THE EFFECT OF THE SOIL LAYER’S EIGENFREQUENCY TO SOIL-STRUCTURE INTERACTION

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ABSTRACT

In case of seismic design the deformability of the soil should be considered, which can be performed in several ways. The deformation of the soil can be taken into account by springs, where the spring coefficients are calculated by static analysis. A more sophisticated way is the substructure method. In this case the effect of the soil is modelled with an impedance matrix, which means instead of the soil, spring and dashpot elements are used. The most accurate method is the direct method, where the structure and the soil are examined together with 3D finite element method.

Most of the methods do not take into account the semi-infinite extent of the soil of the soil, which results significantly different behavior than the spring-dashpot systems. For an infinite medium, which is used in many cases, there are no eigenmodes, however in practical applications the soft soil is always bounded by rocks. For these cases the soil has eigenmodes and the resonance may influence considerably the response of the system. This question was investigated numerically by FE calculations, and it was found that in certain cases the resonance, which is neglected in the common design process, may significantly enhance the earthquake loads. In this paper this phenomenon is investigated and the parameter range is defined when this effect must be taken into account.

Keywords: SSI, resonance, horizontal displacement, numerical analysis

1. INTRODUCTION

In case of static design, the effect of the soil can be neglected (Figure 1a), but for earthquake resistance design (dynamic analysis) the effect of the soil-structure interaction (SSI) must be taken into account. This can be achieved by different methods (Figure 1). The deformability of the soil can be taken into count by elastic support (Figure 1b). There are several formulas in the literature for the stiffness characteristics of an infinite half-space (Kausel 2010), and the stiffness of a finite soil layer can be calculated by the finite element method. In case of a half space radiation damping occurs (Lai and Martinelli 2013), to model this, dashpot elements are added to the model (Figure 1c). A more sophisticated method is the substructure approach, where the response of the structure is calculated by superposition; First the excitation on the free soil surface is determined from the excitation on the bed rock, then the impedance of the soil-structure system is calculated. The soil-structure interaction is obtained by superposition; and hence the method is applicable for linear systems only. Finally, the most exact approach is the direct method (Figure 1d), where the soil and the structure are modeled together. In this case nonlinearities can be considered, however it requires significant computational time and effort.
To calculate the impedance function the soil with a weightless foundation has to be considered, the foundation is excited by a harmonic force (Figure 2). The ratio of this force and the displacement of the foundation is the impedance function, which depends on the excitation frequency. This function consists of an amplitude and a phase angle (the shift of the force and displacement). These can be interpreted as a spring stiffness and a damping value, but these parameters depend on the excitation frequency. As a simplified method, these functions are often approximated by constant values (usually the initial values).

\[
\frac{Q(t)}{u(t)} = G r_0 \left[ F_1 + i F_2 \right],
\]

where \(Q\) is the harmonic force, \(u\) is the vertical displacement, \(G\) is the shear modulus of the soil and \(r_0\) is the radius of the foundation. Sung (1953) gave similar functions for this problem, but they investigated different stress distributions under the foundation. Hsieh (1962) determined the frequency dependent spring stiffness and damping values from the \(F_1\) and \(F_2\) functions. Lysmer and Richart (1966) approximated these with one constant spring stiffness and damping values. For horizontal and rocking motion (Figure 2c and d) Bycroft (1956) gave constant spring stiffness and damping values. The torsional excitation (Figure 2e) was investigated by Reissner and Sagoci (1944). Shah (1968) analyzed both circular and strip foundations for different excitations.

The cone model (Meek and Veletsos 1973), (J.P. Wolf and Deeks 2004) also gives constant spring stiffness and damping coefficients according to the analytical solution of a semi-infinite cone under harmonic excitation. Barros and Luco (1990) and John P Wolf (1997) examined systems consisting of several spring stiffnesses, dashpots and masses by determining the different parameters by the least square method.
2. PROBLEM STATEMENT

In case of practical earthquake resistant design the applicability of frequency dependent impedance function is very limited, because its complexity. Rather engineers are applying constant spring stiffnesses and damping values according to one of the formulas in the literature which are based on the impedance function of a soil half-space, or to calculate a constant spring stiffness by static finite element calculation. None of these are taking into account the possible resonance which may occur in case of the dynamic loading of a finite soil layer (Figure 3).

![Figure 3](image)

In this article we wish to analyze the possible effect of the resonance, and give the parameter range, where this effect cannot be neglected.

3. APPROACH

A finite soil layer with a rigid foundation (Figure 3a) and the simplified models (Figure 3b) are analyzed numerically to determine the effect of resonance. The analyses are limited to horizontal excitation. The numerical analysis was performed by the ANSYS computer code. Two different signals were investigated:

- harmonic (sine) function,
- real earthquake records.

For the sine function the harmonic analysis of ANSYS was also applied.

It is well known that an earthquake record measured at the bedrock and at the surface of the soil differ significantly. Softwares are available in the literature (Kottke and Rathje 2009) with the aid of which, (usually assuming a simple soil column) the record at the surface can be calculated from the record at the bedrock. In the presented analysis the source records will be used at both locations, because we wish to investigate the difference caused by the resonance effect, but we emphasize that the amplitudes of the latter one can be 15-50\% higher than the previous one.

4. HORIZONTAL EXCITATION OF THE STRUCTURE

As it was stated in Section 3, the horizontal motion of a finite soil layer is investigated. In the next sections the importance of the resonance for harmonic and earthquake excitation will be demonstrated.

4.1 Harmonic analysis

In case of soil-structure interaction the impedance function is the ratio of the harmonic force excitation of the weightless foundation and the steady-state solution of the displacement. The resonant frequencies of a free soil layer can be approximated by the resonant frequencies of a sheared beam (Chen 1997):

\[ f_r = \frac{c_s}{2h} \left( r - \frac{1}{2} \right), \]  

(2)

where \( c_s \) is the shear wave velocity in the soil and \( h \) is the thickness of the soil layer (Figure 3).

The natural frequency of a soil layer with a weightless foundation will be different, since this formula is based on a 1D model, but in the case of a rigid (strip) foundation the displacements will be 2D.

In Figure 4, Figure 5 and Figure 6 the inverse of the impedance functions of soil layers with different
parameters are presented. The inverse of the impedance function is given on two diagrams \((u(t)/Q(t))\), the first one shows the amplitude, the second one shows the phase angle, which is the shift between the force and displacement. (This shift represents the energy dissipation of the system, e.g. when only a mass-dashpot system is considered, the phase angle in the impedance function is 90°, while for zero shift the energy dissipation is zero.)

Figure 4 shows the impedance function for different soil layer thicknesses \((h)\). As it can be seen the bigger the value of \(h\) is, the smaller the first natural frequency is (as in Eq. (2)). This means that the effect of the resonance for the amplitude of the displacement will be smaller for thicker soil layers. The dominant frequency of an earthquake is between 0.5 and 3 Hz (according to the analyzed 44 records (FEMA P695 2009)), in this range, the phase angle is around 90°. The phase angle of a spring-dashpot system is different in this range (Figure 7b), which means the energy dissipation in the two systems are different in this range.

![Figure 4. Impedance function of soil layer for different thickness \((h)\) \((c_s=100 \text{ m/s}, \nu=0.3, \rho=1800 \text{ kg/m}^3, \xi=0.05, r=5 \text{ m})\)](image)

In Figure 5 the effect of the shear wave velocity (i.e. stiffness of the soil) is shown. The diagram of the amplitude shows that in case of soft soils (small \(c_s\)) the effect of the resonance is much more dominant than in case of stiffer soils.

![Figure 5. Impedance function of soil layer for different shear wave velocities \((c)\) \((h=40 \text{ m}, \nu=0.3, \rho=1800 \text{ kg/m}^3, \xi=0.05, r=5 \text{ m})\)](image)

Figure 6 shows the impedance for different Poisson ratios. In this case the curves for the amplitude and phase angle are almost on the top of each other, for smaller \(\nu\), the amplitude is slightly bigger.

![Figure 6. Impedance function for different Poisson ratios \((\nu)\) \((h=100 \text{ m}, \rho=1800 \text{ kg/m}^3, \xi=0.05, r=5 \text{ m})\)](image)
In the literature there are several solutions for the impedance functions of a half space with circular or strip foundation (Bycroft 1956), (Shah 1968). As it was mentioned in the introduction, the impedance can be given with a complex function (Eq. (1)). In case of the inverse impedance \( Z = u(t)/Q(t) \) \( f_1 \) represents the real part and \( f_2 \) the complex part of the function. These can be also given as an amplitude and a phase angle:

\[
Z_0 = \sqrt{f_1^2 + f_2^2}, \quad \varphi = \tan^{-1} \left( \frac{f_2}{f_1} \right).
\]

Furthermore, the function can also be given as frequency dependent spring stiffness and damping (Figure 7a) (Hsieh 1962):

\[
k(\omega) = G_r \frac{f_1}{f_1^2 + f_2^2}, \quad c(\omega) = G_r \frac{-f_2}{\omega f_1^2 + f_2^2}.
\]

In the simplified model these frequency dependent functions are approximated by constant values, for example Bycroft (1956) considered the following values in case of horizontal excitation (Figure 7a):

\[
k_x = \frac{32(1-\nu)G_r}{7-8\nu}, \quad c_x = \frac{18.4(1-\nu)G_r^2}{7-8\nu} \sqrt{\rho G}.
\]

The inverse impedance function of a spring-dashpot system as an amplitude and phase angle are given in Figure 7b:

\[
u_0 = \frac{1}{k_x + \frac{c_x \omega^2}{k_x}}, \quad \varphi = -\frac{c_x \omega}{k_x}.
\]

If we compare the functions in Figure 4, Figure 5 and Figure 6 to the function in Figure 7b, the difference between the amplitude is extremely high, and the phase angle is also different in this frequency range.

A common method in practical earthquake resistant design is to consider the effect of the soil by calculating a constant spring stiffness \( k_{\text{static}} \) with the aid of a static finite element calculation. In that case the amplitude if the inverse impedance function is \( 1/ k_{\text{static}} (2.32 \cdot 10^{-8} \text{ m/N}) \) for the same parameters as in Figure 7.
Figure 7. (a) The frequency dependent spring stiffness and damping value and their approximation (Bycroft 1956), where $\omega_s = \omega \sqrt{c}$, is the dimensionless frequency and the constant spring stiffness and damping value are given by Eq. (5). (b) The impedance of a spring-dashpot system (Eq. (6))

(c) = 100 m/s, $\rho$ = 1800 kg/m$^3$, $r$ = 5 m

In the next section the difference between the two models (Figure 3) will be investigated in case of time-history analysis.

4.2 Transient analysis

Time-history analyses were also performed to determine the effect of resonance. Now, the rigid foundation on the finite soil layer has mass $M$, and the horizontal displacement of the foundation is calculated from the base excitation (Figure 8). The constant spring stiffness and damping value are calculated by Eq. (5), both harmonic excitation and earthquake records will be investigated. The base excitation is $u_g(t)$, the horizontal displacement of the structure is $u_{direct}(t)$ in case of the analysis of the finite soil layer (Figure 8a) and $u_{simplified}(t)$ for the simplified model (Figure 8b).

Figure 8. (a) Finite soil layer with thickness of $h$ and rigid foundation with mass $M$, (b) Simplified model

4.2.1 Harmonic excitation

First the two models (Figure 8) for harmonic excitation are analyzed. Figure 9 shows the difference between the horizontal displacement of the direct and simplified model and also for the case, when the spring stiffness is calculated by static FEM.
The left diagram of Figure 9 shows the steady-state solution for \( f = 0.4 \) Hz sine excitation, as it can be seen the maximum horizontal displacement of the direct model is 2.25 times bigger than the displacement in case of the model when the spring stiffness is calculated by static FEM, and 73 times bigger than the simplified model. The right diagram of Figure 9 shows the amplitude for different frequencies, the difference between the models can be even higher for the frequencies close to the resonant point. The right diagram of Figure 9 also shows that the resonant frequency of the three models are different, in this case \( f_0 = 0.46 \) Hz for the direct model, \( f_0 = 3.22 \) Hz for the simplified model and \( f_0 = 0.65 \) Hz for the case when the spring stiffness is calculated by static FEM.

4.2.2 Real earthquake records

To determine the effect of resonance in case of real earthquake records 44 records are investigated (FEMAP695 2009). First, the frequency content of these records are calculated by Fourier transform. Since our goal was to demonstrate the importance of resonance, the thicknesses of the soil layer were chosen in such a way that the eigenfrequency of the soil and the dominant frequencies of the earthquakes are close to each other (Figure 10).

The effect of resonance is shown in case of the record number 32 (Figure 10), for soft soil (\( c_s = 100 \) m/s). The difference between the horizontal displacement response of the two models (Figure 8) are shown in Figure 11. The thickness of the soil layer was chosen according to Figure 10 (62.5 m), the mass of the structure in the given example is 45 t. As it can be seen the difference is huge for real earthquake record too, the displacement of the direct model is five times higher than the displacement of the simplified model.

Figure 9. Horizontal displacement of the foundation for harmonic excitation
\( (c_s = 100 \text{ m/s}, \rho = 1800 \text{ kg/m}^3, r = 5 \text{ m}, h = 40 \text{ m}, M = 10^6 \text{ kg}, \xi = 0.05) \)

Figure 10. Soil layer thicknesses in case of different soil stiffnesses (shear wave velocities) chosen in such a way that the eigenfrequency of the soil and the dominant frequencies of the earthquakes are close to each other
5. CONCLUSIONS

In this paper we investigated the possible errors due to the different modelling of soil structure interaction. Some of the applied soil models, which are extensively used in practical design, the soil substructure has no eigenfrequency, which may lead to significant error and to a not conservative design. For harmonic excitation the error of using the simplified model can be an order of magnitude, and for real earthquakes, where the eigenfrequency of the soil-structure system is close to the dominant frequency of the earthquakes, the predicted maximum displacement of the simplified model can be 4-6 times smaller than that calculated with the more sophisticated (3D) models. The magnitude of the error depends on the soil parameters (Figure 11).

The numerical analyses showed that two cases have to be investigated to determine the effect of resonance, the natural frequency of the soil layer and the natural frequency of the soil layer–structure system. The circular frequency content of typical earthquakes is in the range of $0.45 < f < 2.82$ $1/s$. In Figure 12 it is investigated whether for practical cases it can be close to the natural frequency. The density of the soil varies typically in the 1700–1900 kg/m³ range, we considered the average value. Two curves are given as a function of $c_s$ and $h$, which belong to $f_0=0.45$ $1/s$, and $f_0=2.82$ $1/s$. It can be observed that for softer soils the smaller thicknesses, while for stiffer soils also bigger thicknesses can be in the resonance-sensitive zone.

The first eigenfrequency of the system (soil layer and the foundation with the weight of the rigid structure), can also result in resonance. Figure 13-Figure 15 show the resonance-sensitive zones for different masses.
Figure 13. Parameter range \((c_s \text{ and } h)\), where the natural frequency of the system is in the range of the dominant frequency of earthquakes, \(M=10t\)

Figure 14. Parameter range \((c_s \text{ and } h)\), where the natural frequency of the system is in the range of the dominant frequency of earthquakes, \(M=1000t\)

Figure 15. Parameter range \((c_s \text{ and } h)\) where the natural frequency of the system is in the range of the dominant frequency of earthquakes, \(M=5000t\)

It can be observed that in case of bigger masses and soft soils there is no critical zone, but again for stiffer soils the larger soil thicknesses can also be resonance-sensitive.
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7. REFERENCES


