IN-SITU MEASURED AND NUMERICALLY PREDICTED DYNAMIC AND SEISMIC RESPONSE OF STONE MASONRY BRIDGES

George C. MANOS¹, Evaggelos KOZIKOPOULOS², Lambros KOTOULAS³, Nick SIMOS⁴

ABSTRACT

Stone masonry bridges are studied as part of the built cultural heritage, utilizing in-situ measurements in order to identify their dynamic characteristics in terms of eigen-frequencies, eigen-modes and damping properties. Such information is a valuable basis for building realistic numerical simulations of the structural behaviour of such bridges as well as for their structural health monitoring. Such simplified numerical analyses yielded numerical predictions of bridge deformations and stresses that are useful in understanding the structural behaviour and the structural damage potential for such masonry structures. The structural performance of Plaka bridge that collapsed due to flooding in February 2015 is examined when subjected to the gravitational forces combined with either simplified flooding loads or a design earthquake. Moreover, the structural damage that this bridge sustained during an explosion in 1944, was also examined combined with the flooding loads. It is shown that the flexibility of the foundation, amplified by long or short term erosion of the bridge-footings, results in a more detrimental response for the Plaka bridge, in terms of tensile stress fields, than the applied simplified flooding loads. The obtained numerical predictions demonstrate that the structural damaged sustained by this bridge during the 1944 explosion leads to a dominant tensile stress field at the regions of the main and secondary arches on top of the mid-pier. These predicted peak tensile stress values are well beyond the stone masonry strength. Consequently, they could well have contributed towards the 2nd of February 2015 collapse.

Keywords: Stone masonry bridges; Dynamic Measurements; Numerical Simulation; Foundation Deformability; Flooding; Damage from Explosion

1. INTRODUCTION

In what follows selective results are presented from an extensive study, which focused on old stone masonry bridges that are located mainly in the prefectures of Western Macedonia and Ipiros in Greece (Manos et al. 2016). These bridges are examples of outstanding stone-masonry construction that was dominant for a long period in these parts of Greece. The old stone masonry bridges that survive today have been built during the 18th and 19th century. They were used to connect villages located in rough mountainous terrain bridging currents that could be quite turbulent during part of the year. This type of transporting people, animals and goods was accomplished using a relatively narrow deck with width varying from 2.0m to 3.0m. On the contrary, their size spans from 8m to 40m when a single arch is employed (Konitsa) or over 70m for multi-arch structures. More information on the geometry, construction characteristics and mechanical properties of the employed materials are given by Manos et al. (2016). Today, these structures have retained but only a small part of this primary function, as new roads and bridges have been built to facilitate the contemporary transportation needs. Despite this

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fact they have recently attracted considerable attention as cultural heritage structures together with an effort to become parts of a network of mountain trails. Because a number of people use these structures as visitors a conservation effort was also initiated for their maintenance. Our study has also partly such an objective. That is to obtain information through in-situ instrumentation and measurements on the vibration characteristics of these structures at a given time in a health monitoring framework, as described in section 2. In addition, these in-situ measurements are complemented with a numerical investigation that has as first target to form realistic numerical simulations of such old stone masonry structures. Moreover, to employ next such realistic numerical simulations in order to assess the behaviour of these old stone masonry bridges to a combination of loads that include accidental actions, such as forces generated from earthquakes or flooding.

2. IN-SITU MEASUREMENTS OF THE DYNAMIC CHARACTERISTICS OF THE STUDIED STONE BRIDGES.

In measuring the dynamic response of all the studied stone bridges two types of excitation were mobilized. The first mobilized the wind (figure 1a), despite the variation of the wind velocity in amplitude and orientation during the various tests. The second type of excitation that was employed,
namely vertical in-plane excitation, was produced from a sudden drop of a weight on the deck (figure 1b) of the stone masonry bridge. Measurements from either wind or drop weight excitation were repeated many times. Moreover, the velocity response was measured at by placing the sensors at three locations, as shown in figures 1a and 1b (left sensor, mid sensor, right sensor). Due to the topography of the areas where these stone bridge are located, usually a relatively narrow gorge, the orientation of the wind resulted in a considerable component perpendicular to the longitudinal bridge axis. This fact combined with the resistance offered to this wind component by the façade of each bridge produced sufficient excitation source resulting in small amplitude vibrations that could be recorded by the employed instrumentation. Similarly, the drop weight excitation resulted in velocity response in all three directions which could be recorded by the employed SysCom tri-axial velocity sensors. They had a sensitivity of 0.001mm/sec and were combined with a SysCom data acquisition system with a sampling frequency of 400Hz. All the obtained data were subsequently studied in the frequency domain through available FFT software (Manos et al. 2016, 2015 and 2017). This wind orientation relative to the geometry of each bridge structure coupled with the bridge stiffness properties could excite mainly the 1st symmetric out-of-plane eigen-modes. The variability of the wind orientation could also excite, although to a lesser extent, some of the other in-plane and out-of-plane eigen-modes.

The drop weight excitation produced vibrations of amplitude larger than those of the wind excitation. Moreover, the variation of the location of the drop weight excitation was used to excite in-plane as well as out-of-plane eigen-modes. This process will be presented for the old masonry Konitsa bridge depicted in figures 2a and 2b. This stone masonry bridge has a main arch with span and height quite similar to the Plaka stone masonry bridge depicted in figures 3a and 3b. One of the main differences between the Konitsa and the Plaka bridge is that the main arch of the former is supported through its abutments to the rocky slopes of the river shore whereas the later is supported by secondary arches that span the distance to the rocky shores (Figures 3a and 3b). It is also important to note that the right secondary arch has a pier that is founded within the river bed.

![Figure 4. Vibration measurements from drop weight excitation at the crown of the bridge recorded by the tri-axial velocity sensor located at the middle of the Konitsa Bridge.](image)
In figure 4 the velocity measurements are depicted along the three axes (x-x horizontal out-of-plane, y-y horizontal in-plane and z-z vertical) as they were recorded during a typical sampling with the drop weight excitation. From these measurements an attempt was also made to obtain an estimate of the damping ratio for the dominant in-plane and out-of-plane frequencies. This is depicted in figure 5 for the main symmetric in-plane eigen-mode that is excited by the drop weight and has a dominant period of 7.715Hz. The corresponding damping ratio values is approximately 2.7%. All vibration measurements of the dynamic response of the Konitsa bridge for either type of excitation were utilized to extract the eigen-frequencies depicted in figure 16c together with the approximate shape of the corresponding eigen-modes.

Figure 5. Vibration measurements from drop weight excitation at the middle of the bridge obtained from the tri-axial velocity sensor also located at the middle of the Konitsa bridge.

3. NUMERICAL SIMULATION OF THE VIBRATORY EIGEN-MODES FOR KONITSA BRIDGE

In this section the dynamic characteristics of two studied stone masonry bridges will be predicted through a numerical simulation process. Initially, this numerical simulation will be based on elastic behaviour, assuming the stone masonry as an orthotropic continuous medium and limiting these numerical models at approximately the interface between the end abutments and the rocky river banks thus introducing boundaries at these locations. For simplicity purposes, the bulk of these numerical simulations is made in the 3-D domain representing these bridge structures with their mid-surface employing thick shell finite elements. The various main parts of these stone masonry bridges, that is the primary and the secondary arches, the abutments, the deck, the spandrel walls and the parapets were simulated in such a way that narrow contact surfaces could be introduced between them. All
available information, measured during the in-situ campaign, on the geometry of each one of these parts for every bridge was used in building up these numerical simulations. The mechanical property values obtained from the stone and mortar sample tests, which were presented by Manos et al. (2016). The approximation adopted in this study is a process of back simulation. That is, adopting values for these unknown mechanical stone masonry properties, respecting at the same time all the measured geometric details, which result in reasonably good agreement between the measured and predicted in this way eigen-frequency values. The comparison of the results of these numerical simulations for Konitsa bridge, in terms of eigen-frequencies and eigen-modes depicted in figures 7, is shown in figure 8. It can be seen that in most cases the predicted eigen-frequency values are in reasonably good agreement with the measured values.

Out-of-plane modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fixed Numer.</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st OOP Symmetric</td>
<td>2.526Hz</td>
<td>2.539Hz</td>
</tr>
<tr>
<td>2nd OOP Asymmetric</td>
<td>4.759Hz</td>
<td>4.883Hz</td>
</tr>
<tr>
<td>3rd OOP Symmetric</td>
<td>7.06Hz</td>
<td>7.129Hz</td>
</tr>
</tbody>
</table>

In-plane modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fixed Numer.</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st IP Asymmetric</td>
<td>6.733Hz</td>
<td>5.176Hz</td>
</tr>
<tr>
<td>2nd IP Symmetric</td>
<td>7.078Hz</td>
<td>7.715Hz</td>
</tr>
<tr>
<td>3rd IP Symmetric</td>
<td>10.085Hz</td>
<td>12.549Hz</td>
</tr>
</tbody>
</table>

Figure 8. Numerical and observed eigen-values for the Konitsa bridge.

On the basis of this comparison an additional numerical simulation was performed for the Plaka bridge (figure 9), despite the lack of measured response in this case, adopting the same assumptions that were described before specifically for the Konitsa bridge. As can be seen by comparing the numerical eigen-frequency values of the Konitsa bridge (figure 8) with those of the Plaka bridge (figure 9) the latter, as expected, is more flexible both in the in-plane as well as in the out-of-plane direction. Due to the size of this bridge as well as because of the presence of the left and right secondary arches the 1st symmetric out-of-plane eigen-mode has relatively low eigen-frequency values equal to 2.068Hz as
compared to and 2.526Hz for the Konitsa bridge. This is also true for the higher order out-of-plane eigen modes as well as for the in-plane eigen-modes. This is a clear indication of the flexibility of the Plaka bridge, mainly in the out-of-plane direction which is also the direction of loading during river flooding as well as from the horizontal component of an earthquake excitation in a direction perpendicular to the longitudinal (y-y) axis of this bridge.

**Out-of-plane modes**

1st OOP Symmetric
- Fixed Numer. = 2.068Hz

2nd OOP Asymmetric
- Fixed Numer. = 3.994Hz

3rd OOP Symmetric
- Fixed Numer. = 6.351Hz

**In-plane modes**

1st IP Asymmetric
- Fixed Numer. = 5.583Hz

2nd IP Symmetric
- Fixed Numer. = 6.573Hz

3rd IP Symmetric
- Fixed Numer. = 9.034Hz

**Figure 9. Numerical eigen-values for the Plaka bridge.**

4. SIMPLIFIED NUMERICAL SIMULATION OF THE BEHAVIOUR OF THE PLAKA BRIDGE.

This section includes results of a series of numerical simulations of the Plaka bridge when it is subjected to a combination of actions that include the dead weight (D) combined with accidental forces. This bridge replaced an older bridge at the same place that collapsed from flooding of Aracthos river in 1860. The first attempt in rebuilding this collapsed bridge also collapsed in 1863 during the opening day. The Plaka bridge depicted in figure 3a, one of the most spectacular stone masonry bridges was built in 1866 by the chief mason Kostas Mpekas and survived till January 2015 despite the structural damage that suffered during the World War II. It collapsed on the 2nd of February 2015 again from flooding of Aractos river (figures 14a and 14b). The total length of this bridge was over 60m and the width of the deck at mid-span of the central arch 3.20m (see figure 3b). As already
mentioned a secondary arch is constructed at the right side of the main arch resulting in a mid-pier that is founded on the river bed. Moreover, a large arch-shape opening is present at the left abutment with its foundation also partly supported on the river bed. The presence of these two openings (right secondary arch and the left relief) was not sufficient to protect the structure from the forces caused by the recent flooding. Because almost all the stone masonry bridges in Greece have been built mostly for relatively light live load levels resulting from the crossing of pedestrians or animal flocks their structural vulnerability due to traffic conditions is not an issue. Instead, seismic forces or flooding of the narrow gorge currents that these bridges cross are causes of structural damage that may lead to collapse, as demonstrated from the Plaka bridge (see figures 14a and 14b). Apart from the hydrodynamic loads that a stone masonry bridge is subjected to from a flooded current, one of the main sources of distress that may lead to partial or total collapse is the scouring of its foundation. This seems to be the case for the mid-right pier of Plaka bridge. Initially, through a simplified numerical simulation the flooding and the structural damage from the 1944 explosion are studied by numerically simulating the state of stress of the bridge for these loading conditions. Finally, an additional numerical simulation will try to study the state of stress in the case that this bridge, before its collapse, would have been subjected to a design earthquake.

Figure 14a. Flooding of river Arachtos on 2nd February 2015

Figure 14b. The collapsed Plaka bridge during the visit of December 2015.

4.1 Simplified numerical simulation of the Plaka stone masonry bridge for flooding conditions.

In this section the results of a simplified numerical simulation of the Plaka bridge under flooding conditions are presented. Towards this objective use is made of the numerical simulations discussed before in obtaining the eigen-modes and eigen-frequencies for this structure. This was done as it was concluded that these numerically obtained dynamic characteristics constitute a realistic approximation. In the present study, due to lack of data that can ascertain the velocity of the flow of the river at the bridge and the maximum level of the water during the 2nd of February flooding no attempt was made to estimate the distribution of the hydrodynamic pressures on these parts of the Plaka bridge that was submerged in the flooded river (see figure 14a). Instead, a simplified representation of the flooding load was adopted in the present study as being applied in the lower part of the middle pier and the right abutment. This load was applied as a uniformly distributed pressure acting in these parts of the bridge from the level of the river-bed to foundation of the mid-pier interface till the low rim of the right opening. The amplitude of this water pressure was equal to 0.04MPa. In light of the nature of these assumptions it can be argued that the used static load to approximate the hydrodynamic forces underestimates the actual effect that the severe 2nd of February flooding had on the Plaka bridge. This investigation, apart from considering in the described simplified way the hydrodynamic loads on the Plaka bridge, attempted to also study the deformability of the foundation of the mid-pier, which is believed to be of significance (Manos et al. 2015 and 2016). The support condition of the mid-pier foundation, which as already described lies in the river bed, could also be influenced from erosion due to the river flow in the long term as well as in the short term during the strong and turbulent flow prior to the collapse of 2nd February 2015. In order to examine this parameter in the framework of the current simplified numerical investigation three different stiffness conditions were examined in order
to simulate the deformability of the mid-pier foundation. Due to space limitations, results only from the third case are presented here whereby the mid-pier foundation was considered to be quite flexible (see Manos et al. 2017). The obtained Plaka bridge deformation and stress (tensile as well as compressive) response is shown in figures 15 and 16. From the obtained response it can be concluded that the flexibility of the foundation results in more detrimental response for the Plaka bridge than the applied flooding loadings. For the case of quite flexible foundation conditions the tensile stress field at either the left or right mid-pier regions results in peak values equal to 2.5 MPa and 0.43 MPa, respectively (right part of figure 16). These values are well beyond the tensile strength of stone masonry, thus signifying potential structural damage.

Mid-pier supported by links with variable stiffness from 40 KN/mm to 7 KN/mm

Max vert. deflection main arch above mid-pier (right) = -19.40 mm with horizontal displ. 3.6 mm in y-y, and 0.85 mm hor. displ. (out-of-plane, x-x direction, flow of river)

Minimum compressive stress = -2.2 MPa

Figure 15. Variation of maximum compressive stresses with the deformability of the mid-pier. Gravity Forces together with assumed flooding loads.

Mid-pier supported by links with variable stiffness from 40 KN/mm to 7 KN/mm

Max vert. deflection main arch above mid-pier (right) = -19.40 mm with horizontal displ. 3.6 mm in y-y, and 0.85 mm hor. displ. (out-of-plane, x-x direction, flow of river)

Maximum tensile stress = 2.5 MPa

Maximum tensile stress = 0.43 MPa at right side of main arch

Figure 16. Variation of maximum tensile stresses with the deformability of the mid-pier. Gravity Forces together with assumed flooded loads.

4.2 Simplified numerical simulation of the damage that Plaka stone masonry bridge sustained from an explosion.

An additional part of this numerical study attempted to study the effect that the main arch of Plaka bridge sustained structural damage during an explosion that occurred in 1944, shown in some detail at the top of figure 17a. The numerical model of the bridge with the most flexible foundation conditions
of the mid-pier was extended to simulate this structural damage at the region of the primary arch of the central main arch. This was done by introducing to the damaged region of the primary central arch finite elements with bending and membrane stiffness values equal to 10% of the corresponding values used in the same model without the simulation of this damage. The obtained tensile stress peak response is depicted in the top right side of figure 17.

1944 Explosion

Mid-pier supported by links with variable stiffness from 40KN/mm to 7KN/mm
Damage part of big arch simulated with membrane and bending stiffness values 10% of the original values

Max vert. deflection main arch above mid-pier (right) = -21.95mm with horizontal displ. 4.5mm in y-y, and 0.87mm hor. displ. (out-of-plane, x-x direction, flow of river)

Maximum tensile stress = 2.579MPa at the left side of the mid-pier

Maximum tensile stress = 0.764MPa at the damaged region located at the right side of main arch

1st symmetric out-of-plane eigen-mode f=1.769Hz

1st asymmetric in-plane eigen-mode f=3.913Hz

Figure 24. Numerical simulation of the effect of the damage from the 1944 explosion

By comparing the tensile response of figure 17 with that of figure 16, it can be concluded that this type of simulating the damage resulted in a 79% increase of the maximum tensile stress in the region of the main arch that coincides with the damaged region (from 0.43MPa to 0.769MPa), located at the right side on the top of the mid-pier. It must be pointed out that some years after the explosion the damaged part was repaired with poured in place reinforced concrete. However, the effectiveness of this repair after a long period till the collapse of February 2105 is disputable. In the bottom of figure
The eigen-frequency values for the first two out-of-plane eigen-modes and the first in-plane eigen-mode are shown for the Plaka bridge model simulating numerically the explosion structural damage. Next, by comparing these values with the corresponding values shown in figure 9 for the non-damaged structure with a non-deformable mid-pier foundation it can be seen that both the introduced damage and the mid-pier foundation flexibility reduces considerably the stiffness of the Plaka bridge both in the out-of-plane as well as in the in-plane direction. Whether this stiffness reduction coupled with the resulting values of the eigen-frequencies results in an amplification of the fluid-structure interaction for the 2nd of February 2015 flooding is a subject of further investigation.

4.3 Simplified numerical simulation of the Plaka stone masonry bridge for design earthquake conditions.

The seismic forces were defined by making use of the current definition of the seismic forces by EURO-Code 8. Towards this horizontal and vertical design spectral curves were derived based on the horizontal design ground acceleration. This value, as it is defined by the zoning map of the current Seismic Code of Greece, is equal to 0.24g (g the acceleration of gravity) for the location of the Plaka bridge. Furthermore, it is assumed that the soil conditions belong to category A because of the rocky site where this bridge is founded, that the importance and foundation coefficients have values equal to one (1.0), the damping ratio is considered equal to 5% and the behaviour factor is equal to 1.5 (unreinforced masonry). The design acceleration spectral curves obtained in this way are depicted in figures 18a and 18b for the horizontal and vertical direction, respectively. In these two figures (18a and 18b) the eigen-period range of the first twelve eigen-modes is also indicated (ranging between the low and the high modal period, see also figure 9). More details of the magnitude of the seismic forces generated at the Plaka bridge by these design response spectra are given by Manos et al (2016).

Figure 18a. Horizontal spectral curves for type-1 Euro-Code to be applied in Plaka Bridge.

Figure 18b. Vertical spectral curves for type-1 Euro-Code to be applied in Plaka Bridge.

Figure 19. Deformed shape of the Plaka bridge for the design earthquake. $U_1 = -51.87$mm (out-of-plane, hor. $x-x$), $U_2 = 3.4$mm (in-plane hor. $y-y$), $U_3 = -23.0$mm (in-plane ver. $z-z$).
It must be pointed out that the flexibility of the foundation that was taken into account in the previous section was not considered here. Thus, the results presented and discussed in figures 19 and 20 were obtained for non-deformable mid-pier foundation. Moreover, the results were found from the combination of the gravity forces acting together with the seismic forces, as they resulted from the design spectra of figures 18a and 18b and a dynamic spectral analysis. The deformed shape of the Plaka bridge for the design earthquake is shown in figure 19. As can be seen, the out-of-plane horizontal response attains large values at the crest of the main central arch. The peak value at mid-span (U1=51.87m) is larger than the corresponding value that was found when the bridge was subjected to gravity loads and flooding forces. The peak value at mid-span in the vertical direction (U3=23mm) is of similar amplitude to the one found when the bridge was subjected to gravity loads and flooding forces (see figures 15 and 16).

The distribution of the maximum tensile and the minimum compressive stresses (S11 and S22) when the Plaka stone masonry bridge is subjected to gravity forces combined with the design earthquake, as defined before, are depicted in figures 20a to 20d. As can be seen from figures 20a and 20c the peak maximum tensile stress field develops at the crest of the main central arch (S11=5.73MPa) as well as at the foot of the mid-pier (S22=3.40MPa, at the inner side). Similarly, the peak minimum compressive stress field develops again at the crest of the main central arch (S11=-6.14MPa) as well as at the foot of the mid-pier (S22=-4.9MPa). These peak compressive values are well below the compressive strength of the stone masonry. On the contrary the peak tensile values are many times above the tensile strength of the stone masonry thus indicating the severe potential structural damage of the Plaka bridge from the design earthquake. The most vulnerable parts of the bridge, as obtained from the location of the peak tensile stress concentration, is a wide area at the crest of the main central arch of the bridge as well as the mid-pier internal side footing.

5. CONCLUSIONS

1. Stone masonry construction has a long tradition in many places worldwide. Stone masonry bridges built many centuries ago are one such example. Despite the rigidity and resilience of stone masonry bridges they are in need of maintenance in order to preserve them as part of the built cultural heritage. Towards this end in-situ measurement campaigns were conducted on a number of stone masonry
bridges in order to identify their dynamic characteristics in terms of eigen-frequencies, eigen-modes and damping properties. This information is believed to represent a valuable basis for building realistic numerical simulations of the structural behaviour of such bridges as well as for their structural health monitoring.

2. The simplified numerical analyses yielded numerical predictions of bridge deformations and stresses that are useful in understanding the structural behaviour and the structural damage potential for such masonry structures. The structural performance of Plaka bridge that collapsed due to flooding in February 2015 was also studied. The structural behaviour of this bridge was examined when subjected to the gravitational forces combined with either simplified flooding loads or a design earthquake. Moreover, the structural damage that this bridge sustained during an explosion in 1944, was also examined together with the flooding loads.

3. The long or short term erosion of the bridge footings, as was introduced in this simplified numerical analysis, results in more detrimental response for the Plaka bridge than the applied simplified flooding loads. However, this may be due to the fact that the current investigation ignored any dynamic effects from fluid-structure interaction. The obtained numerical predictions from the numerical simulation of the structural damaged sustained by this bridge during the 1944 explosion leads to a dominant tensile stress field at the regions of the main and secondary arches on top of the mid-pier. These predicted peak tensile stress values are well beyond the stone masonry strength. Consequently, they could well have contributed towards the 2nd of February 2015 collapse.

4. The peak tensile values predicted when design seismic forces were applied for the Plaka bridge are many times above the tensile strength of the stone masonry thus indicating the severe potential structural damage for this old stone masonry bridge from the design earthquake. The most vulnerable parts of the bridge, as obtained from the location of the peak tensile stress concentration, is a wide area at the crest of the main central arch of the bridge as well as the mid-pier internal side footing. Thus, these earthquake vulnerability predictions for the Plaka bridge indicate that had the bridge not collapsed from flooding the design earthquake would have led to its structural damage.

5. The integrity of the stone masonry in various parts of the bridge is an additional maintenance issue of considerable importance. Intervention recommendations for such bridge or attempts to rebuilt, as is the case for the Plaka bridge, should include clauses for applying preparatory actions of measurements and analyses like the ones included here together with established principles that govern a major retrofitting / maintenance effort for cultural heritage together with effective retrofitting / maintenance techniques that are proven to be durable.

6. IN MEMORY

• This work is dedicated to the memory of Ray W. Clough, Professor Emeritus of the University of California, at Berkeley, U.S.A.

7. REFERENCES


