A COMMENT ON NONLINEAR TIME HISTORY ANALYSIS REGULATIONS OF SEISMIC CODE OF NEW ZEALAND APPLICABLE IN EUROCODE 8 AND MANY OTHER SEISMIC CODES

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ABSTRACT

Time history analysis, addressed in almost all seismic codes, is the most versatile tool in analysis of seismic behaviors. The seismic code of New Zealand is an advanced code, with unique details on time history analysis. In this paper, the details of the seismic code of New Zealand, on step size selection and nonlinear time history analysis are reviewed. It is displayed that the reliability of the final response may deteriorate in the sense that the resulting responses may be considerably sensitive to analysis parameters. Meanwhile, it is claimed that this shortcoming depends on the severity of the nonlinear behavior (as well as the earthquake record). Two methods to overcome the shortcoming are suggested and briefly compared. The good performance of the superior method is displayed and its consistency with Eurocode 8 as well as other seismic codes is explained.

Keywords: Time history analysis; Seismic code; Integration step size; Nonlinearity tolerance; Accuracy

1. INTRODUCTION

For time history analysis, the structural dynamic behavior should be expressed as a mathematical model, and using a method for discretization in space, e.g. finite elements, the mathematical model should be simplified to a set of second order differential equation, typically stated as the governing equation:

\[ M \ddot{u} + f_{int} = -M \Gamma \ddot{u}_g \quad 0 \leq t \leq t_{end} \]  \hspace{1cm} (1)

the initial conditions,
\[ u_0 = \dot{u}_0 = f_{int,0} = \dot{O} \]  \hspace{1cm} (2)

and the restrictions originating in nonlinearity, \( Q \),
\[ Q = Q(u, \dot{u}, \ddot{u}, f_{int}) \leq 0 \]  \hspace{1cm} (3)

In Equations 1, 2, and 3, \( M \) and \( f_{int} \) imply the mass and the internal force respectively, \( u \) represents the displacement of different degrees of freedom with regard to the ground, each top dot denotes once differentiation with respect to time, \( t_{end} \) implies the time interval under study, \( \ddot{u}_g \) stands for the ground acceleration (scalar when multi-support excitation is not a concern, and vector otherwise), '0' as a right subscript indicates that the argument is in its initial condition (\( t = 0 \)), and \( \Gamma \) is a vector or matrix displaying the static effect of the ground (support) displacement on the displacements of un-supported degrees of freedom (vector when \( \ddot{u}_g \) is scalar, and matrix when \( \ddot{u}_g \) is vector).

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The next stage of the time history analysis is to consider several records for $\mathbf{\tilde{u}}_g$ consistent with the location of the structure, analyze the problem represented by Equations 1, 2, and 3, numerically, for each $\mathbf{\tilde{u}}_g$, and obtain the target responses from the several results. Most of seismic codes have regulations regarding records selection (number and characteristics), target response, and the way the final target response is obtained from the responses corresponding to each record, e.g. average or maximum of peaks. However, the only code discussing the above-mentioned "analysis" seems to be the seismic code of New Zealand, i.e. (NZS 2004 a) (Amiri and Soroushian 2017, Soroushian 2017). This advancement is to be sincerely acknowledged and the authors express their very congratulations to the authorities in charge of the New Zealand seismic code.

According to this code (NZS 2004 a and b), for analysis against each earthquake record, $\mathbf{\tilde{u}}_g$, the integration step size should be obtained from

$$
\Delta t \leq \text{Min} \left( \frac{\Delta t}{\chi}, \frac{T_1}{T_n}, 0.01, \Delta t_{\text{con}} \right)
$$

(4)

where $\Delta t$ is the step size by which $\mathbf{\tilde{u}}_g$ is digitized, $T_1$ stands for "the largest translational period of the first mode (judged by largest mass contribution) in the direction of principal component of the earthquake", $T_n$ implies the period of the highest mode, in the same direction as $T_1$, required to achieve the 90 % mass as described in the modal response spectrum method, $\chi$ is available from

$$
\chi = \begin{cases} 
100 & \text{when the nonlinear behavior is not involved in impact} \\
1000 & \text{when the nonlinear behavior is involved in impact}
\end{cases}
$$

(5)

and $\Delta t_{\text{con}}$ implies the largest step size, guaranteeing convergence to the exact response. As explained in the commentary of the code of New Zealand (NZS 2004 b), and in view of the practical considerations, Equation 4 can generally be simplified to

$$
\Delta t \leq \text{Min} \left( \frac{\Delta t}{\chi} \right)
$$

(6)

If the analysis is halted before $t=t_{\text{end}}$, the code of New Zealand suggests a second analysis with half steps, and the process is being continued till the target response is obtained in the interval $0 \leq t \leq t_{\text{end}}$. Then, the analysis should be again repeated with half steps once or by times till the peaks of target response in two sequential analyses differ less than 5 %, at which stage, the analysis with larger steps (from the two last analyses) is considered as the final analysis. An assumption in this procedure is that analyses carried out with smaller integration steps are more accurate. In view of various statements in the literature, and as typically displayed in Figure 1, this assumption is not necessarily correct specifically in nonlinear problems, e.g. see (Belytschko and Hughes 1983, Low 1991, Rashidi and Saadeghvaziri 1997, Soroushian et al. 2005, Xie and Steven 1994). The computed responses can hence be sensitive to integration method, nonlinearity tolerance, etc.

![Figure 1. Typical errors change when the integration steps are not large: (a) linear, (b) nonlinear](image-url)
The objective in this paper is to study this shortcoming of the code of New Zealand, and present methods and a practical comment to eliminate it, and study the applicability of the proposed method. First, the reason of the phenomenon is explained, the more severe cases are introduced, and methods are presented for elimination of the phenomenon (shortcoming). The suggested methods are then compared, leading to a practical comment, the consistency of which, with different seismic codes including Eurocode 8, is briefly discussed. Finally, the paper is concluded with a brief set of the achievements.

2. THEORY

2.1 An Explanation for the Phenomenon

Responses obtained from nonlinear time history analysis, with or without implementation of a nonlinearity iteration method, depend on the integration step size, and the nonlinearity residuals. Addressing, the response by \( R \), and the step size and the residual with \( \Delta t \) and \( \delta \),

\[
R = F(\Delta t, \delta)
\]  

(7)

where, \( F \) is a representation of the nonlinear time history analysis, we obtain

\[
\Delta R = \frac{\partial F}{\partial \Delta t} \Delta(\Delta t) + \frac{\partial F}{\partial \delta} \Delta \delta
\]  

(8)

leading to

\[
\frac{\Delta R}{\Delta(\Delta t)} = \frac{\partial F}{\partial \Delta t} + \frac{\partial F}{\partial \delta} \frac{\Delta \delta}{\Delta(\Delta t)}
\]  

(9)

Considering that, when round off is negligible, provided convergence, the exact response is available at the vicinity of zero integration steps, i.e.

\[
\lim_{\Delta t \to 0^+} R = R_{\text{exact}}
\]  

(10)

Equation 9 results in

\[
\frac{R - R_{\text{exact}}}{\Delta t} \bigg|_{\Delta t \to 0^+} = \left( \frac{\partial F}{\partial \Delta t} + \frac{\partial F}{\partial \delta} \frac{\Delta \delta}{\Delta(\Delta t)} \right)
\]  

(11)

In absence of nonlinearity the changes of errors with respect to \( \Delta t \) is broadly discussed, and convergence to exact response (Belytschko and Hughes 1983), and even the proper convergence (Soroushian 2010 a), can be maintained. Accordingly, Equation 11 implies that, in order to guarantee obtaining more accurate responses from nonlinear time integration analysis with smaller steps, some considerations should be taken into account regarding the nonlinearity. The lack of such considerations in the code of New Zealand results in the probable incorrectness of the assumption of more accuracy for responses obtained from time integration analysis with smaller steps. The consequence can be sensitivity of the responses to different analysis parameters including the integration method and the nonlinearity analysis details.

2.2 More Severe Cases

Obviously, the core of the explanation in the previous section is the role and contribution of \( \delta \) in the response; see Equation 7. Therefore, in view of the conventional broadly accepted relations below (Allgower and Georg 1980, Belytschko et al. 2000, Soroushian et al. 2013, Wriggers 2008):
\[ \| \delta_i \| < \delta_i \leq K \]

at each detection of nonlinearity, where the new parameters \( i \), \( \delta \), and \( K \), respectively stand for the number of iterations, the nonlinearity tolerance, and the maximum acceptable number of iterations, the assumption implied in the code of New Zealand (NZS 2004 a and b) would be more correct, when \( \delta \) is smaller and \( K \) is larger. In other words, better modeling of nonlinearity and more closeness of the mathematical and numerical models enhance the validity of the assumption of correspondence between more accuracy and analysis with smaller steps; of course depending on the computational facilities and round-off, limitations exist on the smallness of \( \delta \) and largeness of \( K \).

Returning to the mathematical model and the behavior of the structural system (see Equations 1, 2, and 3), it is reasonable to expect more contribution of \( \delta \) in Equations 7 and 8, when nonlinearities are more severe. (Although, no broadly accepted measure exists for the severity of nonlinear behavior, measures are introduced in the literature e.g. see (Soroushian et al. 2006).) Accordingly, in analyses involved in more severe nonlinearity, the assumption may be violated more. Consequently, the assumption of more accuracy in analyses with smaller integration steps might be doubted, specifically when, (1) \( \delta \) is larger, (2) \( K \) is smaller, and (3) the nonlinear behavior is more severe.

### 2.3 Two Methods to Circumvent the Weak Point

In view of the consequence of the previous section, a way to circumvent the weak point is to use sufficiently small values of \( \delta \). For single degree of freedom systems, the event-to-event nonlinearity solution method (Bernal 1991) can guarantee \( \| \delta \| = \delta = 0 \) and no role for nonlinearity residuals in Equation 7 leading to the validity of the assumption of more accuracy in analysis with smaller steps. Nevertheless, in general MDOF analysis \( \delta \neq 0 \). Meanwhile, "sufficiently small" is a vague expression, implying differently in different analyses. Therefore, implementation of sufficiently small nonlinearity tolerances can be materialized only with trial and error, taking into account the equivalence between convergence and pseudo convergence (Soroushian et al. 2013).

Alternatively, in order to circumvent the ambiguities in arriving at more accuracies, in analysis with smaller steps, we can control the nonlinearity residuals, \( \delta \), throughout the repetitions of the analysis (requested in the code of New Zealand). In this regard, a suggestion is to assign values to \( \delta \) entailing

\[ \delta \propto \Delta t \]

such that the relative contribution of the two terms at the right hand side of Equation 8 (or equivalently Equation 11) can be kept unchanging throughout the sequential analyses (Soroushian et al. 2013). Consequently, the validity of the assumption would be as linear problems. To arrive at such a relation between \( \delta \) and \( \Delta t \), a way is to define the \( \delta \) of each analysis repetition based on the value of \( \delta \) in the previous repetition, e.g. as discussed in (Soroushian et al. 2013),

\[ j \delta_i = \frac{1}{2^q} \min_{\Omega_j} \delta_{i-1} \]

where, \( \Omega_j \) is the \( j^{th} \) section of the imaginary space consisted of all degrees of freedom (and structural members) and the probable nonlinearities, i.e.

\[ \Omega = \Omega_1 \cup \Omega_2 \cup \ldots \cup \Omega_j \]

\[ \forall j_1, j_2, 1 \leq j_1, j_2 \leq J: \quad \Omega_{j_1} \cap \Omega_{j_2} = \emptyset \]

\( q \) represents the order of accuracy (Belytschko and Hughes 1983, Lambert 1973), \( j \delta_i \) denotes the tolerance
in the $i^{th}$ repetition of time integration analysis (according to the code of New Zealand) taken into account in $\Omega_j$, $\Omega$ stands for the total space of degrees of freedom and probable nonlinearities, and $J$ is the number of the sections in $\Omega$.

### 2.4 Comparing the Proposed Methods and Presenting a Practical Comment

Since, in consistence with the code of New Zealand (NZS 2004a), in implementation of either of the two methods suggested in Section 2.4, the analyses should be carried out sequentially, continuation of the analyses till the end of the time interval, i.e. $t_{end}$, is of high significance in both methods. However, this is not necessarily the case for all nonlinear analyses. (This also implies a drawback in the code of New Zealand; see (Amiri and Soroushian 2017, Soroushian 2017).) To overcome this obstacle, an approach is to pay attention to the notion of nonlinearity residuals in nonlinear dynamic analysis and do not stop the analysis, even when Equation (12) is not satisfied (Soroushian et al. 2015).

Then, in comparison of the proposed two methods, while in order to completely define the first method the value of sufficiently small tolerance should be cleared, for the second method, the value of nonlinearity tolerance in the first analysis should be set. Considering these, and that, in view of Equations 12 and 14, the unknown value of the first method is much more effectual compared to that of the second method, the second method sounds more practicable and hence superior. With no specific sensitivity, the unknown value of the second method can be simply set as

$$j, 0 = 10^{-2} \quad j = 1, 2, \ldots, J$$

(16)

Meanwhile, to eliminate the need to compute and record the $\delta_{i+1}$ in Equation 14, in view of Equation 12 and the non-stop analysis explained in the start of this section, it is reasonable to replace Equation 14 with

$$j, 0 = \frac{1}{2^q+1} j, 0 = 1, 2, \ldots, J$$

(17)

or since the right hand side of Equation (16) is independent from $j$,

$$j, 0 = 10^{-2}$$

(18)

Consequently, as a comment regarding the New Zealand seismic code NZS (2004 a and b):
(1) In the first nonlinear time integration analysis and its repetitions, the tolerance is to be set according to Equations 14 and 16 or according to Equation 18.

(2) In the nonlinear time integration analyses, even if Equation 12 is not satisfied, the analyses are to be continued till $t = t_{end}$.

### 3. NUMERICAL EVIDENCE

Consider the shear frame model and the earthquake excitation record addressed in Figure 2, where the details of the model are reported in Table 1, $g = 9.81 m/sec^2$, and the classical damping matrix is defined based on the constant mass $M$ and the actual (changing) stiffness matrix $K$. In view of Figure 3, the behavior is nonlinear. The top drift is considered as the target response, and the time history analysis is carried out according to the New Zealand code, several times, taking into account three integration methods and three values of nonlinearity tolerance. The analyses are carried out once considering the Modified Newton Raphson method (Allgower and Georg 1980, Cook et al. 2002) and once using the Fractional Time Stepping method (Mahin and Lin 1983, Nau 1983). The results are as reported in Table 2 and 3, where the final values of the target response according to the New Zealand code are stated in bold. As apparent in Tables 2 and 3 depending on
Figure 2. Structural system under numerical study: (a): Shear frame, (b): Earthquake record

Table 1. Properties of the shear frame in Figure 2.

<table>
<thead>
<tr>
<th>Floor (with 3 meters height each)</th>
<th>Mass (\times 10^{-3})</th>
<th>Stiffness (\times 10^{-8})</th>
<th>Yield displacement (\times 10^3)</th>
<th>Damping [C = \mu M + \lambda K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1036</td>
<td>129</td>
<td>1.12500</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1034</td>
<td>11.5</td>
<td>1.09375</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1032</td>
<td>102.5</td>
<td>1.06250</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1030</td>
<td>78.75</td>
<td>1.03125</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1028</td>
<td>59.50</td>
<td>1.00000</td>
<td>(\mu = 0.3801808471)</td>
</tr>
<tr>
<td>6</td>
<td>1026</td>
<td>49.50</td>
<td>0.96875</td>
<td>(\lambda = 0.0053331127)</td>
</tr>
<tr>
<td>7</td>
<td>1024</td>
<td>40</td>
<td>0.93750</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1022</td>
<td>31</td>
<td>0.90625</td>
<td></td>
</tr>
</tbody>
</table>

Twice the top drift after multiplying the excitation history by 0.5

Figure 3. Checking the nonlinearity of the behavior in view of the superposition principle
Table 2. Target values obtained from time history analysis of the structural system introduced in Figure 2 and Table 1 according to the seismic code of New Zealand, using Modified Newton Raphson nonlinearity iterations.

<table>
<thead>
<tr>
<th>Integration Method</th>
<th>$\delta = 10^{-2}$</th>
<th>$\delta = 10^{-5}$</th>
<th>$\delta = 10^{-7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Acceleration (Newmark 1959)</td>
<td>0.002144</td>
<td>0.002145</td>
<td>0.002143</td>
</tr>
<tr>
<td>HHT ($\alpha = 0.1$, $\gamma = 0.6$, $\beta = 0.3025$) (Hilber et al. 1977)</td>
<td>0.002112</td>
<td>0.002113</td>
<td>0.002113</td>
</tr>
<tr>
<td>CH ($\rho_\omega = 0.5$) (Chung and Hulbert 1993)</td>
<td>0.002099</td>
<td>0.002092</td>
<td>0.002092</td>
</tr>
</tbody>
</table>

Table 3. Target values obtained from time history analysis of the structural system introduced in Figure 2 and Table 1 according to the seismic code of New Zealand, using Fractional Time Stepping nonlinearity iterations.

<table>
<thead>
<tr>
<th>Integration Method</th>
<th>$\delta = 10^{-2}$</th>
<th>$\delta = 10^{-5}$</th>
<th>$\delta = 10^{-7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Acceleration (Newmark 1959)</td>
<td>0.00099</td>
<td>0.00357</td>
<td>0.00221</td>
</tr>
<tr>
<td>HHT ($\alpha = 0.1$, $\gamma = 0.6$, $\beta = 0.3025$) (Hilber et al. 1977)</td>
<td>0.00143</td>
<td>0.00226</td>
<td>0.00226</td>
</tr>
<tr>
<td>CH ($\rho_\omega = 0.5$) (Chung and Hulbert 1993)</td>
<td>0.00052</td>
<td>0.00182</td>
<td>0.00052</td>
</tr>
</tbody>
</table>

the integration method and tolerance, the final target response may differ considerably, e.g. the deviation in Table 3, is as stated below:

$$\frac{0.00209 - 0.00225}{0.5(0.00209 + 0.00225)} \times 100 = 7.4\%$$

while it is also worth noting that the repetition of the analysis might be excessive, e.g. see $\delta = 10^{-2}$ in Table 3. Implementation of the proposed comment, while considering the behavior of each floor in the total time interval as one of the sections of $\Omega$ (considering nonlinearities in different floors in different time instants as $\Omega$, $\Omega$, ..., $\Omega$ corresponding to nonlinearities in different floors.), replaces Table 3 with Table 4.
Table 4. Performance of the proposed comment in analysis of the structural system introduced in Figure 2 and Table 1 with different integration methods.

<table>
<thead>
<tr>
<th></th>
<th>Average Acceleration</th>
<th>HHT ($\alpha = 0.1$, $\gamma = 0.6$, $\beta = 0.3025$)</th>
<th>CH ($\rho = 0.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Newmark 1959)</td>
<td>(Hilber et al. 1977)</td>
<td>(Chung and Hulbert 1993)</td>
</tr>
<tr>
<td></td>
<td>0.000994</td>
<td>0.00143</td>
<td>0.000524</td>
</tr>
<tr>
<td>0.00217</td>
<td>0.00225</td>
<td>0.002190</td>
<td></td>
</tr>
<tr>
<td>0.00220</td>
<td>0.00223</td>
<td>0.002180</td>
<td></td>
</tr>
</tbody>
</table>

Obviously by implementation of the proposed comment, not only the ambiguity in selection of $\delta$ and hence the sensitivity is eliminated, but also the sensitivity to the integration method is lessened; see Equation 19, and in view of Table 4, that the sensitivity implied in Table 4 can be addressed as:

$$\frac{0.00217 - 0.00225}{0.5(0.00217 + 0.00225)} \times 100 = 3.6\%$$

(20)

4. CONSISTENCY WITH SEISMIC CODES

Fifteen seismic codes including the seismic code of Canada, China, Chile, Eurocode 8, Greece, India, Iran, Italy, Japan, US, Mexico, New Zealand, Romania, Taiwan, and Turkey, are reviewed for time history analysis (Amiri and Soroushian 2017, Soroushian 2017). The analysis procedure of the New Zealand code specifically when starting from Equation 16 and the comment introduced in Section 2 seem simply implementable in all seismic codes of practice that consider time history analysis as an seismic analysis alternative.

5. CONCLUSION

The details of time history analysis in the seismic code of New Zealand are very advanced and worth sincere appreciation. In this paper, it is displayed that the results of time integration analysis according to the seismic code of New Zealand can be sensitive, especially with respect to analysis parameters. A comment is put forward to eliminate/lessen the sensitivity. Based on the proposed comment, the halt of analysis, because of unsuccessful nonlinearity iterations, need to be omitted, and the nonlinearity tolerances need to be decreased thoughtfully (e.g. based on Equation 18) while repeating the analyses. The proposed comments, as well as the procedure of time history analysis according to the seismic code of New Zealand, seem simply applicable in different seismic codes.

6. REFERENCES


