( \mathbf{W}_k^{(i)} \) for \( i = 0, \ldots, 2n \)
\[ \hat{\mathbf{y}}_k^{(i)} = \mathbf{T} \left( \hat{\boldsymbol{\theta}}_k^{(i)}, \mathbf{u}_k^{(i)} \right) \]
\[ \hat{\mathbf{y}}_k = \sum_{i=0}^{2n} \mathbf{W}_k^{(i)} \hat{\mathbf{y}}_k^{(i)} \]
\[ \hat{\mathbf{y}}_k^+ = \sum_{i=0}^{2n} \mathbf{W}_k^{(i)} \left( \hat{\mathbf{y}}_k^{(i)} - \hat{\mathbf{y}}_k \right) \left( \hat{\mathbf{y}}_k^{(i)} - \hat{\mathbf{y}}_k \right)^T + \hat{\boldsymbol{\theta}}_k^+ \]

**Correction**

\[ \mathbf{K}_k = \hat{\boldsymbol{P}}_k^+ \left( \hat{\boldsymbol{P}}_k^+ \right)^{-1} \]
\[ \hat{\boldsymbol{\theta}}_k^+ = \hat{\boldsymbol{\theta}}_k - \mathbf{K}_k (\hat{\mathbf{y}}_k - \hat{\mathbf{y}}_k) \]
\[ \hat{\boldsymbol{P}}_{k+}^+ = \hat{\boldsymbol{P}}_k^+ - \mathbf{K}_k \hat{\mathbf{P}}_k^+ \mathbf{K}_k \]

End for

\( \hat{\boldsymbol{\theta}}_{est} = \hat{\boldsymbol{\theta}}_{n_k}^+ \quad \hat{\boldsymbol{P}}_{est}^+ = \hat{\boldsymbol{P}}_{n_k}^+ \) Final estimate of the mean and covariance

Figure 2: The constrained unscented Kalman filtering algorithm used for estimation process.

4. VERIFICATION STUDY

In order to eliminate the modeling assumption effects on the performance of the implemented algorithm, we first study two pseudo problems using the layering profile of one of the KiK-net sites KSRH10 (see Table 1).

4.1 Low strain shear wave velocity and damping

Assuming that the true shear wave velocity profile is the same as those available from the KiK-net database, we consider eight layers for the site KSRH10 and assume that the coefficient of the stiffness proportional damping matrix is \( a_1 = 0.002 \). As such, we define the true parameter vector as \( \boldsymbol{\theta}_{\text{true}} = [V_{S1}, V_{S2}, \ldots, V_{S8}, a_1] = [90, 130, 210, 300, 1400, 2000, 1500, 1700, 0.002] \) where \( V_{S_i} \) is the shear wave velocity at layer \( i \) in m/s. Then, we initialize the estimation process by setting \( \hat{\boldsymbol{\theta}}_0 = 1.3 \boldsymbol{\theta}_{\text{true}}, \hat{\boldsymbol{\theta}}_0^+ = \text{diag}\left[\left(0.15 \hat{\theta}_0^+\right)^2\right], \boldsymbol{Q} = \text{diag}\left(10^{-4} \hat{\theta}_0^+\right), \) and \( \boldsymbol{R} = (0.0025g)^2 \) where \( \text{diag}(\mathbf{x}) \) returns a diagonal matrix with the elements of the vector \( \mathbf{x} \) on the main diagonal and \( \mathbf{I} \) is the identity matrix.

Figure 3 shows the evolution of the mean ± one standard deviation (±SD) of the estimated parameters by iteration \( k \), and

Figure 4 shows the final estimates for the shear wave velocity profile along with the true profile and the one chosen a priori for initialization.

Figure 5 shows the response time histories at the ground surface as well as the site transfer functions (i.e., the spectral ratio computed by dividing the Fourier transform of the surface motion to the one in the borehole) when we use the final and a priori estimates for site response analysis. As shown, all parameters are closely converged to their true values and both true ground surface response and site
transfer function are captured well using the final estimates.

![Figure 3: Evolution of parameters with iteration for estimation of shear wave velocity profile.](image)

**Figure 3**: Evolution of parameters with iteration for estimation of shear wave velocity profile.

![Figure 4: Final estimate for the shear wave velocity profile compared against the true profile and a priori profile at iteration \(k = 0\).](image)

**Figure 4**: Final estimate for the shear wave velocity profile compared against the true profile and a priori profile at iteration \(k = 0\).

![Figure 5: Ground surface acceleration time history and site transfer function obtained from using the final and a priori estimates compared to the true response.](image)

**Figure 5**: Ground surface acceleration time history and site transfer function obtained from using the final and a priori estimates compared to the true response.

### 4.2 Free parameters of the soil model

In order to verify the method for estimation of the free parameters of the constitutive model used in this study, we consider the same site KSRH10 excited by the input that has peak acceleration of about 0.5g for which nonlinearity is expected to be triggered. We consider \(\theta_{\text{true}} = [a_h, m, a_{R0}, a_{R1}] = [0.075, 1.25, 1.25, 1.25]\) and after performing the forward simulation to obtain the true soil surface
response $y_t$, we pollute the signal by adding a Gaussian white noise with the root-mean-square amplitude of $0.0025g$. Then, we start the estimation process by setting $\hat{\theta}_0 = 1.5\theta_{\text{true}}$, $P_0 = \text{diag}\left\{(0.5\theta_0)^2\right\}$, $Q = \text{diag}\left\{(10^{-4}\theta_0)^2\right\}$, and $R = (0.005g)^2I$. It should be noted that the shear wave velocity profile and the small strain damping are assumed to be known and we set their values to those used in Section 4.1. Again, as shown in Figure 6, all parameters are converged to their true values, which shows the robustness of the implemented algorithm. Accordingly, Figure 7 shows the ground surface responses as well as the strain-stress curves at depth $z = 0.5$ m using the true, final, and initial (prior) estimates of the parameters.

![Figure 6](image6.png)

**Figure 6:** Evolution of parameters with iteration for estimation of nonlinear soil model parameters.

![Figure 7](image7.png)

**Figure 7:** Ground surface acceleration time history and strain-stress curve at depth $z = 0.5$ m from using the final and a priori estimates compared to their true counterparts.

### 5. APPLICATION TO KIK-NET DOWNHOLE ARRAY

The KiK-net strong motion seismograph network in Japan consists of about 670 stations with a ground surface and downhole array instrument pair (Aoi et al. 2004). To evaluate the performance of the framework, we use six stations for which, according to the taxonomy proposed by Thompson et al. (2012), 1D wave propagation is a valid assumption for site response analyses. A subset of motions in these sites with PGA>0.01g was recently studied by Shi and Asimaki (2017), who employed the following processing steps: for each record, the east-west and north-south components were first rotated into SH and SV components according to the azimuth between the station and the epicenter, and successively, the SH borehole and ground surface components were used as input motion and benchmark response, respectively. We use the same dataset in this study.

#### 5.1 Inversion for the shear wave velocity and damping

Although the shear wave velocity profiles are available at the KiK-net stations from suspension logging and downhole tests, previous studies showed that these profiles are sometimes too coarse to capture the

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5
effects of site amplification on the higher frequency components of ground shaking, even for very weak ground motions (Shi and Asimaki 2017). To improve this, Shi and Asimaki (2017) employed a deterministic waveform inversion technique proposed by Assimaki et al. (2006) for nine KiK-net stations using the $V_s$ profiles provided by KiK-net as initial trial profiles and five weak ground motions across which the $V_s$ results were averaged.

In this study, we use the described probabilistic framework to determine the shear wave velocity profile as well as the Rayleigh damping ratio for a subset of those KiK-net stations, as summarized in Table 1, using a series of weak motions at each site. The estimated shear wave velocity profiles at each site along with the mean of the final estimates (estimated (mean) profile) and the initial guess (a priori estimate at iteration $k = 0$) are shown in Figure 8 and the corresponding Rayleigh damping ratios at frequency $f = 1$ Hz are tabulated in the last column of Table 1. Figure 9 shows the comparison of the site transfer function obtained from using the estimated (mean) profile and from averaging the empirical transfer functions, which are smoothed using a Konno–Ohmachi smoothing window (Konno and Ohmachi 1998). As shown, the estimated (mean) transfer functions are able to capture the mean empirical transfer functions reasonably at all considered sites except site FKSH11. We expect that the performance of the algorithm will improve at higher frequencies if we employ a higher order Rayleigh damping model (or any nearly frequency-independent formulation) for the estimation of the small-strain damping. This refinement, that requires a larger number of parameters to be estimated, has been reserved for future extensions of the algorithm presented here.

5.2 Inversion for shear modulus reduction curve

To estimate the modulus reduction curves, we next use the verified Bayesian framework at three sites FKSH14, IWTH08 and TKCH08. Error! Reference source not found. summarizes the events that we use for estimation of the nonlinear soil model parameters. For each station, we (arbitrarily) chose the motion with the highest east-west PGA in the dataset used by Shi and Asimaki (2017) to ensure that a wide range of strain-dependent properties would be probed. In all cases, we use the estimated low-strain properties from the previous section for defining the shear wave velocity profile and damping. Figure 10 shows the resulting modulus reduction curves obtained from using final estimates of the soil model parameters in computing $G/G_{\text{max}}$ from Eq. (1). It is worth mentioning that, although the inferred $G/G_{\text{max}}$ curves are shifted to the right with depth for stations FKSH14 and TKCH08, they show negligible shifts for the station IWTH08. The ground surface responses using the final estimates are also plotted in Figure 11 to be compared against the actual recordings. Moreover, in order to see the effects of using the nonlinear soil model, for each event, we computed the empirical, linear and nonlinear transfer functions, which are shown in Figure 12. These comparisons show the good performance of the trained model despite existing noise and modeling uncertainties.

Finally, to test the predictive capability of the trained constitutive models for each site, we choose a set of motions that are not used in the estimation process and perform nonlinear analyses to obtain ground surface responses. Figure 13 shows three examples of the predicted acceleration responses compared against the measured ones. Although the results of other events are not provided, it should be noted that in all case a good performance of the trained model in predicting the surface response is observed.

<table>
<thead>
<tr>
<th>Site Code</th>
<th>Site Name</th>
<th>Depth (m)</th>
<th>Number of Weak Motions</th>
<th>PGA (g)</th>
<th>$M_n$</th>
<th>$\xi^\text{est}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FKSH11</td>
<td>YABUKI</td>
<td>115</td>
<td>38</td>
<td>0.02-0.1</td>
<td>≤ 5</td>
<td>0.55</td>
</tr>
<tr>
<td>FKSH14</td>
<td>IWAKI-E</td>
<td>147</td>
<td>37</td>
<td>0.02-0.1</td>
<td>≤ 5</td>
<td>0.48</td>
</tr>
<tr>
<td>KSZH10</td>
<td>HAMANAKA</td>
<td>255</td>
<td>23</td>
<td>0.02-0.1</td>
<td>≤ 5</td>
<td>0.49</td>
</tr>
<tr>
<td>IWTH08</td>
<td>KUJI-N</td>
<td>100</td>
<td>10</td>
<td>0.05-0.1</td>
<td>≤ 5</td>
<td>0.30</td>
</tr>
<tr>
<td>IWTH27</td>
<td>RIKUZENTAKATA</td>
<td>100</td>
<td>42</td>
<td>0.02-0.1</td>
<td>≤ 5</td>
<td>1.01</td>
</tr>
<tr>
<td>TKCH08</td>
<td>TAIKI</td>
<td>100</td>
<td>32</td>
<td>0.015-0.03</td>
<td>≤ 5</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 1: Summary of the KiK-net sites and motions that are used for estimation of $V_s$ and damping.
Figure 8: Estimated shear wave velocity profiles at six KiK-net sites used in this study.

Figure 9: Estimated (mean) site transfer functions compared to empirical ones calculated from the considered weak motions for Bayesian inference in this study.

Figure 10: Modulus reduction curves at different depths obtained from final estimates of soil model parameters.
Table 2: Summary of the KiK-net events that are used for estimation of soil model parameters.

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>PGA-EW (g)</th>
<th>M&lt;sub&gt;W&lt;/sub&gt;</th>
<th>Depth (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FKSH141103111446</td>
<td>03/11/2011</td>
<td>0.40</td>
<td>9.0</td>
<td>24</td>
</tr>
<tr>
<td>IWTH080807240026</td>
<td>07/24/2008</td>
<td>0.39</td>
<td>6.8</td>
<td>108</td>
</tr>
<tr>
<td>TKCH080309260450</td>
<td>09/26/2003</td>
<td>0.51</td>
<td>8.0</td>
<td>42</td>
</tr>
</tbody>
</table>

Figure 11: Comparison of the ground surface acceleration time histories obtained from nonlinear analysis using the trained model against their true counterparts.

Figure 12: Comparison of the site transfer functions obtained empirically, from nonlinear analysis using the trained model, and from linear analysis.

Figure 13: Examples of using the trained model to predict ground surface acceleration response in events that are not used for estimation.

6. CONCLUSIONS

In this study, we provided details of a nonlinear Bayesian technique, which is based on the unscented Kalman filtering, for nonlinear site characterization using downhole array data. We considered two steps of first using the weak motions recorded at the site for recovering the low-strain shear wave velocity profile and damping, and then using the strong motions for recovering the free parameters of the constitutive model that we used to approximate the nonlinear soil behavior at the site. By solving two pseudo examples, we showed that the implemented algorithm is successful in learning dynamic soil properties in both linear and nonlinear regimes. Then, we tested the algorithm using real data from the KiK-net downhole arrays in Japan and showed the good performance of the recovered constitutive model in predicting the soil surface responses for the events that were not used for training the model.
7. REFERENCES


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