INFLUENCE OF MATERIAL HETEROGENEITY AND RESERVOIR WATER LEVEL ON THE SEISMIC RESPONSE OF AN ARCH DAM-WATER-FOUNDATION SYSTEM

Maroua HAMMAMI¹, Régis COTTEREAU², Christine NORET³, Xavier MOLIN⁴, François HALGAND⁵

ABSTRACT

The dynamic response of an arch dam-foundation system to a recorded accelerogram is investigated considering randomly fluctuating material properties and different reservoir water levels. In the first section, only the foundation is modeled. Spatial variability of seismic ground motions is represented by a coherency function for both homogeneous and heterogeneous media. Results show that at zero frequency, the lagged coherency is different from unity, which can be explained by the influence of the valley topography. At low frequencies, the seismic response is more coherent in the left bank-right bank direction. However, at large frequencies, it is more similar in the direction of the valley. Randomly fluctuating material properties reduce the coherency function in all directions.

In the second section, the foundation is coupled to an arch dam-reservoir system. It is demonstrated that heterogeneity, typically ignored in engineering practice, can have an influence on the modal response of the structure and the stress distribution on its upstream and downstream faces. Indeed, even if randomization almost does not affect the first mode, it clearly induces a densification of the other modes whether the reservoir is empty or full. In particular, different values of the Young’s modulus on the two banks lead to more hoop stress on the dam crest.

Keywords: arch dam; rock foundation; coherency; heterogeneity; reservoir water level

1. INTRODUCTION

There always exist uncertainties in defining geomaterial mechanical parameters. This results from natural heterogeneity and limited availability of information about material properties. Indeed, using well-log data collected in various areas of the world, several authors [1-3] have constructed random models for these fields of mechanical properties. Not only deep reflection surveys and investigations of the continental crust in a variety of tectonic settings [4-5], but also travel-time tomography campaigns over the years [6] have proven that the crustal structure is laterally inhomogeneous. Besides, cone penetrometer tests CPT, extensive database of triaxial test results and realistic estimates of soil parameters variability [7-9] have identified horizontal correlation distance and consequently a scale of soil properties fluctuation between 1 m and 300 m and a vertical scale of fluctuation in the range of 1 m to 6 m. Thus, to cope with these inherent uncertainties, we used a probabilistic modeling in our seismic analysis.

Moreover, when a dam-reservoir is implemented, changes in the reservoir water level affect also the dynamic behaviour of the structure [10-11] since it changes the global mass of the system that influences the dynamic interaction between the water mass and both rock and structure. Furthermore, spatial variability of seismic ground motion can influence significantly the three-dimensional response of the foundation system to seismic excitation.
The objective of the paper is to present the seismic response of an arch dam-foundation reservoir system coupling the two aspects “randomly fluctuating material properties” and “different reservoir water levels”. It focuses on how they can impact the modal response of the structure and the stress distribution on the upstream and downstream faces. No other study combining these phenomena can be found in the literature. In the first section of our contribution, only the rock foundation is modeled. The spatial variability of seismic ground motions [13] due to dispersion is analysed. In the second section, the foundation is coupled to an arch dam and both of them are considered as isotropic and homogeneous. Then, material parameters are modeled as random fields [14]. The seismic response of the arch dam-water-foundation rock is evaluated and compared with the homogeneous case coupling the influence of material heterogeneities and the effect of the reservoir water level. Modal response and stress distribution are investigated.

2. NUMERICAL MODELING OF DAM-RESERVOIR-FOUNDATION INTERACTION IN THE TIME DOMAIN

2.1. Motion equations

The equilibrium equation is obtained by expressing that the sum of the rate of work by external forces and the rate of work by internal forces is equal to the rate of work by inertial forces.

\[ \nabla \cdot \sigma + f = \rho \frac{\partial^2 u}{\partial t^2} \]  

(1)

where \( \sigma \) is the Cauchy stress tensor, \( f \) the body forces, \( \rho \) the density and \( u \) the displacement. The isotropic linear elastic constitutive law for small strains is given by the Hooke's law:

\[ \sigma(u) = \lambda \text{div}(u)I + 2\mu \varepsilon(u) \]  

(2)

where \( \lambda \) and \( \mu \) denote the Lame constants, \( I \) the identity 2nd order tensor and \( \varepsilon \) the strain tensor. Considering Poisson's theorem, it is always possible to decompose the total displacement field \( u \) as the sum of a gradient of a scalar potential \( \Phi \) and the curl of a vectorial potential \( \Psi \):

\[ u(x, t) = \nabla \Phi(x, t) + \nabla \times \Psi(x, t) \]  

(3)

The pressure wave velocity and the shear wave velocity are given respectively by \( V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} \) and \( V_s = \sqrt{\frac{\mu}{\rho}} \).

2.2. Spectral element formulation

The code SEM (developed by CEA, IPGP and the laboratory MSSMAT of CentraleSupelec) is a finite element software based on a spectral formulation [15]. The displacement field \( u \) is given in each element in a basis of Lagrange polynomials \( \phi_i(x) \) at order \( N \). Using high-order polynomials guarantees the spectral convergence. These polynomials are defined on Gauss Lobatto-Legendre (GLL) points. Thus, \( \phi_1(x) = 1 \) on the GLLs and 0 on the other GLLs. The displacement field is given by:

\[ u(x, y, z) = \sum_{i,j,k} u_{i,j,k} \phi_i(x) \phi_j(y) \phi_k(z) \]  

(4)

The estimation of all integrals in each finite element yields the equation of motion in the following matrix form:

\[ M \ddot{U} + KU = F \]  

(5)
where $K$ is the stiffness matrix deriving from the elastic energy, $M$ the mass matrix derived from the kinetic energy, $U$ and $\dot{U}$ the vectors of displacement and acceleration. The mass matrix is given by:

$$M_{i,j} = \sum_k \alpha_k \phi_i(x_k) \phi_j(x_k)$$  \hspace{1cm} (6)

The originality of the spectral method results from the choice of the quadrature for the numerical evaluation of the finite element formulation. Gauss points are wisely identical to the nodes of the basic functions. Therefore, the mass matrix is diagonal.

$$\phi_i(x_k) \phi_j(x_k) = \delta_{ij}$$  \hspace{1cm} (7)

2.3. Explicit Newmark velocity scheme

We discretize the time interval of interest using a time step $\Delta t$. The semi-discrete momentum equation is written in the conservative form $t_{n+\frac{1}{2}}$ [16] :

$$\frac{1}{\Delta t} M(u_{n+1} - u_n) = F^\text{ext}_{n+\frac{1}{2}} - F^\text{int}_{n+\frac{1}{2}}(u_{n+\frac{1}{2}}, v_{n+\frac{1}{2}})$$  \hspace{1cm} (8)

where $u_{n+\frac{1}{2}} = u_n + \frac{\Delta t}{2} [v_n + v_{n+1}]$, $v_{n+\frac{1}{2}} = \frac{1}{2} [v_n + v_{n+1}]$ and $F^\text{ext}_{n+\frac{1}{2}} = \frac{1}{2} (F^\text{ext}_{n+1} + F^\text{ext}_n)$

$M$, $u$, $v$, $F^\text{ext}$ and $F^\text{int}$ denote respectively the mass matrix, displacement, velocity, external forces and internal forces. Unlike the implicit scheme, there is no need to assemble and invert the global mass matrix. The time step $\Delta t$ has to be smaller than the critical time $\Delta t_{cr}$ which in an undamped homogeneous system depends on the Courant number $c$ and the in the element size [17]:

$$\Delta t \leq \Delta t_{cr} = c \frac{\Delta t}{C_L}$$  \hspace{1cm} (9)

where $\Delta L$ is the smallest element size and $C_L$ the maximum wave velocity.

3. INFLUENCE OF SPATIAL VARIABILITY OF SEISMIC GROUND MOTION ON A ROCK FOUNDATION RESPONSE

This section is devoted to the analyses of the influence of spatial variability of seismic ground motions on the rock foundation (figure 2a) response. Neither the dam nor the reservoir is considered.

3.1. Model of the foundation

Foundation characteristics are given in the table below:

<table>
<thead>
<tr>
<th></th>
<th>$V_p$ (m/s)</th>
<th>$V_s$ (m/s)</th>
<th>$\rho$ (Kg/m$^3$)</th>
<th>Dimension (m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foundation</td>
<td>3353</td>
<td>2175</td>
<td>2200</td>
<td>3700 x 3505 x 2220</td>
</tr>
</tbody>
</table>

$V_p$ and $V_s$ denote respectively the pressure velocity and the shear velocity.

In this work, the seismic loading is derived from a recorded accelerogram (Bam earthquake, Iran 2003) corrected to a PGA equal to 0.2g in the valley. The input signal was calculated using a deconvolution between the bottom of the valley and the base of the model. The acceleration history of the seismic input is given through three accelerograms presented below.
Figure 1. Acceleration time histories in the directions X (left bank-right bank), Y (direction of the valley) and Z (vertical direction).

The seismic load is applied at the base of the foundation. To reduce spurious reflections at the medium boundaries perfectly matched layers "PMLs" are used [18-19]. An attenuation of 3% in the rock foundation is considered for the pressure waves. The quality factor of the shear waves is about half that of the pressure waves (irreversible behaviour of the shear waves) [20].

3.2. Dynamic analysis of a homogeneous rock foundation response

The spatial incoherence of seismic ground motions due to dispersion can generally be modelled in such analysis by a "coherency function" in frequency domain [21]. By definition, the coherency function $\gamma_{ij}$ is a complex normalized function obtained by the ratio between the cross spectral density (cross spectrum $S_{ij}$) and the power spectral density (power spectra: $S_{ii}$ and $S_{jj}$) of the observed accelerograms at i and j sensors:

\[
S_{ij}(w) = \frac{2\pi}{T} A_i^*(w) A_j(w) \tag{10}
\]
\[
S_{ii}(w) = \frac{2\pi}{T} A_i^*(w) A_i(w) \tag{11}
\]

where for a sensor $i$, $A_i(w)$ represents the Fourier transform of the acceleration time histories $a_i(t)$. $^*$ denotes the complex conjugate.

The coherency function generally depends on the frequency $w$ and station-separation distance $d_{ij}$. Its modulus term (amplitude) is called the "lagged coherency" and is given by:

\[
|\gamma_{ij}(d_{ij}, w)| = \left| \frac{S_{ij}(w)}{\sqrt{S_{ii}(w)S_{jj}(w)}} \right| \tag{12}
\]

This latter is a measure of "similarity or correlation" in the seismic motions that indicates the degree to which the data recorded at two stations i and j are related. It tends to unity as frequency tends to zero or motions are similar. To evaluate the spatial coherency of seismic ground motions in the rock foundation, we chose 24 sensors at the bottom of the valley, distributed 8 by 8 on 3 circles (C1, C2 and C3) with respective radius of 290 m, 980 m and 1280 m. The lagged coherency in the left bank-right bank direction is calculated considering acceleration time histories in this direction and is denoted by $|$Coherency – X$. That in the direction of the valley is denoted by $|$Coherency – Y$. 


Figure 2. (a) Vertical cross-section of the rock foundation. (b) Sensors position for the coherency study.

Figure 3. Lagged coherency of the foundation’s seismic response 1st circle (b) 2nd circle (c) 3rd circle.

Figure 3 shows that at zero frequency, the lagged coherency is different from unity. This can be explained by the influence of the valley topography.

Looking at figure 3.a, we note that at low frequencies the seismic response of the foundation is more coherent in the left bank-right bank direction (X). However, at large frequencies, it is more similar in the direction of the valley (Y).

The more sensors are far from the valley (figure 3.b and 3.c), the more motions become coherent in the direction of the valley than in the left bank-right bank direction.

The peaks of frequency at 3Hz (figure 3a), 3.7 Hz (figure 3b) and 3.2 Hz (figure 3c) correspond to the resonant mode of the valley.

Comparing figures 3a, 3.b and 3.c, one observes that coherency decays for long station separation distances (2nd and 3rd circles).

3.3. Influence of heterogeneity in the seismic response of the foundation

To quantify the influence of heterogeneity in the medium, probabilistic mechanical parameters can be used. The spectral representation is a classic way to sample gaussian random field. This latter is generated using the formula of discretization proposed by Shinozuka and Deodatis [22].

An example of 1V-2D (i.e 1 variable 2 dimensions) is given in the equation bellow:

\[
\begin{align*}
\text{f}_{\text{Gf}}(x_p, y_q) = \Re \left[ \sqrt{2} \sum_{k=0}^{M_x-1} \left( \sum_{l=0}^{M_y-1} \sqrt{2} S(k_{x,k}, k_{y,l}) \Delta k_x \Delta k_y e^{i \phi_{kl}} e^{i k_{y,l} y_q} \right) e^{i k_x x_p} \right] + \\
\Re \left[ \sqrt{2} \sum_{k=0}^{M_x-1} \left( \sum_{l=0}^{M_y-1} \sqrt{2} S(k_{x,k}, k_{y,l}) \Delta k_x \Delta k_y e^{i \psi_{kl}} e^{i k_{y,l} y_q} \right) e^{i k_x x_p} \right] 
\end{align*}
\]

(13)

where \((x_p, y_q)\) represent the position in the mesh \((x_p = p \Delta x \text{ and } y_q = q \Delta y)\). \((\Delta x, \Delta y)\) the dimensions of the mesh in the spatial domain, \(M_x\) and \(M_y\) the numbers of points in the spatial domain in both directions x and y, \(\phi\) and \(\psi\) the random independent phases, \((k_{x,k}, k_{y,l})\) the coordinates of points in
the frequency domain, \((\Delta k_x, \Delta k_y)\) the dimensions of the mesh in the frequency domain and \(S\) the spectral density function given in all points in the frequency domain. The function \(f_G\) is used successively to generate the Lamé coefficients and the isotropic model. The evaluation of the response variability due to system stochasticity consists of performing the response analysis of structural systems with gaussian correlation and log-normal first-order marginal distribution in their material properties. Standard deviations \(\sigma\) and correlation lengths \(L_c\) are given in the table below.

<table>
<thead>
<tr>
<th>(L_c (m))</th>
<th>(\sigma (%))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foundation</td>
<td>300</td>
</tr>
</tbody>
</table>

The same accelerograms, the same seismic attenuation and the same procedure of calculation as in the homogeneous case are considered. Only heterogeneous foundations where there are almost the same orders of size of the Young modulus in the middle of the valley are chosen to avoid dispersions due to heterogeneities when it will be coupled with the arch dam.

Figure 4. Young Modulus on a vertical cross-section for 8 chosen realizations.

Figure 5. Lagged coherency of the foundation’s seismic response considering heterogeneity C1 (a) in the left bank-right bank direction (X) (b) in the direction of the valley (Y)

Figure 5 shows the variation of the coherency both in the homogeneous (black curve) and the heterogeneous (colored curves) rock foundation models. Comparing with results in the homogeneous case, one can remark the decay of coherency in the random cases. We note also the peak of frequency around 3 Hz in most of the models which corresponds probably to a resonance mode in the left bank-right bank direction.
4. SEISMIC RESPONSE OF A HOMOGENEOUS DAM-FOUNDATION-RESERVOIR SYSTEM

This section is devoted to the dynamic analysis of an arch dam - foundation system considering different reservoir water levels and homogeneous material properties.

4.1. Physical model

Material characteristics and geometry are listed in the table below:

<table>
<thead>
<tr>
<th>Material</th>
<th>$V_p$ (m/s)</th>
<th>$V_s$ (m/s)</th>
<th>$\rho$ (Kg/m$^3$)</th>
<th>Dimension (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foundation</td>
<td>3353</td>
<td>2175</td>
<td>2200</td>
<td>3700 x 3505 x 2220</td>
</tr>
<tr>
<td>Arch dam</td>
<td>3472</td>
<td>2195</td>
<td>2400</td>
<td>60 base x 220 height x 300 crest</td>
</tr>
<tr>
<td>Reservoir</td>
<td>1500</td>
<td>-</td>
<td>1000</td>
<td>200 x 2500 x 220</td>
</tr>
<tr>
<td>Bedrock</td>
<td>4500</td>
<td>2470</td>
<td>2600</td>
<td>3700 x 3505 x 1000</td>
</tr>
</tbody>
</table>

The same accelerograms, the same procedure of calculation as in the 3$^{rd}$ section are considered. The seismic load is applied at the base of the foundation. An attenuation of 3\% in the foundation and 5\% in the dam is considered for the pressure waves. The quality factor of the shear waves is about the half that of the pressure waves (irreversible behaviour of the shear waves).

4.2. Mesh properties

The hexahedral mesh was generated using the free software Gmsh [23]. The model contains over 25 millions elements. Computations were run in parallel with 168 processors.

![Mesh model](image)

Figure 6. Mesh model.

4.3. Results and interpretations

4.3.1. Analysis’ method

The seismic load is usually measured at the base of the dam. To take into account the interaction between the dam and the foundation, we need to quantify the acceleration at the base of the model that generates the same response at the base of the arch dam. Thus, first, the measured accelerations are applied at the base of the foundation (neither the dam nor the reservoir is considered). Then, the response obtained across the valley is deconvoluted. Finally, the deconvoluted signals are applied to the arch dam-foundation-reservoir system for all models (homogeneous / heterogeneous realizations).

4.3.2. Seismic response at the bending axis of the arch dam’s main section

Comparing the acceleration response at the base and on the dam crest, one observes an amplification on the crest (see figure 3).
It reaches 2 times the response at the base in the left bank-right bank direction for both models (empty / full reservoir). In the normal direction of the arch, the highest acceleration on the crest is roughly 13 times that at the base when the reservoir is empty and 10 times when it is full.

4.3.3. Modal analysis

In order to analyze the behaviour of the dam under the seismic load, time responses are converted in the frequency domain by performing the Fast Fourier Transform. We choose 42 equidistant sensors on the dam crest and a reference station at its base (see figure below).

![Sensor positions](image)

Figure 8: Sensors positions.

For the evaluation of the modal frequencies of the structure, the spectrum ratio between each sensor and the reference station were calculated. Then, drawing the sum of these ratios, function of frequency, it is possible to select the peaks that correspond to eigenfrequencies. This procedure was done separately for all directions: left bank-right bank, downstream-upstream and vertically (refer to our paper presented in the benchmark workshop of the ICOLD 2017 [24]. The first 10 eigenfrequencies are reported in the table below.

<table>
<thead>
<tr>
<th></th>
<th>F1 (Hz)</th>
<th>F2 (Hz)</th>
<th>F3 (Hz)</th>
<th>F4 (Hz)</th>
<th>F5 (Hz)</th>
<th>F6 (Hz)</th>
<th>F7 (Hz)</th>
<th>F8 (Hz)</th>
<th>F9 (Hz)</th>
<th>F10 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full reservoir</td>
<td>0.44</td>
<td>0.63</td>
<td>0.78</td>
<td>0.83</td>
<td>1.07</td>
<td>1.42</td>
<td>1.51</td>
<td>1.76</td>
<td>1.85</td>
<td>2.15</td>
</tr>
<tr>
<td>Empty reservoir</td>
<td>0.78</td>
<td>1.51</td>
<td>1.76</td>
<td>1.81</td>
<td>2.00</td>
<td>2.15</td>
<td>2.20</td>
<td>2.34</td>
<td>2.83</td>
<td>3.66</td>
</tr>
</tbody>
</table>

Looking at the eigenfrequencies, we note that they are higher in the case of empty reservoir. So that the arch dam vibrate more rapidly. That highlights the influence of the reservoir water level on the dynamic behaviour of the structure. The fuller it is, the heavier becomes the system dam-foundation-reservoir. Thus, the structure vibrates slowly.

4.3.4. Stress envelopes
The next diagrams give some representative comparisons of stress envelopes at the main section of the arch between results for the different reservoir levels (full / empty) and considering homogeneous dam and foundation.

Hoop stresses (SXX) are maximum on the dam crest. One observes that on the top of the dam in the case of full reservoir, it is roughly two and a half times the stress when the reservoir is empty.

On the upstream face of the dam, envelopes of orthogonal stress to the arch (SYY) obtained with full reservoir are quite larger that those with empty reservoir. This can be explained by the influence of water on the interface arch-reservoir. The vertical stress (SZZ) reaches its maximum at the upper part of the dam’s downstream face (3H/4) for both water reservoir levels.

5. SEISMIC RESPONSE OF AN ARCH DAM-Foundation-Reservoir SYSTEM CONSIDERING RANDOMLY FLUCTUATING MATERIAL PROPERTIES

Standard deviations and correlation lengths are given in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Lc (m)</th>
<th>σ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foundation</td>
<td>300</td>
<td>50</td>
</tr>
<tr>
<td>Arch dam</td>
<td>20</td>
<td>12</td>
</tr>
</tbody>
</table>

The same accelerograms, the same seismic attenuation and the same procedure of calculation as in the homogeneous case are considered.

5.1. Results and interpretations

This section reports and discusses the influence of heterogeneities on the modal response of the dam and the stress envelopes on its main section.

5.1.1. Modal analysis
Proceeding in the same manner as in the homogeneous case, eigenfrequencies of the first four random realizations are summarized in the tables below.

The results show that for both reservoir water levels, heterogeneities affect significantly the modal response of the dam.

Comparing heterogeneous cases with the homogeneous one in the dam-foundation-reservoir model, one observes for the two water reservoir levels:

- 1st eigenfrequency is almost not influenced by heterogeneities.
- Heterogeneity induces a densification of the modes (many modes with similar eigenvalue)

We note also that frequencies are higher when the reservoir is empty than those when it is full of water.

### Table 6. Modal frequencies of the arch dam-foundation-empty reservoir system.

<table>
<thead>
<tr>
<th></th>
<th>F1 (Hz)</th>
<th>F2 (Hz)</th>
<th>F3 (Hz)</th>
<th>F4 (Hz)</th>
<th>F5 (Hz)</th>
<th>F6 (Hz)</th>
<th>F7 (Hz)</th>
<th>F8 (Hz)</th>
<th>F9 (Hz)</th>
<th>F10 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous</td>
<td>0.78</td>
<td>1.51</td>
<td>1.76</td>
<td>1.81</td>
<td>2.00</td>
<td>2.15</td>
<td>2.20</td>
<td>2.34</td>
<td>2.83</td>
<td>3.66</td>
</tr>
<tr>
<td>Heterogeneous 1</td>
<td>0.78</td>
<td>1.53</td>
<td>1.82</td>
<td>1.87</td>
<td>1.97</td>
<td>2.02</td>
<td>2.17</td>
<td>2.22</td>
<td>2.34</td>
<td>2.83</td>
</tr>
<tr>
<td>Heterogeneous 2</td>
<td>0.78</td>
<td>1.51</td>
<td>1.77</td>
<td>1.87</td>
<td>2.03</td>
<td>2.13</td>
<td>2.24</td>
<td>2.34</td>
<td>2.39</td>
<td>3.48</td>
</tr>
<tr>
<td>Heterogeneous 3</td>
<td>0.78</td>
<td>1.51</td>
<td>1.75</td>
<td>2.15</td>
<td>2.34</td>
<td>2.44</td>
<td>2.73</td>
<td>2.88</td>
<td>3.17</td>
<td>3.32</td>
</tr>
<tr>
<td>Heterogeneous 4</td>
<td>0.79</td>
<td>1.40</td>
<td>1.56</td>
<td>1.66</td>
<td>1.77</td>
<td>1.87</td>
<td>2.13</td>
<td>2.18</td>
<td>2.03</td>
<td>2.24</td>
</tr>
</tbody>
</table>

### Table 7. Modal frequencies of the arch dam-foundation-full reservoir system.

<table>
<thead>
<tr>
<th></th>
<th>F1 (Hz)</th>
<th>F2 (Hz)</th>
<th>F3 (Hz)</th>
<th>F4 (Hz)</th>
<th>F5 (Hz)</th>
<th>F6 (Hz)</th>
<th>F7 (Hz)</th>
<th>F8 (Hz)</th>
<th>F9 (Hz)</th>
<th>F10 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous</td>
<td>0.44</td>
<td>0.63</td>
<td>0.78</td>
<td>0.83</td>
<td>1.07</td>
<td>1.42</td>
<td>1.51</td>
<td>1.76</td>
<td>1.85</td>
<td>2.15</td>
</tr>
<tr>
<td>Heterogeneous 1</td>
<td>0.44</td>
<td>0.64</td>
<td>0.79</td>
<td>0.84</td>
<td>1.07</td>
<td>1.18</td>
<td>1.28</td>
<td>1.42</td>
<td>1.53</td>
<td>1.82</td>
</tr>
<tr>
<td>Heterogeneous 2</td>
<td>0.44</td>
<td>0.78</td>
<td>0.83</td>
<td>1.07</td>
<td>1.09</td>
<td>1.25</td>
<td>1.40</td>
<td>1.51</td>
<td>1.76</td>
<td>1.87</td>
</tr>
<tr>
<td>Heterogeneous 3</td>
<td>0.44</td>
<td>0.63</td>
<td>0.78</td>
<td>1.07</td>
<td>1.42</td>
<td>1.51</td>
<td>1.86</td>
<td>2.20</td>
<td>2.34</td>
<td>2.44</td>
</tr>
<tr>
<td>Heterogeneous 4</td>
<td>0.44</td>
<td>0.78</td>
<td>0.83</td>
<td>0.92</td>
<td>1.07</td>
<td>1.09</td>
<td>1.25</td>
<td>1.40</td>
<td>1.51</td>
<td>1.87</td>
</tr>
</tbody>
</table>

5.1.2. **Stress envelopes**

This section is focusing on the maximum stress values on the upstream face of the dam. The next diagrams give some representative comparisons between results for homogeneous and heterogeneous models considering different water reservoir levels (empty/full). Looking at the envelopes below, one observes:

- The stress fields at the base of the dam are not affected by heterogeneities.
- The maximum of stress (hoop ones) are reached on the dam crest in homogeneous and heterogeneous cases for both reservoir water levels.
- In homogeneous models, hoop stresses on the dam’s crest in the full model are two and a half times those in the empty one.
- On the upstream face of the dam, the upper $\frac{1}{3}$ of the structure is influenced by heterogeneities. However, when the reservoir is full, only the response on the crest is influenced.
In order to evaluate the influence of Young modulus on the maximum hoop stress, we calculated the mean value in three surfaces as it is shown in the figure below:

![Figure 13. Young Modulus at the base, left bank and right bank of the dam.](image)

$E_b$, $E_1$ and $E_2$ represent respectively the mean values of Young modulus at the base of the dam, on the left bank and on the right bank. The Young modulus in the homogeneous case is 23 GPa. The difference between the maximum hoop stress in the heterogeneous model and that in the homogeneous one is quantified by “Hoop stress%”.

### Table 8. Influence of Young modulus mean values on the maximum hoop stress in the full model.

<table>
<thead>
<tr>
<th></th>
<th>real1</th>
<th>real2</th>
<th>real3</th>
<th>real4</th>
<th>real5</th>
<th>real6</th>
<th>real7</th>
<th>real8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>E_1 - E_2</td>
<td>$ [GPa]</td>
<td>3.01</td>
<td>1.04</td>
<td>0.50</td>
<td>1.85</td>
<td>0.02</td>
<td>0.08</td>
</tr>
<tr>
<td>Hoop stress %</td>
<td>+8.57</td>
<td>-17.19</td>
<td>-3.79</td>
<td>-19.95</td>
<td>0.89</td>
<td>-2.72</td>
<td>+23.85</td>
<td>+8.78</td>
</tr>
</tbody>
</table>

Comparing results illustrated in the table below and regarding the Young modulus cross sections presented in figure 4, one note that:

- All random models have almost the same mean value of Young modulus at the base of the dam. As it was mentioned in the 3rd section, realizations were chosen to avoid dispersions due to the heterogeneities at the interface dam/foundation.
- The more the foundation is symmetrically homogeneous on the right and the left banks, the more hoop stresses are minimized comparing with the homogeneous case.
Influence of heterogeneity coupled with the reservoir water level on the maximum hoop stress on the upstream of the dam is summarized in the table below:

<table>
<thead>
<tr>
<th></th>
<th>real1</th>
<th>real2</th>
<th>real3</th>
<th>real4</th>
<th>real5</th>
<th>real6</th>
<th>real7</th>
<th>real8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full (%)</td>
<td>+8.57</td>
<td>-17.19</td>
<td>-3.79</td>
<td>-19.95</td>
<td>0.89</td>
<td>-2.72</td>
<td>+23.85</td>
<td>+8.78</td>
</tr>
<tr>
<td>Empty (%)</td>
<td>+43</td>
<td>-16</td>
<td>-22</td>
<td>+13</td>
<td>+11</td>
<td>+30.8</td>
<td>+43.7</td>
<td>+14</td>
</tr>
</tbody>
</table>

Looking at the influence of heterogeneities on the maximum hoop stress on the dam crest, one observes that it is more significant when the reservoir is completely empty. The increase of the reservoir water level adds an inertia to the system that reduces the effect of material properties randomization.

6. CONCLUSIONS

A dynamic analysis of an arch dam-foundation-reservoir system was performed using the spectral element method. The influence of coupling randomly fluctuating material properties on the seismic response of the dam was studied considering two reservoir water levels (full and empty). Results confirmed that heterogeneities could change the mechanical response of the structure (underestimate or overestimate its response depending on the spatial variability of the mechanical properties). In this contribution, our conclusions are based on linear material behaviour. Considering contraction joints is an interesting issue to the earthquake design since they reproduce nonlinearities in the response of the structure subjected to strong motions.

7. REFERENCES


