SOLUTION OF FAR FIELD PROBLEMS IN TIME DOMAIN BY DIRECT INFINITE ELEMENT PROCEDURE

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ABSTRACT

In dynamic soil structure interaction, most of the problems treating the near field discretization are usually solved by using finite element method which cannot be applied to simulate far field radiation energy in the infinite media.

This paper presents a new procedure to simulate far-field problems under non-vanishing time average loading in time-domain. Based on infinite element method using direct scheme with viscous boundary in absorbing layer form, this approach adds wave impedances to the adjacent finite element on the interface "finite-infinite element". In the proposed system, the interface is modelled by finite element ensuring the wave dissipation while the infinite element ensures the elastic recovery.

Correlations of the obtained results with published analytical and numerical solutions are presented. Using implicit integration scheme in time domain good performance is obtained compared to the viscous boundary or with extrapolation algorithm.

Keywords: Soil Structure Interaction; Infinite element; standard viscous boundary; Finite element method.

1. INTRODUCTION

In dynamic soil-structure-interaction, the simulation of the soil-structure system raises inevitably the question of the type of boundary to be used for the model. Indeed, in the absence of special precautions, these boundaries can become reflective surfaces that return the waves to the structure instead of transmitting them outside the model. Several numerical strategies such as boundary element method, viscous boundaries, local transmitting boundaries, superposition method, and infinite elements have been developed to treat these problems.

The coupling of infinite and finite elements can be used to simulate complex geometrical and material modeling in a realistic way by providing good alternative for boundary simulation in static cases. However, in dynamics the existence of multiple wave types invalidates the direct application of elasto static infinite elements. In this case the basic formulation must be coupled to terms representing dynamic part whose characteristics must satisfy the condition of radiation wave.

Most research related to infinite elements for wave problems has been done in the frequency domain (Medina and Penzien 1982, Chow and Smith 1981, Medina and Taylor 1983, Zienkiewicz et al 1983, Bettess & Bettess 1991, Yun et al 1995, 2007, Zhao and Valliappan 1993, Park and Watanabe 2004, Seo et al 2007, and Kazakov 2010, 2012), requiring cloning method for nonlinear dynamic problems involving the inverse of the fast Fourier transform to obtain the time domain results.

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The main research carried out in time-domain is due to Haggblad and Nordgreen (1987) for solving nonlinear problems. In this approach, an absorbing layer is added to an infinite element in the form of damping. Developed by Su and Wang (2013) and Edip et al (2013), this new concept has been validated through several dynamic applications.

Based mainly on Haggblad's approach, this paper presents the formulation and implementation of a new procedure using direct infinite element coupled with viscous boundary. From the viscous formulation, wave impedances are added to the adjacent finite element on the interface "finite-infinite element" ensuring the dissipation while the infinite element ensures the elastic recovery.

Using axisymmetric finite element, the performance of this method is tested under non-vanishing time average loading encountered in several dynamic problems.

2. FORMULATION OF DIRECT INFINITE ELEMENTS

Direct infinite element's interpolation functions can be obtained from a standard three noded finite element based on Lagrange polynomials (Bettess 1977, 1980, 1984) defined for i=1 to n-1. These shape functions are combined with decay functions to form the displacement functions. In 2D isoparametric representation, the formulation can be expressed by (Equations 1 and 2) :

$$u = \sum_{i=1}^{4} H^i u_{i,} \quad \text{and} \quad v = \sum_{i=1}^{4} H^i v_i \tag{1}$$

$$x = \sum_{i=1}^{4} N^i x_{i,} \quad \text{and} \quad z = \sum_{i=1}^{4} N^i z_i \tag{2}$$

Where x, y describe the geometry, represented by nodal coordinates x_i and z_i in the global system coordinate using standard interpolation functions $N^i(\xi,\eta)$, while the displacements u, v are expressed from nodal displacements u_i and v_i using functions of displacement $H^i(\xi,\eta)$.

According to the conventions of the Figure (1), the standard interpolation functions $N^i(\xi,\eta)$ can be expressed by (Equation 3) :

$$[N] = [-\xi \frac{(1-\eta)}{2} \; ; -\xi \frac{(1+\eta)}{2} ; \; \frac{(1+\xi)(1+\eta)}{2} ; \; \frac{(1+\xi)(1-\eta)}{2}] \tag{3}$$

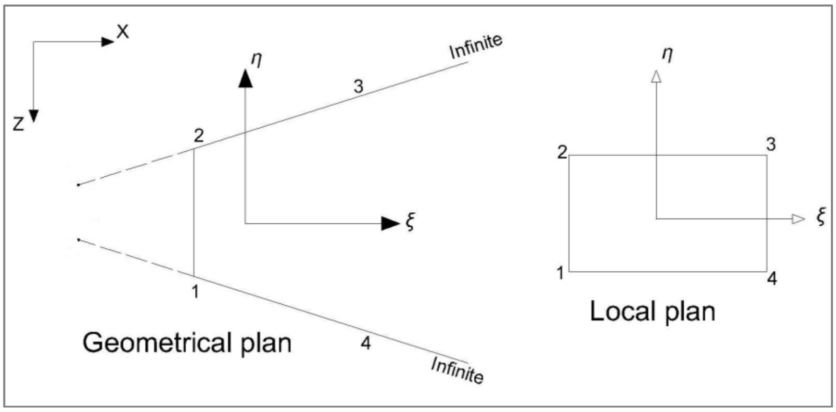


Figure 1. Local and global representation of an infinite element

Fixing the decay functions fi(ξ) in exponential form, the displacement functions of direct infinite element $H^i(\xi,\eta)$ are expressed by (Bettess 1977, 1980, 1984) (Equation 4) :

$$[H] = [\,-\xi\frac{1-\eta}{2}e^{\frac{-1-\xi}{L}} \quad -\xi\frac{1+\eta}{2}e^{\frac{-1-\xi}{L}} \quad (1+\xi)\frac{1+\eta}{2}e^{\frac{-\xi}{L}} \quad (1+\xi)\frac{1-\eta}{2}e^{\frac{-\xi}{L}}\,] \qquad (4)$$

Local coordinates η and ξ are defined such as $\eta \in [-1,+1]$ and $\xi \in [-1,+\infty]$, and related numerical integrations is done using Gauss-Laguerre quadrature (Pissanetzky 1983).

Shape functions must satisfy the following conditions:
- Radiation energy at infinity
- Completeness and consistency
- Finite values at infinity

That imposes to verify (Equations 5) :

$$\left[\begin{array}{l} f^i(\xi \to +\infty) = 0 \\ H^i(\xi_i,\eta_i) = 1 \\ \xi \in [-1,0] \Rightarrow \sum_{i=1}^{4} H^i \cong 1 \\ \xi \to +\infty \Rightarrow \sum_{i=1}^{4} H^i = 0 \end{array}\right. \qquad (5)$$

The severity of the decay is determined by the exponential decay length, L. The value of the exponential decay length can be obtained if something is known about the behavior of the exterior solution for example its Green's function. By simplification, L can be set to unity, or as the distance between the nodes (Bettess 1980).

Stiffness and mass matrices for the direct infinite elements

In the same way as for the finite elements, the stiffness matrix $[K^e]$ for an element is given by the virtual work theorem (Equation 6) :

$$[K^e] = \int_v [B]^T [D][B]\,dv = \int_{-1}^{+1}\int_{-1}^{+\infty} [B]^T [D][B] \det[J]\,t.d\eta d\xi \qquad (6)$$

[B]: deformation matrix.
[D]: material matrix.

Referring to the approach proposed by Hinton et al. (1976), the evaluation of the mass to be allocated to each node of the infinite element is based on the concept of consistent mass matrix [Mc]. The numerical integration is done by the quadrature Gauss-Laguerre (Pissanetzky 1983).

3. IMPLEMENTATION OF THE VISCOUS BOUNDARY IN THE METHOD PROPOSED

Developed and implemented with success by Lysmer and Kuhlemeyer (1969), Viscous boundaries became widely used for the simulation of soil-structure interaction. To avoid reflection of inducing waves, normal and shear stresses are used for this purpose in the two-dimensional plane strain model (Figure 2). The stresses on the nodes of the border are expressed by (Equation 7):

$$\sigma = -a\rho.V_p.\dot{u}_n$$
$$\tau = -b\rho.V_s.\dot{u}_t \qquad (7)$$

Written in following nodal form (Equation 8):

$$\begin{bmatrix}\sigma\\ \tau\end{bmatrix} = [D^*][\dot{U}_N] = \begin{bmatrix}a\rho V_P & 0\\ 0 & b\rho V_S\end{bmatrix}\begin{bmatrix}\dot{u}_n\\ \dot{v}_s\end{bmatrix} \qquad (8)$$

Vp and Vs are the velocities of P (compression) and S (Shear) waves, respectively.
[D*] is the matrix of nodal damping and $[\dot{U}_N]$ is the matrix of normal and tangential velocity.

Coefficients a and b suggested by White et al (1977), depend of the direction of the incident wave. To maximize the absorption rate of P and S wave of the viscous boundary we take a = b=1.

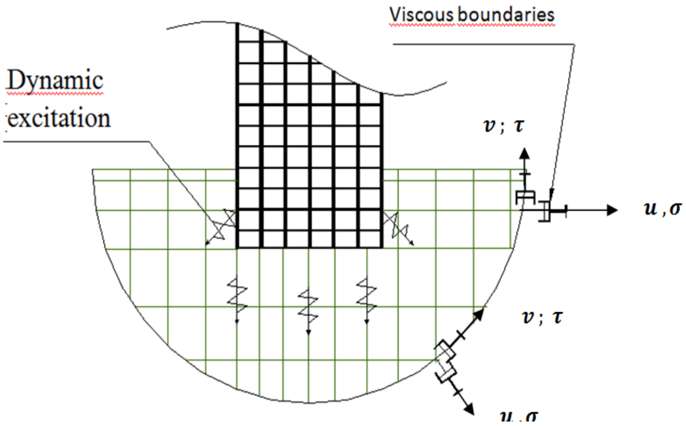


Figure 2. planar viscous boundary

Based on Haggblad and Nordgren (1987) formulation used recently by Su and Wang (2013) and Edip et al. (2013) for mapping infinite elements, the present scheme uses an absorbing layer in damping form with a direct infinite element to materialize the new boundary. The absorbing layer ensures wave dissipation while the elastic recovery is done through the direct infinite element. On the absorbing interface (Figure 3), the wave is transmitted from finite to the infinite element so the continuity of the stresses σ and τ is insured. Established for each side of the interface elements facing the waves propagating from the origin, the coherent damping matrix (representing the absorbing layer) is given by (Equation 9):

$$[C] = \int_s [H]^T [P]^T [D^*][P][H] ds \qquad (9)$$

Where P is the projection matrix relating normal and tangential velocities to the global Cartesian velocities components, while [H] is the matrix of displacement interpolation functions of interface finite element. The damping elements are calculated using a 2 points Gauss-Legendre numerical integration on each concerned side. Using diagonal damping matrix form, similar method basing on Hinton scheme can be used.

Hence, each type of dissipative system can be implicitly combined in the formulation of the global equations.

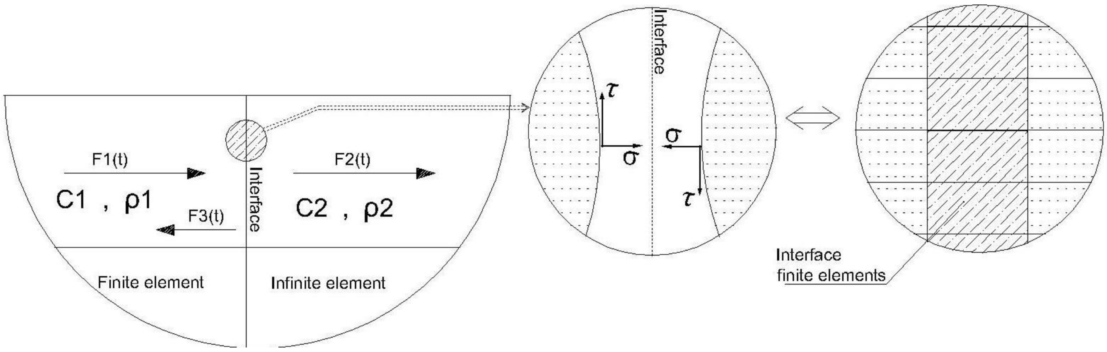


Figure 3. Absorbing interface schematization

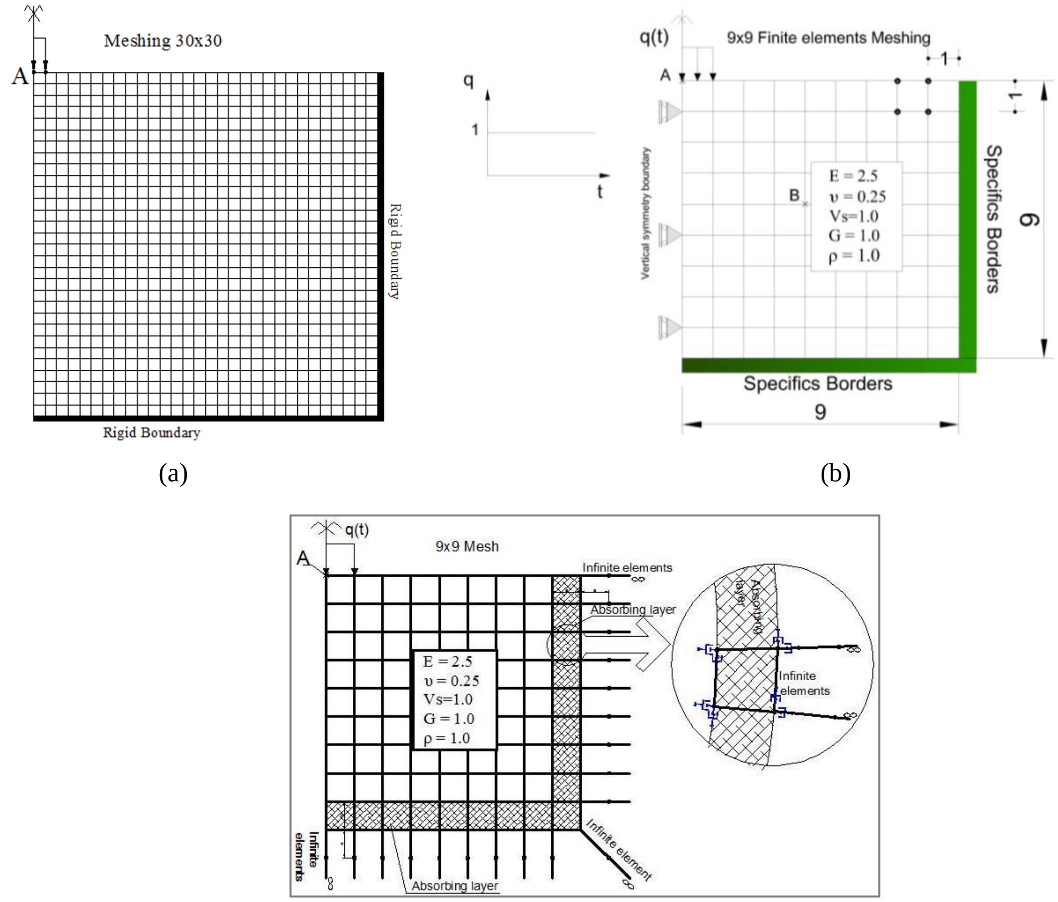
4. APPLICATIONS

The performance of the procedure under non-vanishing time average loading is carried out through original software developed with FORTRAN language. The region of interest is modeled by finite elements, while the far field domain is discretized using infinite elements coupled with an absorbing layer applied on the finite element interface. In the proposed software step by step integration algorithm with implicit procedure is implemented. Using axisymmetric finite element, the domain is modeled by four nodes isoparametric finite elements while the masses and rigidities are evaluated using Gaussian numerical integration.

Using the results from published work of Simons and Randolph (1986), Wolf (1988), Shridar and Chandrasekaran (1995), and Nour and Alam (2002), the present test applications consist of evaluating the response of a soil profile with a flexible base under vertical load having a non-vanishing time average. This loading can be encountered in some dynamic problems, such as the foundation for a transmission line tower when the cables on one side snap or pile driving analysis. For both selected applications, the loading is suddenly applied at time t=0 and maintained static thereafter. According to Fourier transform, the loading (heavy-side) gives low-frequency components. The corresponding wavelengths are large and in this case, finite element models are suitable for the solution of the problem.

4.1. First Application: homogeneous elastic half-space case

A sudden vertical load is applied over a circular area of unit radius on the surface of a homogeneous linear elastic half-space and then is maintained constant thereafter. This pulse could represent the effect of a sudden load on an elastic infinite medium.



(c)

Figure 4. (a) The large model, (b) The used model, (c) model of method proposed

The analytic solution given by Timoshenko and Godier (1951) is (Equation 10):

$$\delta = \frac{2qr(1-\upsilon^2)}{E} = 0.75 \qquad (10)$$

The reference model used to simulate far field medium is obtained by extending the mesh (extended model) with rigid boundary conditions, so that the wave after hitting the wall of reflection will not have time to disrupt the solution. For the extended model a 30x30 mesh is used (Figure 4.a) while for boundary testing, the referential model consists of a 9x9 mesh (Figure 4.b) on which various boundary conditions will be tested, such as rigid condition, viscous boundaries, infinite boundaries and the proposed procedure (Figure 4.c).

The finite element size Δx=Δz is taken equal to unity. With a propagation velocity Vp = 1.732 m/s and Vs = 1m/s.

The time integration is carried out with Δt=0.025 s. The complementary data used are α = 0.25, E = 2.5, Density = 1, and Thickness = 1.

From the vertical displacement at point A (Figure 5) given by the extended model, the response reaches a peak value and then settles to the static displacement value. Compared with the large model, the discrepancy with the rigid model (on mesh 9X9) appears clearly from the 12^{th} second and the solution becomes distorted therafter, confirming the inefficiency of this type of boundary in dynamic problems. The response given by the infinite elements oscillate around the static solution but the discrepancies start from the 22nd seconds. This solution tends to be linked to the reference solution with important peaks differences.

Starting from 25 seconds, the response obtained by viscous boundaries exhibits until 20 seconds an excellent behavior. After that the solution starts to deviate asymptotically from the true solution. Results given by the present procedure shows good agreement with the reference solution, and provides an excellent behavior until 50 seconds.

On the Figure 6, we can observe evolution of discrepancies through responses at point B. the obtained displacement confirm the stability of the solution by the proposed procedure compared to the extended model, while the viscous boundary's response diverges asymptotically.

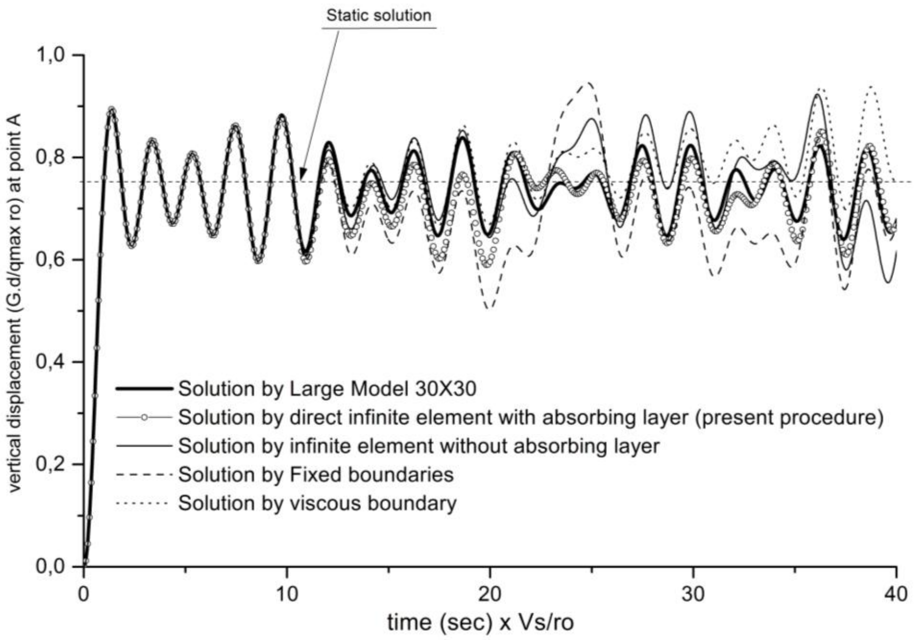


Figure 5. Comparison of verticals responses at point A

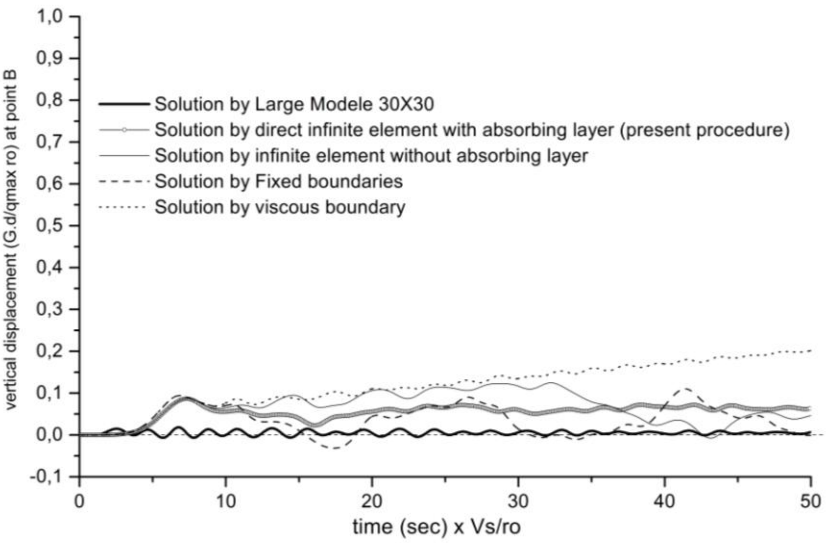


Figure 6. Comparison of verticals responses at point B

Using Simon and Randolf (1986) and Shridar and Chandrasekaran (1995) results, the new solution based on a refined mesh (in order to approach the 8 noded elements used by authors), shows good performance compared to those obtained by viscous boundary and extrapolation algorithm (Figure 7).

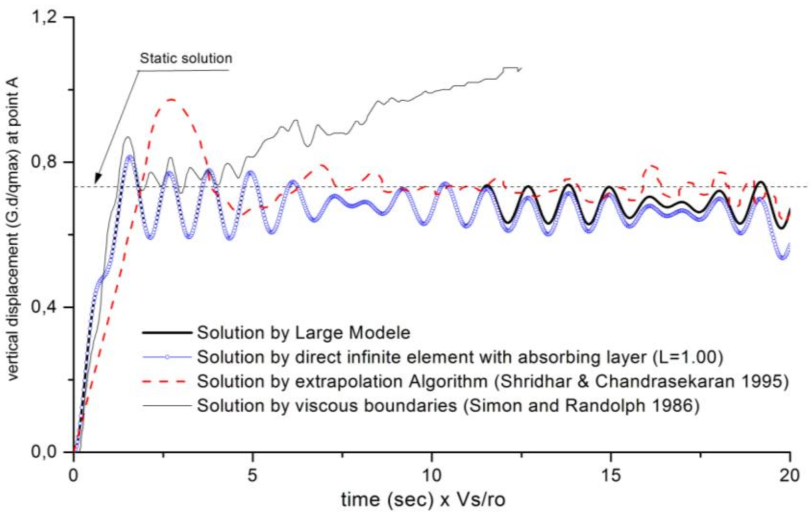


Figure 7. Comparison the responses with other procedures

4.2. Second Application: Response of a two-layered medium

Used by Shridar & Chandrasekaran (1995) for validation testings, this application considers a two-layered medium subjected to the same sudden load applied by a circular area of unit-radius as previously defined. In order to approach the 8 noded elements used by authors, the finite element size is taken as $\Delta x = \Delta z = 0.5$, that gives a 60x60 meshing for the reference model. The testing models used and geotechnical characteristics are given in Figure 8.

Highlighted by results of the previous application, solutions obtained by rigid boundaries and infinite elements without absorbing layers present divergences more important than which obtained with other presented procedures (Figure 9).

From 25 seconds, the response produced by the proposed method is better related to the reference solution than those obtained with the extrapolation algorithm or the viscous boundary (Figure 10).

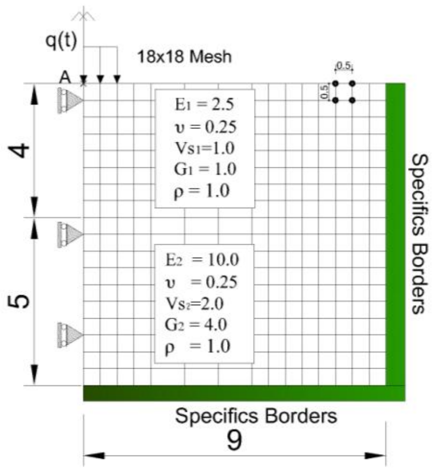


Figure 8. Two-layer model under a heavy side loading

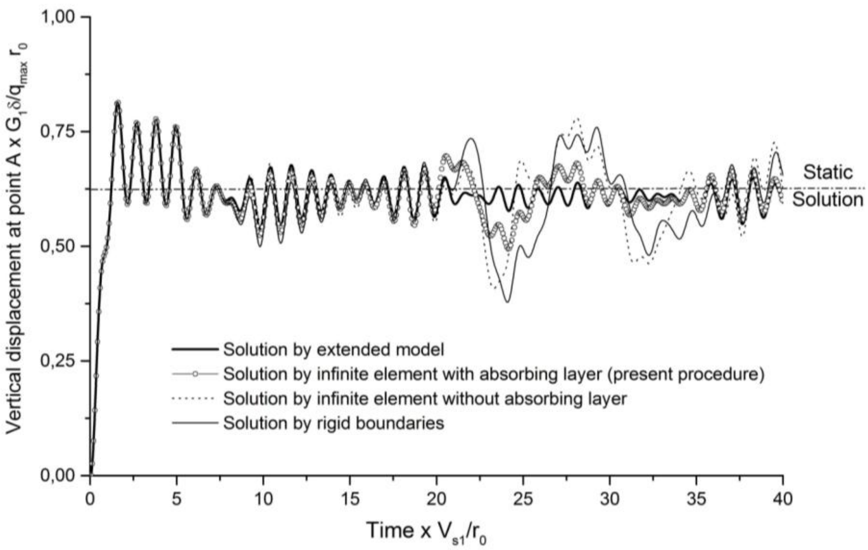


Figure 9. Comparison with responses obtained by direct infinite elements and rigid boundaries

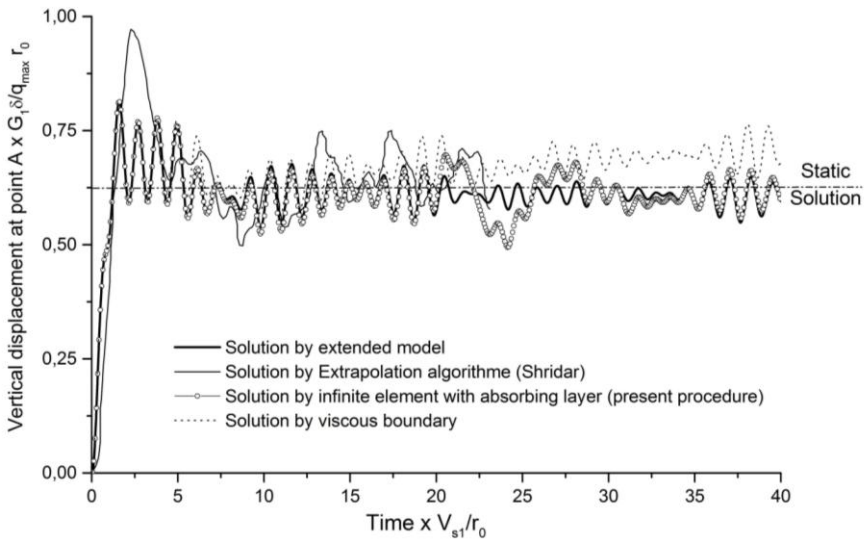


Figure 10. Comparison with responses obtained by extrapolation algorithm and by viscous boundary

5. CONCLUSIONS

The performance of a procedure to simulate far-field problems in time-domain has been examined for homogeneous and layered half-spaces subjected to suddenly applied vertical loading over a circular area on the surface of an elastic half-space. Starting from a direct infinite element and using an absorbing layer from viscous boundary method, the proposed procedure is easy to implement. Good performance with acceptable accuracy for non-vanishing time average loading is obtained.

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