DUCTILITY REDUCTION OF RECTANGULAR R/C MEMBERS DUE TO BIAXIAL BENDING

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ABSTRACT

Inelastic analysis is prescribed by design codes and is used widely for the analysis of new and existing buildings. Codes prescribe inelastic properties of members through calibrated empirical relationships, based on uniaxial experiments. An analytical model has been developed, in order to quantify the effects of biaxial performance of structural members, in terms of capacity, as well as deformability and ductility. From the parametric analysis, it has become evident that biaxial performance plays significant role in the inelastic properties of the structural members.

Keywords: Reinforced Concrete; Deformation Capacity; Inelastic Analysis

1. INTRODUCTION

Eurocode-8/Part-3 (2005), FEMA-356 (2000), ASCE7-05 (2010), ACI-318 (2010), KANEPE (2012) and several national Codes provide empirical expressions for the determination of curvature at yield and at ultimate, also the total chord rotation capacity at ultimate (elastic plus inelastic part) of concrete members. These expressions are based on databases of hundreds of test results (Panagiotakos T, Fardis M, 2001); theoretical models have not been, as yet, developed to predict the experimental behavior of specimens. All the above mentioned tests, however, are limited to uniaxial bending, while it is widely accepted that biaxial loading plays important role in the reduction of both capacity and allowable deformations of structural members; that is a reason why minimization of a structure’s rotation is among the principles of structural design. However, data for biaxial performance, especially for curvature, are not available in Codes and available software is limited to uniaxial response only (towards each principle direction). In the present paper a model has been developed, in order to quantify the effect of biaxial bending to the yield and ultimate state of concrete members, in terms of capacity and deformability.

2. FORMULATION OF THE ANALYTICAL MODEL

2.1 Computation of the axial force and the bending moments

The axial force and the bending moments of the cross section can be computed from the stresses of the concrete and the rebars (Vougioukas and Stamos 2016⁴⁻⁵):

\[ N = N_{co} + N_{st} = \iint \sigma_{co}(x,y) \, dx \, dy + \sum \sigma_{st}(x_i, y_i) \, \pi d_i^2 / 4 \]  

\[ M_x = M_{x,co} + M_{x,st} = \iint y \sigma_{co}(x,y) \, dx \, dy + \sum y_i \sigma_{st}(x_i, y_i) \, \pi d_i^2 / 4 \]  

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\[ M_y = M_{y,co} + M_{y,st} = \int x \sigma_{co}(x,y) \, dx \, dy + \sum x_i \sigma_{st}(x_i, y_i) \pi d_i^2 / 4 \]  

(3)

The integrals are computed using numerical integration (Press et al 1992) taking into account the tilted neutral axis (\( x' \)) and linear strains with respect to \( y' \) axis. The compressive strain \( \varepsilon_{up} \) of the highest point of the cross section, and the tensile strain \( \varepsilon_{down} \) of the lowest rebar of the cross section are considered known. On the other hand, the yield moments \( M_x^y \) and \( M_y^x \) are computed setting \( \varepsilon_{up} \) equal to the yield strain of concrete, or setting \( \varepsilon_{down} \) equal to the yield strain of steel. The other strain is computed so that Equation 1 holds. The maximum moments \( M_x^R \) and \( M_y^R \) are computed similarly setting \( \varepsilon_{up} \) or \( \varepsilon_{down} \) to the maximum strains of the concrete and steel respectively. More details can be found in Vougioukas and Stamos (2016).

2.2 Computation of the bending slope ignoring cracks

The curvature at any point of a column is given by:

\[ \frac{1}{R} = \nu''(z) = \frac{\varepsilon_{down} - \varepsilon_{up}}{d} \]  

(4)

where \( z \) is the axis of the column, compressive strains are negative and \( d \) is the distance between the 2 strains along the \( y' \) axis. Integration of the curvature along the \( z \)-axis, gives the bending slope at any point of the column. Assuming a column of length \( L \) with symmetrical, user-defined bending moments \( M_{max} \) at its ends, the bending moment varies linearly along the \( z \)-axis and it is zero at the middle of the column:

\[ M(z) = \frac{M_{max}}{L/2} z \]  

(5)

where the origin of the \( z \)-axis is at the middle of the column and \( M_{max} \) is a constant. Thus, if the relationship between curvature and bending moment \( \nu''(M) \) is established, then the relationship between curvature and \( z \)-axis is simply:

\[ \nu''(z) = \nu''(M) = \nu''(M(z)) = \nu''\left(\frac{M_{max}}{L/2} z\right) \]  

(6)

In order to establish the relationship between curvature and bending moment, a lot of compressive strain values \( \varepsilon_{up} \) between zero and the maximum strain of concrete (0.0035) are tried. For each \( \varepsilon_{up} \) the \( \varepsilon_{down} \) strain is computed so that Equation 1 holds for a user defined axial force \( N_c \). The computation is done using a combination of the exhaustive search and the binary search algorithms (Press et al 1992). Essentially the axial force \( N \) is computed for many consecutive \( \varepsilon_{down} \) strains ranging between \( \varepsilon_{up} \) and the maximum strain of steel (0.07) are tried, and the one that yields \( N=N_f \) is selected. It must be noted that not all combinations of \( \varepsilon_{up} \) and \( N_f \) are valid; for the given \( \varepsilon_{up} \) compressive strain the axial force \( N_f \) may be too high and the cross section does not withstand it, or too low and the cross section in not in equilibrium.

Once \( \varepsilon_{down} \) is computed, the curvature is computed by Equation 4 and the bending moment by Equation 2. Thus a (numerical) relationship \( M=M(\nu'') \) is established. A typical diagram of the relationship is given in Figure 1.

The bending slope is given by the integral of equation 6:

\[ \nu'(z) = \int_0^z \nu''(u) \, du = \int_0^z \nu''\left(\frac{M_{max}}{L/2} u\right) \, du \]  

(7)
The integral of Equation 7 is computed numerically for many consecutive values of z between 0 and L/2, so that the (numerical) relationship $\nu'(z)$ is established.

### 2.3 Computation of the chord rotation angle ignoring cracks

The bending arrow can be computed by the numerical integration of Equation 7:

$$\nu = \nu(M_{\text{max}}) = \int_0^{L/2} \nu'(z)dz$$

Note that $\nu'(z)$ is realized by many $z_i, \nu'_i$ pairs where $z_i$ varies between 0 and L/2. Additionally, the bending moment varies from zero to $M_{\text{max}}$ (Equation 5), and thus the bending arrow and the chord rotation angle are functions of $M_{\text{max}}$. The chord rotation angle is computed by:

$$\theta_{\text{chord}} = \theta_{\text{chord}}(M_{\text{max}}) = \frac{\nu(M_{\text{max}})}{L/2}$$

In order to realize the relationship between bending moment and chord rotation angle, $\theta_{\text{chord}}$ is computed for many (for example 200) consecutive values of $M_{\text{max}}$ between zero and $M_R$, including $M_y$, as defined in Section 2.1.

### 2.4 Computation of the chord rotation angle with one crack

When the tensile stress surpasses the tensile strength of concrete, cracks form. The total length of all the cracks is the total displacement of the steel rebars, minus the total displacement of the concrete due...
to the (tensile) strains which correspond to its tensile strength. The latter is negligible and can be ignored without little error. The total displacement of the steel rebars is the integral of the (mostly tensile) steel strains:

$$\Delta L = \Delta L(M_{\text{max}}) = \int_{0}^{L/2} \varepsilon_{\text{down}}(z)dz$$  \hspace{1cm} (10)

Note that $\varepsilon_{\text{down}}$ is computed as a byproduct of the computation of the bending moment and curvature, as described in Section 2.2. Thus the $\varepsilon_{\text{down}}(M)$ relationship is readily available, and it can be converted to $\varepsilon_{\text{down}}(z)$ by an equation similar to Equation 6.

At first it is assumed that displacement $\Delta L$ creates one large crack at the position of maximum bending moment $M_{\text{max}}$ ($z=L/2$), and thus the length of the crack is $\Delta L$ (Equation 10). The crack propagates from the bottom of the cross section until its compressive zone, which begins at the neutral axis of the section. The height $d_{\text{tens}}$ of the cross section (parallel to y'-axis) from the bottom to the neutral axis is the height of the tensile zone. Under the assumption that the crack is roughly triangular, the crack length $\Delta L$ creates additional chord angle:

$$\theta_{\text{chord,crack}} = \theta_{\text{chord,crack}}(M_{\text{max}}) = \frac{\Delta L}{d_{\text{tens}}}$$  \hspace{1cm} (11)

which must be added to that of Equation 9. The effect of the crack is remarkable as shown in Figure 2.

2.5 Computation of the chord rotation angle with distributed cracks

If n cracks are formed, the separate the length of the column L/2 into n intervals. Each interval has length $L/(2n)$ and spans the z-axis from $z_{i-1}$ to $z_i$, where

$$z_i = \frac{L}{2n} i$$  \hspace{1cm} (12)

assuming uniform distribution of cracks. The displacement of the steel rebars along the interval is given by:

$$\Delta L_i = \Delta L_i(M_{\text{max}}) = \int_{z_{i-1}}^{z_i} \varepsilon_{\text{down}}(z)dz$$  \hspace{1cm} (13)

Half of this displacement is assumed to contribute to the formation of the crack on the left side of the interval and the other half to contribute to the crack on the right side. Thus the length $l_i$ of each crack is determined, and the tensile zone height $d_{\text{tens},i}$ at the position of each crack is determined as described in the previous section. The cracks produced additional chord rotation angle:

$$\theta_{\text{chord,crack}} = \theta_{\text{chord,crack}}(M_{\text{max}}) = \sum_{i=1}^{n} \frac{l_i}{d_{\text{tens},i}}$$  \hspace{1cm} (14)

The effect of distributed cracks with various interval lengths is shown in Figure 2. Consideration of cracks results to increment of $\theta_{\text{chord}}$ more than three times, while the number or cracks (related to their distribution) doesn’t seem to play any important role.
3. INFLUENCE OF THE BOND FORCES

3.1 Estimation of the cracks position and length due to bond forces

The bond stress slip between concrete and steel rebars depends on the slip as shown in Figure 3 (Zhao et al 2012). The bond stress practically vanishes for slip (thus elongation of the rebars) of the order of 1 mm.

Without bond, no $\Delta F$ (difference in axial force of the rebars) could be developed between adjacent sections of concrete, thus no moment increment could take place. It can be thus assumed that if the
relative displacement (due to elongation) of the rebars between any 2 positions along the axis of the
column is 1 mm or more, a crack of length 1mm is formed. The displacement is given by Equation 13.
It can also be reasonably assumed that the position \( z_1 \) of the first crack is the position of the maximum
bending moment. If follows that the position \( z_2 \) of the second crack is such that the displacement
between \( z_1 \) and \( z_2 \) is also 1 mm. The position \( z_3 \) of the third crack is accordingly computed and so on
until the displacement of the last position and the beginning of the column is less than 1mm (this is
also the length of the last crack). The total length of the cracks is also given by Equation 10 and their
effect is similar to that of Figure 2.

3.2 Estimation of the additional displacement inside the support

The rebars are anchored inside the support, and the anchor length \( l_{bd} \) is given by Eurocode 2 section
8.4. Inside the support the strain of the rebars is assumed to vary linearly from zero to the strain \( \varepsilon_{down} \)
due to the maximum bending moment, which is computed as described in Section 2.1. The
displacement inside the support is thus given by:

\[
\Delta L_s = \int_0^{l_{bd}} \varepsilon(z) dz = \frac{l_{bd} \varepsilon_{down}}{2}
\] (15)

This displacement is added to the displacement of the first crack; this also produces additional chord
rotation angle.

Procedure stops at the maximum bending moment reached and does not cover the degradation stage of
the section.

4. NUMERICAL RESULTS

Two series of typical sections of columns under uniaxial and biaxial bending, and under axial force
have been examined, with various reinforcement, concrete and steel properties.

First series are the sections presented in Figure 4. These sections have been examined by López-López
et al (2016) and their material properties are C25/30, B500C. Results are presented in Figures 5 and 6.

Second series are two typical sections that have been used in Greece at the period 1960-1985: 200/100
C16, B400, 8\( \phi \)18 and 25/55 C16, B400, 4\( \phi \)14 both under uniaxial and biaxial bending, and under axial
force. Results are presented in Figure 8.

The length of the column is 3 m, in all cases. Safety factors were taken equal to 1, the cover 3 cm and
the diameter of stirrups 8 mm.
4.1 Results for series A

The numerical results are presented in Figures 5 and 6. Figure 7 is used for comparison purposes. For illustration purposes the diagrams are also shown in Figure 9, normalized to the maximum moment and chord angle per section.

Figure 5. Calculated $M_x-\theta_{\text{chord}}$ relations for sections 300x300 and 350x350, 8\(\Phi\)16, C25/30, B500C (Series 1, Part A)
Figure 6. Calculated $M_x-\theta_{chord}$ relations for sections 400/400 $8\Phi 16$ and 450/450, 12$\Phi 16$, C25/30, B500C (Series 1, Part B)
4.2 Discussion of results of series A

Figure 7 presents comparison results among three different approaches and it is used for comparison with the results obtained by the proposed analysis method, for the case of uniaxial bending.

Comparison or results indicates that the presented model is in good accordance with the provisions of FEMA-356. It has to be mentioned, however, that the degradation effects are not included; obviously the margins for inelastic deformation of the other two models (Biskinis & Fardis, 2013, and López et al, 2016) are larger due to this reason.

Figure 7. Calculated $M$-$\theta$ chord relations for sections by A. López-López et al (2016) and, in parenthesis, comparison to numerical results obtained by current model (Materials C25/30, B500C)

- 300/300, 8\(\Phi\)16 (upper left), ($M=115$, $\theta=0.018$ for $N=225$ and $M_2=0$)
- 350/350, 8\(\Phi\)16 (upper right), ($M=150$, $\theta=0.016$ for $N=306$ and $M_2=0$)
- 400/400, 8\(\Phi\)16 (lower left), ($M=280$, $\theta=0.012$ for $N=800$ and $M_2=0$)
- 450/450, 12\(\Phi\)16 (lower left), ($M=135$, $\theta=0.010$ for $N=1012$ and $M_2=0$)
4.3 Results for series B

Second series consists of two typical sections that have been used in Greece at the period 1960-1985: 20/100 cm, C16, B400, 8Φ18 and 25/55 cm C16, B400, 4Φ14, both under uniaxial and biaxial bending and under axial force. For illustration purposes the diagrams are also shown in Figure 9, normalized to the maximum moment and chord angle per section.

Figure 8. Calculated $M_x$-$\theta_{chord}$ relations for sections 200/1000 (4Φ14) and 250/550 (4Φ14), C16/20, B400 (Series 2)
5. CONCLUSIONS

From the analysis of the square sections, it becomes evident that, for relatively small moment coefficient in direction 2 (out of plane), e.g. of the order of 15%-30%, not only $M_{\text{yield}}$ and $M_{\text{R}}$ are reduced by an equivalent ratio, but also the chord rotation capacity is reduced by the same ratio (Figures 5 and 6).

For rectangular sections, with sides’ ratio about 2:1 reduction in ductility is significant, even for small
values of out of plane moment coefficient. In Figure 8 (right), it becomes evident that, for small axial force, 10% of out of plane moment may reduce ductility by 50%. For larger axial force, reduction of ductility is almost negligible.

For rectangular sections, with sides’ ratio about 5:1 (wallets) there is no reduction in ductility for the expected small values of out of plane moment coefficient. In Figure 8 (left), it becomes evident that, actually an increase of ductility is noticed. But, in this case, the $M_Y$ and $M_R$ capacity are reduced significantly, by a ratio which is double the out of plane moment coefficient.

Thus, chord rotation capacity, a section property with significant role in the ductility of R/C structures, is found to be strongly effected by out-of-plane bending. It seems that the available experimental data for uniaxial bending cannot be extrapolated for biaxial bending.

Biaxial ductility is a very significant parameter and has to be investigated in detail with performance of properly designed biaxial tests, focusing in ductility properties of r/c members.

6. REFERENCES


ACI SP-17M-09 (2010). ACI design handbook – design of structural reinforced concrete elements in accordance with ACI 31 SM-05. ACI American Concrete Institute, Farmington Hills, Michigan


KANEPE (2012), Greek code of retrofitting of reinforced concrete buildings, Organization of Seismic Design & Protection (OASP), Hellenic Ministry of Infrastructure, Transport and Network, Athens. (in Greek)


Vougioukas E, Stamos A-A, (2016). Influence of biaxial bending to the yield curvature and capacity and to the chord rotation angles of R/C members, Proceedings of the 17th Greek Concrete Conference “Concrete Structures”, 10-12 September, Thessaloniki, Greece. (in Greek)