KERNEL SMOOTHING METHODS FOR NON-POISSONIAN SEISMIC HAZARD ANALYSIS

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ABSTRACT

For fifty years, the mainstay of probabilistic seismic hazard analysis has been the methodology developed by Cornell, which assumes that earthquake occurrence is a Poisson process, and that the spatial distribution of epicentres can be represented by a set of polygonal source zones, within which seismicity is uniform. Based on Vere-Jones’ use of kernel smoothing methods for earthquake forecasting, these methods were adapted in 1994 by the author for application to probabilistic seismic hazard analysis. There is no need for ambiguous boundaries of polygonal source zones, nor for the hypothesis of time independence of earthquake sequences.

In Europe, there are many regions where seismotectonic zones are not well delineated, and where there is a dynamic stress interaction between events, which are not independent samples from a Poisson process. From the Amatrice earthquake of 24 August, 2016, the subsequent damaging earthquakes in Central Italy over months were not independent events. Because of the spatial dispersion of epicentres, and the clustering of magnitudes for the largest events in a sequence, which might all be around magnitude 6, the specific event causing the highest ground motion can vary from one site location to another. Where significant active faults have been clearly identified geologically, they should be modelled as individual seismic sources. The remaining background seismicity may be modelled as non-Poissonian using statistical kernel smoothing methods.

Keywords: seismic; hazard; kernel; non-Poissonian; aftershocks

1. INTRODUCTION

In his standard textbook on earthquake hazard analysis, Reiter (1990) noted that concern over enhanced damage from aftershocks must be taken into account in a separate analysis. He further remarked that few such analyses had been carried out. In the quarter century since this book was published, it can still be stated that rather few such analyses have been carried out. To the extent that the ground motions of foreshocks and aftershocks are enveloped by the ground motion generated by the larger main shock, these lesser events may be ignored in calculating the exceedance probability of ground shaking at a given site. Similarly, to the extent that the damage inflicted by foreshocks and aftershocks is subsumed by the larger main shock damage, these lesser events may be ignored in calculating the exceedance probability of earthquake loss.

However, such situations do not always prevail, especially if there is only a small difference in magnitude between the main shock and the largest aftershock or foreshock, or if there is a sizeable difference in their ground shaking area footprints. Furthermore, other than foreshocks and aftershocks, an earthquake may trigger other events, which are dynamically dependent to some extent. The M6.3 Christchurch, New Zealand, earthquake of 22 February 2011 is a salient example of a major event dynamically linked in some manner with an earlier larger main shock, the M7.1 Darfield, Canterbury, earthquake of 4 September 2010, causing massive economic loss in a different area, which is more densely populated.

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The inter-dependence between earthquakes may induce a wide variety of spatio-temporal seismic sequences, which are not statistically consistent with simple representation by a time-independent Poisson process. An especially notable, and practically significant, form of spatio-temporal sequence manifests in a progressive migration of seismicity over time. For example, in 2010-2011, there was a west-east migration of seismicity in Canterbury, New Zealand.

Such migration may potentially cross the borders between designated seismic area zones, which may be poorly constrained. Indeed, the artificial character of some zone boundaries reflects an irreducible degree of subjectivity in their definition by seismic hazard analysts. It is well known that the ambiguity in defining seismic zone boundaries is a major source of epistemic uncertainty in seismic hazard assessment, resulting in the proliferation of alternative area zonations in logic-trees.

Migration, or other manifestations of spatio-temporal seismicity correlation, may also be observed within an area seismic zone, as shown in Figure 1. Especially in regions of rather moderate seismicity, the observed pattern of observed historical seismicity may be discordant with pre-conceived geological notions used to delineate seismic area zones. It has not been routine standard practice in seismic hazard modelling to undertake a spatio-temporal statistical analysis of the epicentres within each area zone to check for consistency with the assumption that events occur randomly both geographically and temporally. Nearest neighbour analysis to test for spatial clustering within a zone was first developed in the context of seismic hazard to a nuclear power plant by the UK Seismic Hazard Working Party (SHWP, 1987).

Geological conviction may over-ride the empirical seismological evidence; the argument being that, in the geological long-term, seismicity will eventually fill out the area zone. Except for applications such as the disposal of nuclear waste, the usual time horizon for a seismic hazard study is a few decades rather than centuries or millennia. The definition of return period as the reciprocal of the annual exceedance probability has been a cause of common geological misunderstanding.

![Figure 1. Migration of seismic activity within an area seismic zone](image)

A practical difficulty in parameterizing an area seismic zone arises where the observability of events varies significantly both geographically and temporally. This is commonplace before the modern era of seismic networks, and is especially challenging with seismic zones that straddle a region of low population density, or a coastal area, as indicated schematically in Figure 2. For any given magnitude, the time window of catalogue completeness will be shorter offshore than on land. This is problematic for regions where major offshore earthquakes pose the predominant seismic threat, as it is along some coastlines of Europe. This is an important earthquake engineering issue, since there are many critical facilities sited on or near coasts, with ready access to ports and transport infrastructure.
Another practical difficulty is associated with the uncertainty in the location of earthquake epicentres. For some historical events, the uncertainty may be of the dimension of an area seismic zone, or even greater. For older events, the uncertainty may be so large as to render almost intractable any reasoned assignment to one specific zone, rather than an adjacent zone. An illustration is shown in Figure 3. This is a major challenge when such events include some of the most important earthquakes in a sparse historical catalogue, dating back to the Middle Ages in Europe. For example, as judged by its felt area, the largest documented earthquake close to the coast of Wales is the event of 20 February 1247. However, the uncertainty in the epicentre exceeds 100 km. Perplexed by the irreducible ambiguity in zone assignment, seismic zonation modellers for Wales may decide to neglect this important event altogether, since it is hard to know in which zone to locate it. Probabilistic seismic hazard assessment is an exercise in the quantitative treatment of uncertainty. It should be possible to accommodate very large error bars on the epicentre of a major regional earthquake.

The use of area seismic source zones was an essential pragmatic simplification fifty years ago (Cornell, 1968), when computers were numbered in the thousands rather than the hundreds of millions in the 21st century. Reducing the number of degrees of freedom of the earthquake generation system to a small handful was necessary when computing power was a very scarce costly resource, but much more computer-intensive approaches are now practical. Whilst commemorating the 50th anniversary of seismic hazard modelling, progressively the representation of seismic sources as area zone sources is becoming less defensible in regions with tenuous geological constraints on zone boundaries.
1.1 Seismicity Smoothing

The basic principle underlying a zonal partition is that, whereas significant differences may exist between zones, the characteristics of seismicity within each zone are supposed to be sufficiently homogeneous, or the uncertainty over future spatial patterns of activity is perceived to be sufficiently great, for seismological parameters to be treated as uniform within the designated zones. As non-seismological criteria for delineating zones, geological data have been used with varying degrees of scientific conviction.

The sizeable variations in seismic hazard which stem from alternative plausible choices of area zonation model reflect the conundrum of deciding on zone boundaries, when geological constraints are weak. For site-specific seismic hazard assessment, the location of the site close to a boundary between zones of markedly contrasting seismicity poses model stability concerns. A significant difference in design ground motions may depend on expert judgement on where a boundary should be drawn. For seismic hazard mapping on a national or continental scale, these concerns are amplified according to population density. Towns either side of a zone boundary may end up with significantly different seismic building code ground motion levels, for no other reason than a subjective drawing of a seismic area zone boundary, which may be as arbitrary as some political borders.

To address this problem for US seismic hazard mapping, Frankel et al. (1996) developed a seismicity smoothing approach, based on Gaussian smoothing of activity rates across a spatial grid of points with fine spacing of 0.1 degrees of Latitude and Longitude. The number of events greater than the minimum magnitude is counted on this grid. The logarithm of this number is the maximum likelihood a-value for each grid cell. In the decades since 1995, seismicity smoothing has become a common supplement to fault modelling in characterizing a seismic source model (Felzer, 2013). The spatial degree of smoothing, measured in km, is a parameter that is chosen to avoid excessive jaggedness and over-smoothing in seismic hazard maps. Frankel found a good balance in a choice of 50km.

This seismicity smoothing approach avoids the need to identify and parameterize area seismic zones, and so gets round the boundary sensitivity problem. However, the smoothing of activity rates is susceptible to bias associated with temporal variations in the detectability of events, and the epicentre location uncertainties. Counting the number of events could be subject to considerable error during past historical time windows. This error is particularly serious in regions of moderate or low seismicity, where the omission of a few key events could make a notable difference to the hazard.

This bias issue is remedied by reverting from the aggregation of events to treating events individually, accounting for each event’s detectability over time, and the uncertainty in its magnitude and epicentre. Rather than the coarse smoothing of activity rate, smoothing is carried out on a finer more granular event by event basis, such as described below.

2. KERNEL SMOOTHING

An alternative approach to modelling seismic area zone sources and seismicity smoothing involves the use of kernel estimation techniques for representing the seismic activity rate density (Kagan and Jackson, 1994; Woo, 1995; 1996). The author was motivated to develop this approach to modelling seismic sources from the review of statistical methods for the description and display of earthquake catalogues, undertaken by the seismological statistician Vere-Jones (1992). In his expert view, the kernel method is the most straightforward and widely used of the standard methods, and for exploratory purposes in particular, he considered it hard to see substantial reasons for going beyond it. Vere-Jones noted the remarkable development of smoothing methods for estimating a probability density after a long period of neglect. The book by Silverman (1986) chronicles the progress in smoothing methods, in particular in the domain of practical applications. However, this progress did not include the statistical methodology of seismic hazard analysis.
Within zonal seismic source models, typically the sizes and locations of zones are adjusted manually and subjectively in a fashion akin to drawing histograms. The coarsest and simplest approach to estimating a probability density is to draw histograms. In non-parametric statistical data analysis, a major advance over histograms is the technique of kernel smoothing, and it is natural to extend the application of this more sophisticated technique to the statistical analysis of spatial seismicity data. The smoothing is two-dimensional, and hence applies to catalogues of shallow crustal earthquakes.

The essential observation underlying the new approach is the recognition that the geometry of earthquake epicentres hardly ever satisfies the constraints of spatial uniformity, as presumed by the standard Euclidean zonation method, but rather has a far richer, more structured, fractal characterization. In order to represent this fractal geometry, a seismic source model is most conveniently constructed from the spatial pattern of earthquake epicentres, drawn both from the historical catalogue of earthquakes, and available geological information on neotectonic fault movements.

Where there is a lack of correlation between geological structure and seismicity, it has been canonical practice from the early days of probabilistic hazard computation to assume that earthquakes are equally likely to occur anywhere over the area. However, the assumption of spatial uniformity within a Euclidean area zone is now known from extensive studies of inter-event correlation functions (Kagan and Knopoff, 1980) to conflict with the actual spatial distribution of seismicity, which is capable of arrangement in a far more intricate and complex pattern.

Venturing beyond the restrictions of zonation, the general functional representation of mean activity rate is 

\[ \lambda(M, x) \]  

which is the expected annual number of events of magnitude \( M \) occurring at location \( x \).

From a catalogue of \( N \) historical epicentres \( x_i \), each event being of magnitude \( M \), and associated with an effective observation time period \( T(x_i) \), the activity rate for events of magnitude \( M \) at a general point \( x \) within the region, \( \lambda(M, x) \), can be estimated via a statistical smoothing operation, which recognizes the fundamental probabilistic nature of the discrete sample of historical observations. The smoothing operation involves the introduction of a kernel \( K(M, x) \), which is a magnitude-dependent multivariate probability density function.

The mean activity rate at a point \( x \) is written as a kernel sum over the historical dataset, in which the contribution of the \( i \)'th event is divided by its effective return period \( T(x_i) \). This is the sum of all the historical years since the event occurred, weighted by the prevailing detection likelihood.

In the expression for activity rate, there is no requirement of independence of events in the historical event dataset.

\[ \lambda(M, x) = \sum_{i=1}^{N} K(M, x - x_i) / T(x_i) \]  

(1)

Significant uncertainty exists in both the magnitude and epicentre of each historical event, yet seismic area zonation methods are ill-equipped to deal with this uncertainty. Over the past few decades, the substantial re-evaluation of European historical catalogues has allowed the uncertainties in event magnitude and epicentre to be assessed. But the perceived limited practical usage of such uncertainty data has greatly undervalued this painstaking meticulous historical research. Within the area zonation approach, the spatial distribution of seismicity is presumed to follow a simple parametric distribution, namely uniformity. The uncertainty in epicentre locations only compounds the doubt over where to draw zone boundaries, and undermines the validity of this parametric distribution.

By contrast, within the kernel smoothing approach, these inherent data uncertainties can be accounted for explicitly by assigning probability distributions to them. This data-intensive exercise, which
involves effort proportional to the size of the historical catalogue, has the virtue of maximizing the quantitative use of historical information on past events, which may be crucial for seismic hazard assessment, especially in Europe where historical earthquakes catalogues may date back more than a thousand years.

Various forms of kernel $K(M, x)$ might be chosen. A form suggested by Vere-Jones (1992) has the following infinite-range power-law decay, dependent on the radial separation distance $r(M)$, a fall-off index $a$ and a magnitude-dependent bandwidth parameter $h(M)$:

$$K(M, x) = [(a - 1) / \pi] * h(M)^{-2} * (1 + r^2 / h(M)^2)^{-a}$$

The parametric form of kernel tends to be less important than the bandwidth $h(M)$. For a dissipative system displaying power-law behaviour, the probability of an avalanche-type event occurring within a given spatial extent should scale with the event size. The kernel bandwidth $h(M)$ should therefore scale according to earthquake size, which might be taken in the present context to be fault length $L$. Given the standard form of logarithmic correlation of fault length $L$ with magnitude $M$, the bandwidth $h(M)$ can be parameterized as $h(M) = H \exp(kM)$, where $H$ and $k$ are constants which are region-specific, and can be estimated from regional data. The exponential form of the bandwidth function $h(M)$ precisely mirrors the logarithmic form of the Gutenberg-Richter magnitude-frequency relation.

The above expression for the Kernel function is isotropic, i.e. the smoothing is independent of direction. This is satisfactory in regions where no particular association is discernible between earthquake locations and geological structure. However, in areas where epicentres form identifiable lineaments, some directionality in smoothing may be desirable. Under these circumstances, an appropriate anisotropic Kernel is used. This has the form:

$$K(M, x) = [(a - 1) / \pi] * h(M)^{-2} * (1 + r^2 / h(M)^2)^{-a} * (1 + \delta \cos^2 \theta) / (1 + \delta / 2)$$

In this expression, $\theta$ is the angle subtended at $x$ between the intersection of the fault plane with the Earth's surface and the epicentre location (see Figure 4). $\delta$ is a parameter which modulates the degree of anisotropy; a zero value indicates isotropy and a value of 10 indicates significant anisotropy.

![Figure 4. Ellipsoidal smoothing for kernel directivity](image-url)
This entire computational procedure has been coded up in the open source program KERGRID, available from the author. The basic principle underlying the program is that the epicentre of each past shallow event is smoothed geographically to generate a spatial probability distribution for event recurrence. This is consistent with the seismological observation that, although earthquakes do occur out-of-the-blue from time to time, most often earthquakes tend to be approximate recurrences of events which have occurred before. In contrast with zonal methods, the smoothing procedure can account for fault strike variations on an event-by-event basis. For the larger events of magnitude 6.0 or more, smoothing is anisotropic, being directional about the fault plane axis, reflecting the greater likelihood for events to recur close to the fault plane of a previous event, rather than away from the fault plane. With spatial smoothing undertaken event by event, at any given grid location, the magnitude distribution is defined from the smoothed catalogue, allowing for uncertainty in magnitude estimation.

3. APPLICATION TO SEISMICITY IN ITALY

The standard practice in seismic hazard assessment has been to strip out the foreshocks and aftershocks from the relevant earthquake catalogue. Progress has been made in the past decade to advance beyond this practice. From a declustered catalogue, Boyd (2012) incorporated foreshocks and aftershocks by generating a set of related foreshocks and aftershocks according to a modified Gutenberg-Richter relation for aftershocks. For a high hazard US site, the inclusion of dependent events was found to increase engineering design ground motion levels by about 10%. Iervolino et al. (2013) formulated a methodology for aftershock probabilistic seismic hazard analysis. APSHA models aftershock occurrence via a non-homogeneous Poisson process, whose rate depends on the magnitude of the main shock magnitude.

As an alternative to developing an explicit seismological model for dependent events, the most straightforward practical approach is retain these dependent events within the earthquake catalogue, and treat these as the particular historical realization of a variety of possible outcomes, variants of which can be simulated through the event kernel smoothing procedure. This approach maintains a symmetrical analytical status in the treatment of foreshocks, main shocks, aftershocks, swarms, triggered events etc, which may not always be easy to distinguish. The ellipsoidal smoothing function generates patterns of dependent events displaying directional alignment with the causative primary fault rupture.

The kernel seismicity smoothing approach has been applied in many parts of the world where there is an extensive historical earthquake catalogue. Within Europe, there have been studies for UK, France, Spain, Switzerland and Italy. The application to Italy is especially important because of its high level of seismicity, long historical record, and seismically vulnerable building stock. Earthquake building codes have been upgraded over time, but this still leaves a construction legacy of numerous buildings which do not meet modern standards of earthquake-resistant construction. Primarily, the purpose of probabilistic seismic hazard assessment (PSHA) has been to determine ground motions for civil engineering design. But for existing buildings, PSHA is necessary for rational risk mitigation decision-making, specifically the prioritization of buildings to retrofit, when the budget available is sufficient only to mitigate the seismic risk for a small proportion of vulnerable buildings. This applies to churches, chapels and monasteries as well as homes.

Zuccolo et al. (2013) carried out a seismic hazard mapping study for Italy using the KERGRID program provided by the author. The mean PGA map is shown in Fig. 4. The highest acceleration values (>0.25g) are obtained in the Friuli region, Central–Southern Apennines, and around the Messina Strait, with the maximum value (0.357g) reached in the L’Aquila area. This map reflects the spatial non-uniformity of epicentres and accounts for the expectation that future large, damaging earthquakes are more likely to occur at sites near the epicentres of previous moderate-sized earthquakes. On the contrary, the hazard computed by seismic area zonation is strongly influenced by the polygonal geometry of the seismogenic zones adopted in the computation.
In 2016, a major seismic sequence began on 24 August, enclosing an area of the Apennines bounded by the 2009 L’Aquila sequence to the south and the 1997 Umbria-Marche sequence to the north. The area of Norcia was affected by a seismic sequence in 1979. The 24 August event was Mw 6.0. It caused the deaths of 297 people, of whom 234 were in Amatrice, 11 in Accumoli, and 49 in Arquata del Tronto. It is insightful to review the regional historical record: a distinctive cluster of four earthquakes occurred in 1627, 1639, 1646 and 1672. The 1639 earthquake laid waste to Amatrice and surrounding villages. Subsequently, a Mw 5.9 earthquake occurred about 3 km west of Visso on 26 October. This epicentre was a sufficient distance northwest of the epicentre of the 24 August earthquake for the intensity levels to correspond to the historical maximum for the area.

A few days later, on 30 October, a Mw 6.5 earthquake struck Norcia. The earthquake occurred early on Sunday morning at 7.40am, as those in religious orders were heading for morning prayer services. Had the earthquake occurred later, there would have been a severe death toll in ecclesiastical buildings;
weakened by repeated ground shaking, a number of these were wrecked. As it happened, there were no direct fatalities, although several related heart attack deaths were recorded. A detailed report was produced by INGV (2016), which shows that the epicentre of this earthquake was located close to the epicentres of notable historical earthquakes in 1328, 1719, 1730, 1815 and 1859.

For the primary event on 24 August, the subsequent August events of Magnitude 4.5 or higher are listed in Table 1. These aftershocks compounded the damage from the Mw 6.0 event. The possibility of even a small magnitude 4 earthquake causing the collapse of some buildings in Italy was demonstrated on the island of Ischia on 21 August 2017.

Table 1. Earthquakes of Magnitude 4.5 or higher in August 2016

<table>
<thead>
<tr>
<th>Date</th>
<th>Local Time</th>
<th>Mw</th>
<th>Municipality</th>
<th>Lat.</th>
<th>Lon.</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 August</td>
<td>03:36:32</td>
<td>6.0</td>
<td>Norcia</td>
<td>42.71</td>
<td>13.17</td>
</tr>
<tr>
<td>24 August</td>
<td>03:56:02</td>
<td>4.6</td>
<td>Amatrice</td>
<td>42.61</td>
<td>13.28</td>
</tr>
<tr>
<td>24 August</td>
<td>04:33:29</td>
<td>5.5</td>
<td>Norcia</td>
<td>42.79</td>
<td>13.15</td>
</tr>
<tr>
<td>24 August</td>
<td>13:50:31</td>
<td>4.9</td>
<td>Visso</td>
<td>42.87</td>
<td>13.11</td>
</tr>
<tr>
<td>24 August</td>
<td>19:46:09</td>
<td>4.6</td>
<td>Arquata</td>
<td>42.72</td>
<td>13.19</td>
</tr>
<tr>
<td>25 August</td>
<td>05:17:16</td>
<td>4.7</td>
<td>Norcia</td>
<td>42.78</td>
<td>13.18</td>
</tr>
<tr>
<td>26 August</td>
<td>06:28:27</td>
<td>4.7</td>
<td>Amatrice</td>
<td>42.66</td>
<td>13.25</td>
</tr>
<tr>
<td>28 August</td>
<td>17:55:36</td>
<td>4.6</td>
<td>Norcia</td>
<td>42.78</td>
<td>13.15</td>
</tr>
</tbody>
</table>

In zone-free approaches to seismic hazard analysis, the predicted areas of severe ground motion coincide with the zones subjected to a large number of past earthquakes reported in the seismic catalogue. This makes these approaches intuitively suitable for informing challenging decisions on allocating finance for building seismic safety improvements.

4. PRIORITIZATION OF SEISMIC RETROFIT IN ITALY

In the aftermath of the L’Aquila earthquake of 6 April 2009, the Italian government decided to provide funds for reducing seismic risk on a medium-term time scale. If there were a home earthquake insurance market in Italy, homeowners might decide for themselves and their families to seismically retrofit their property, so mitigating the seismic risk, and earning a reduction in earthquake insurance premiums as well. However, in Italy, the government is committed to paying the cost of damage to homes from natural hazards. Accordingly, the introduction of earthquake insurance premiums would be perceived as some sort of surrogate additional tax. As it turns out, after a major natural hazard event, the Italian government may well levy an additional tax, e.g. on car fuel, in order to raise money to pay for losses. Efforts have been made to explain, in the Italian language, the merits of insurance as an efficient means of encouraging risk mitigation (Woo, 2013). Clearly, the incentive for citizens to spend their own money to reduce seismic risk is greatly diminished if the government is expected to foot the bill for losses anyway.

Given the constraints on Italian public expenditure, and the lack of home earthquake insurance, the budget allocation for retrofitting poses a challenging problem for public policy. In order to be able to justify a chosen allocation, a systematic objective method for prioritization is needed; one which allocates the fixed budget in an optimal way, maximizing the retrofitting benefit. This involves evaluating the respective costs and benefits of alternative risk mitigation actions, taking account of the time horizon for assessing the retrofitting benefits. The mathematical formulation of the prioritization process falls within the class of knapsack problems,
which are well known in combinatorial optimization (Lombardi et al., 2014). Searching for an optimal solution turns out to be both informative and instructive, because some individual retrofit opportunities with high benefit-cost ratios may not be funded. The finding that the optimal allocation is far from obvious or intuitive makes further studies of this kind worthwhile in the future. The principal inputs for an optimal allocation algorithm are the seismic hazard, and the time horizon for a cost-benefit analysis. From a political decision-making perspective, a short time horizon of about 10 years is preferable to a time horizon of several decades, because the benefit would accrue during an extended term of individual political office, which would maximize the political return on safety investment.

A short time horizon encourages the prioritization of high hazard areas of Italy, compared with modest hazard areas, such as around Milan. The kernel smoothing seismic hazard map is especially well suited for use as input for a prioritization cost-benefit study, because the smoothing algorithm is based on the general principle that earthquakes tend to occur in the areas where they have occurred before. Furthermore, whilst the inclusion of active faults is essential for a comprehensive seismic hazard assessment, if the return periods for rupture recurrence are measured in many hundreds or even thousands of years, nearby buildings should cede in the queue for urgent retrofitting to buildings close to areas badly damaged by historical earthquakes and their aftershocks.

Kernel smoothing methods have their origin in earthquake forecasting, and indeed are being adapted for this purpose (Werner et al., 2013). In terms of having a shorter term payoff for seismic risk mitigation expenditure, the kernel smoothed seismic hazard model would be advantageous to use for the prioritization of limited budgets.

In general terms for a short-term 10 year time horizon, retrofitting should be prioritized in areas with a historical record of damaging earthquakes, such as the highest hazard areas shown in Figure 5. The size of event to be mitigated would not be an occasional large earthquake of magnitude 7+, but rather an event of magnitude 6, accompanied by aftershocks and other triggered events of magnitude 5+. Implementation of such a policy in 2010 might already have mitigated some of the damage from the earthquakes of 2016, and might potentially have reduced the tragic casualty toll.

5. CONCLUSIONS

The inclusion of foreshocks, aftershocks and other dependent events should be an integral part of probabilistic seismic hazard assessment. The kernel smoothing methodology provides a straightforward and efficient means for achieving this goal. The primary application of probabilistic seismic hazard assessment has been to calculate ground motion exceedance probabilities for engineering seismic design purposes. But besides this, there is another important practical application, which is to facilitate decisions on prioritizing buildings for retrofit, according to a cost-benefit analysis.

For this urgent practical application, kernel smoothed seismic hazard maps are particularly well adapted, since the underlying seismic source model is designed to characterize historical seismicity quite closely. The kernel smoothing methodology is based on using the full non-Poissonian earthquake catalogue, including aftershocks and triggered dependent events. From a seismic design perspective, the ground motions from subsidiary events might typically be enveloped by those of the main shock. However, from the risk mitigation perspective, the main shock might cause damage to a building, leaving some elements, e.g. parapet or chimney, in a precarious position, which a subsidiary event might destabilize. Earthquake casualties have been caused in this manner. An important country for application of kernel smoothing methods is Italy, where there is a historical legacy of numerous buildings in need of retrofit, located in regions where damaging earthquakes have been documented to occur. The occurrence in 2016 of a complex spatial sequence of dependent Apennine earthquakes is evidence of the need to allow for the inter-event connectivity of seismicity in regions of dispersed population density and significant earthquake vulnerability. The application of this non-Poissonian approach to addressing such complex sequences remains a research topic for the future.
6. REFERENCES


