

SEISMIC LOSS ESTIMATION OF STEEL MOMENT RESISTING FRAMES

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ABSTRACT

Methodology for loss estimation of structures during earthquake has been introduced by the Pacific Earthquake Engineering Research center (PEER), as a new generation of performance-based earthquake engineering (PBEE). In PEER's approach, monetary loss as a quantitative criterion is measured by three means of damage: cost, death, and downtime. The probabilistic framework of monetary economic loss is calculated from the site-specific seismic hazard, structural response, structural and nonstructural components' damage, and the resulting repair costs. In this paper, the seismic loss of Steel Moment-Resisting Frames (SMRFs), which are designed, based on different concepts, has been estimated. In this regard, expected seismic losses of six archetype SMRFs, with different types of ductility, including special, intermediate, and ordinary moment frames, have been estimated using simplified story-based loss estimation method. To simulate the building responses in terms of drift and acceleration, the OpenSees software framework was used to conduct a time history analysis, subjected to far field recorded ground motions. Loss estimation process is then developed using in house MATLAB code. It is observed that special moment resisting frames have the least economic loss during earthquake. In addition, acceleration sensitive members have the most effective contribution in a building loss.

Keywords: Seismic Design; Loss Estimation; Performance-Based Earthquake Engineering; Incremental Dynamic Analysis; Steel Moment Resisting Frame

1. INTRODUCTION

Seismic design codes provide minimum requirements to design structures that can withstand earthquakes in seismically active regions. One of the recent developments in the seismic design of new structures, and rehabilitation of existing structures is performance-based design approach. But, there are two main flaws in current codes.

First, using a number of qualitative performance levels that are not well understood by stakeholders, second, not considering the aim of mitigating economic loss, caused by earthquakes. In order to resolved mentioned flaws, in the new generation of PBEE, (developed by PEER center), economic loss has been considered as a major criterion for assessing building performance.

One of the first report of loss estimation was presented by Scholl et al (1982), in which the author developed methods to improve both empirical and theoretical loss estimation procedures. The theoretical method recommended a probabilistic, component-based method of evaluating damages. In conjunction with their research, Kutsu et al (1982) collected laboratory test data to estimate damage in various high-rise building components.

After considering building-level fragility curves and damage probability matrices (DPMs), proposed by Singhal & Kiremidjian (1996), Porter and Kiremidjian (2001) introduced probabilistic assembly-based

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framework. Later, Porter and Kiremidjian (2001) introduced a probabilistic assembly-based framework. The framework combines the uncertainty stemming in the damage estimation and the associated repair costs. Next, a component-based method with considering collapse probability was developed by Aslani & Miranda (2005). It was followed by a simplified method of PEER framework, presented by Zareian & Krawinkler (2006) [2]. The Zareian & Krawinkler framework allowed building components to be classified in various clusters of story level or the whole building. Reiser & Beck (2007) [3] followed Aslani & Miranda's research [1] using an MATAB Damage and Loss Analysis toolbox (MDLA), in which the PEER framework was implemented.

The story-based loss estimation is one of the most recent seismic loss estimation studies of PEER approach, which has been introduced by Ramirez and Miranda (2009) [4]. The current study is based on aforementioned research.

The aim of this paper is to investigate the effect of seismic design level of Steel Moment-Resisting Frames (SMRFs) on repair cost of seismically induced damages through incremental dynamic analysis. In this regard, six archetype SMRFs, including ordinary, intermediate, and special frames are considered for being analyzed. All SMRFs have been initially designed according to seismic codes [8, 9, 10]. In fact, they are in full compliance with AISC code [9]. Consequently, all SMRFs are analyzed under incremental dynamic analysis using OpenSees software.

1.1 Loss estimation framework

The new generation of performance-based design methodology is represented in this section. As it was mentioned before, the methodology is based on the works of Krawinkler & Miranda (2004) [5], Aslani & Miranda 2005[1], Mitrani-Reiser (2007) [3]. It is a probabilistic framework, which includes the following stages: Hazard Analysis, Structural Analysis, Damage Analysis, Loss Analysis-as Equation 1 [10].

$$\lambda[DV] = \iiint (G(DV|DM)dG(DM|EDP)dG(EDP|IM)d\lambda(IM). d(EDP). d(DM))$$
(1)

where G[X|Y] is Complementary cumulative distribution function of X conditional on Y and $\lambda[X|Y]$ is Mean annual frequency of X given Y.

The following assumptions are made in the present article:1st-Intensity measure (IM) is defined as the spectral acceleration at the fundamental period of building (Sa (T1)).2nd-Engineering demand parameter (EDP), which is provided to survey structural responses, is considered as inter-story drift ratio (IDR) and peak floor acceleration (PFA).3rd-Three quantitative parameters are considered in PEER framework as decision variable (DV), for introducing seismic performance: damage cost, downtime, and death. However, in this paper, DV is only represented by monetary losses.

As aforementioned, a simplified version of PEER's building-specific loss estimation approache, "Story-based method", which has been proposed by Ramirez & Miranda (2009) [4] is utilized in this research. This approach relies on assumptions about the building's cost distribution to calculate loss for each story. In order to implement building-specific relation between the correlating ground motion intensity and the economic monetary loss (i.e. loss function) a probabilistic phrase is applied as Equation 2(Aslani (2005)) [1].

$$E[L_T|IM] = E[L_T|NC.IM]P(NC|IM) + E[L_T|C]P(C|IM)$$
(2)

Expected losses at a given level of ground motion intensity are computed as the sum of expected losses in two states; a) NC state, in which no collapse occurs, and b) C state, in which collapse occurs, and the building is needed to be rebuilt(C). In the right hand side of Equation 2, $E[L_T|NC.IM]$ is calculated by story-based methodology [4], as clarified in Equations 3,4 and 5, where S is the number of stories and N is the number of components in each story [11].

$$E[L_T|NC.IM] = \sum_{i=1}^{S} \sum_{k=1}^{N} E[L_{i,k}|NC.IM]$$
(3)

where,

$$E[L_{i,k}|NC.IM] = \int_0^\infty E[L_{i,k}|NC.EDP_k] | dp (EDP_k > edp_k | NC.IM)|$$
(4)

In which:

 $E[L_{i,k}|NC.IM]$ is the expected loss at the i^{th} story and k^{th} component, conditional on non-collapse state, and IM is seismic intensity measure.

 $E[L_{i,k}|NC.EDP_j]$ is the expected loss at the i^{th} story and k^{th} component conditional on non-collapse state, and EDP_j which is related to k^{th} component.

 $P(EDP_k > edp_k | NC.IM)$ is the probabilistic distribution of EDP_k, based on incremental dynamic analysis (IDA) results.

$$E[L_{i,k}|NC.EDP_j] = \sum_{j=1}^{M} E[L_k|NC.DS_m]|P(DS = ds_m|NC.EDP_k)|$$
(5)

Where, M is the number of damage states in the k^{th} component, $E[k|NC.DS_m]$ is the normalized loss (i.e. ratio of repair value to replacement value) in k^{th} component at DS_m damage state. It is evaluated as a fractional value according to Ramirez and Miranda (2009) [4] that depends on building occupancy.

Moreover, $P(DS = ds_m | NC.EDP_k)$ is the fragility function of k^{th} component at the definite damage state: DS_m in Figure 3, which is resulted from Table 2, demonstrates the fragility function. The mean and standard deviation values of this function is reported by Ramirez and Miranda (2009) [4], based on experimental data.

According to the approach, presented by Zareian and Krawinkler (2006) [2] and Ramirez and Miranda (2009) [4], components are grouped into different categories such as drift sensitive structural members, drift sensitive non-structural members, acceleration sensitive non-structural members, and rugged members. Rugged members are ignored since they are consistent after seismic motions. In order to compute the loss estimation, each component is categorized to one of the above-mentioned group sensitivities. Next, its response, Inter-Story Drift Ratio (IDR) or Peak Floor Acceleration (PFA), for instance, is used in loss estimation.

The story-based approach, as Equation 3 and 4, requires that the replacement value of entire building be distributed among each story and each type of building component in the structure. Therefore, in the considered case-study buildings, a complete inventory of components should be available; otherwise, an assumption should be made in the distribution of replacement value among the stories of a building. It has been recognized that the 1st floor and the roof might have different values than the other floors. Therefore, three categories of distributed values are allocated to the components. Table 1 presents the distributed values for each category, based on RSMeans Square Foot Costs; although, there are slight differences [6,13]. To evaluate Fragility functions; $P(DS = ds_m|NC.EDP_k)$ and cost distribution, assigned with repair cost of the components, some experimental parameters are listed as Table 2, which are approximately similar to data used by Ramirez and Miranda (2009) [4]. It is emphasized that the units for fragility function parameters (e.g., x_m or the median value) depends on engineering demand, and each damage state is assumed to be conditionally independent (given EDP) from all other assembly groups [12].

Table 1. The Distribution value of structural, non-structural and rugged members between separated story by story [6, 13].

	Structural	Non-Structural (IDR)	Non-Structural (PFA)	Rugged
1st				
Floor	10.10	43.90	31.80	14.20
Custom				_
Floor	10.30	42.20	33.20	14.30
Тор				
Floor	8.60	38.50	36.80	16.10

Table 2 . Structural and non-structural components' Fragility functions and expected repair cost parameters (normalized by component replacement cost), DS1: slight damage state, DS2: moderate damage, DS3: extensive damage DS4: complete damage state, that components should be replaced [6, 13]

			Fragility Repair cost		Parameters	
Component	Damage State	Seismic Sensitivity	Median (% for IDR, g for PFA)	Dispersi on	Expected Value	Reference
Partitions (including	DS1		0.21	0.61	0.10	ATC-58
Façade)	DS2	IDR	0.69	0.40	0.60	
	DS3		1.27	0.45	1.20	
Partition-like	DS1	IDR	1.27			[1]
				0.45	1.20	
Windows	DS1 DS2	IDR	1.60	0.29	0.10	[1]
	DS3		3.20	0.29	0.60	
			3.60	0.27	1.20	
Generic-Drift	DS1	IDR	0.55	0.60	0.03	[4]
	DS2		1.00	0.50	0.10	
	DS3		2.20	0.40	0.60	
	DS4		3.50	0.35	1.20	
Ceilings	DS1	PFA	0.30	0.40	0.12	ATC-58
	DS2		0.65	0.5	0.36	
	DS3		1.28	0.55	1.2	
Generic-Acceleration	DS1	PFA	0.7	0.5	0.02	[4]
	DS2		1	0.5	0.12	
	DS3		2.2	0.4	0.36	
	DS4		3.5	0.35	1.2	
Post-1994Welded-	DS1	IDR	3	0.35	0.14	[11]
steelmoment	DS2		4	0.35	0.47	
frame(Structural component)	DS3		5	0.35	0.71	

1.2 Structural Properties of the Models

To evaluate the effect of seismic design level and ductility of frames on estimated seismic loss, six 5story steel moment-resisting frames have been designed for the very high level of relative seismic hazard zone. These buildings are assumed to be located on soil type B, in which the average shear wave velocity at a depth of 30 m would be 360-750 m/s. The buildings are square in plan as in Figure 1, consisting three bays of 5.0 m in each direction, and having a height of 3.2 m. Gravity loads are supposed to be similar to common office buildings [13]. The designed buildings are labeled as SMF-V3, SMF-V2, IMF-V3, OMF-V3, and OMF-V2 for Special, Intermediate, and Ordinary Moment-resisting Frames. The "V3" or "V2" extensions in the aforementioned labels indicates that the corresponding structure has been designed in compliance with the third or second revision of Iranian seismic design code, respectively. The sixth building is a moment-resisting frame, which has been designed in compliance with the first revision of Iranian seismic design code. The values of the response modification factors (i.e. R) have been selected from the seismic design code as shown in Table 3 [6, 7, 8]. Moreover, Table 4 shows section properties of SMF-V3 and OMF-V2 as samples of frame members. It can be seen that overall stiffness is increased in a respective manner as models change from SMF to OMF. Appendix A shows design criteria of the structures subject to earthquake per different revisions of Iranian seismic design code [6, 7, 8].

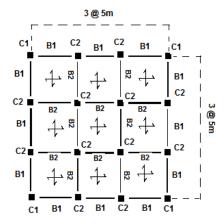


Figure 1. The structural plan of all floors

Table 3. Different types of seismic steel moment-resisting frames [6, 7, 8]

Structure	R
SMF-V3	10
IMF-V3	7
OMF-V3	5
SMF-V2	10
OMF-V2	6
MRF-V1	6

Table 4. The section properties of the members of SMF-V3 & OMF-V2 buildings, provided as samples

SMF-V3 Story	C1	C2	B2	B1
1	180×180×20	300×300×20	IPE400	IPE360
2	180×180×20	300×300×20	IPE400	IPE360
3	180×180×20	300×300×20	IPE400	IPE360
4	160×160×16	20×200×200	IPE330	IPE300
5	160×160×16	20×200×200	IPE330	IPE300

OMF-V2 Story	C 1	C2	B2	B1
1	200×200×16	340×340×20	IPE500	IPE500
2	200×200×16	340×340×20	IPE500	IPE450
3	200×200×16	340×340×20	IPE500	IPE450
4	180×180×16	240×240×20	IPE450	IPE360
5	180×180×16	240×240×20	IPE450	IPE360

1.3 Modeling of structures

Nonlinear analysis of frames has been carried out using OpenSees software. To simulate the behavior of steel material, a bilinear kinematic stress-strain curve has been assigned to the elements from OpenSees library [14]. In addition, displacement-based beam-columns in combination with fiber sections have been used to model cross sections as accurately as possible. Moreover, the corotational method has been accounted for considering geometric stiffness matrix. All models have been analyzed under incremental dynamic analysis (IDA) (Vamvatsicos & Cornell (2002) [17]) in order to evaluate the structure response from elastic to inelastic region.

To impose dynamic loading, 15 far field ground motions have been applied to the structures [18]. The accelerograms have been measured on soil type B, in which the average shear wave velocity at a depth of 30m would be 360-750 m/s. Properties of the imposed ground motions are presented in Table 5.

Table 5. The seismic characteristics of imposed ground motions (FEMA P695 [18])

	Earthquake	Date	Magnitude	Record Station	PGA max(g)	Distance(km)	Soil
				Beverly Hills-			
1	Northridge	1994	6.7	Mulhol	0.52	13.3	D
				Canyon			
				Country-			
2	Northridge	1994	6.7	WLC	0.48	11.9	D
3	Duzce, Turkey	1999	7.1	Bolu	0.82	12.2	D
	Duzee, Turkey	1///	/ . 1	Doiu	0.02	14.4	
4	Imperial Valley	1979	6.5	Delta	0.35	22.25	D

5	Imperial Valley	1979	6.5	El Centro Array#11	0.38	13	D
6	Kobe,Japon	1995	6.9	Shin-Osaka	0.24	23.8	D
7	Kocaeli, Turkey	1999	7.5	Duzce	0.36	14.5	D
8	Landers	1992	7.3	Yermo FirE Station	0.24	23.7	D
9	Landers	1992	7.3	Cool Water	0.42	19.85	D
10	Loma Pierta	1989	6.9	Capitola	0.53	22.1	D
11	Loma Pierta	1989	6.9	Gilroy Array #3	0.56	12.5	D
12	Superstition Hills	1987	6.5	Pore Road(temp)	0.45	11.45	D
13	Cape Mendocino	1992	7	Rio Dell Overpass	0.55	11.1	D
14	Chi-Chi Taiwan	1994	7.6	CHY101	0.44	12.75	D
15	San Fernando	1971	6.6	LA- Hollywood Star	0.21	24.35	D

2. NUMERICAL EXAMPLES

2.1 Analysis Results

Incremental Dynamic Analysis of the designed steel moment-resisting frames with different ductility and seismic design levels have been carried out using OpenSees software. As a sample median, the IDA results of the different type of frames are compared in Figure 2. It can be seen that special moment resisting frames have more capacity compared to ordinary and intermediate moment resisting frames. Besides, the inter-story drift (IDR) and peak floor acceleration (PFA) of SMF-V3 frame at different spectral acceleration (Sa) levels are shown in Figure 3-

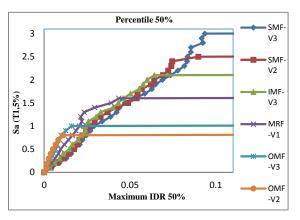
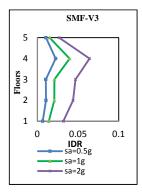


Figure 2. Median of IDA results of frames



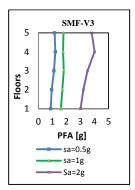


Figure 3. Median peak Inter-Story Drifts (IDR) and Peak floor acceleration(PFA) ratios, of all stories at several specific Sa levels in SMF-V3

The fragility curves of the frames for the collapse prevention (CP) level, based on FEMA 350 guideline, is shown as Figure 4. A lognormal cumulative distribution function is often fit to this data to provide a continuous estimation of collapse probability as a function of Sa.

The equation of this function is $P(C|Sa = x) = \phi(\frac{lnx - \mu}{\beta})$ in which $P(C \mid Sa = x)$ is the collapse probability at a given ground motion of Sa = x, Φ is the normal cumulative distribution function, and μ and β are the mean and standard deviation of lnSa.

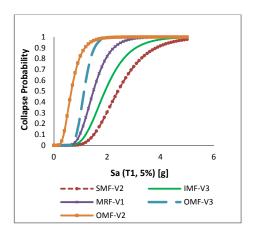
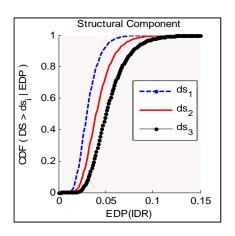


Figure 4. The collapse fragility curves of the frames for the collapse prevention (CP) level

As a sample, the fragility function of each component for a structural drift sensitive component (EDP of IDR) and a non-structural acceleration sensitive component (EDP of PFA), under their damage states, for 2 sample components is shown as in Figure 5. In this figure, CDF is the cumulative distribution of fragility function. To produce the Engineering Demand Parameters- Decision Variable (EDP-DV) function in each story, which is called $(E[L_{i,k}|NC.EDP_j])$, the normalized cost $(E[L_k|NC.DS_m])$ is multiplied by the corresponding fragility function $P[DS = ds_m \mid NC.EDP_k)$

Figure 6 shows a sample of EDP-DV function in SMF-V3 frame, which is the result of Equation 5. By assigning a lognormal distribution to structural response, the result of Equation 4 is achieved in Figure 7. Since it was mentioned before, the Decision Value (DV) is assumed to be identical to loss ratio.



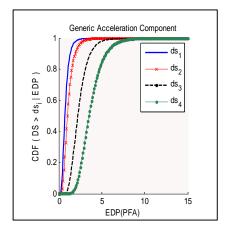


Figure 5. Fragility functions for IDR and PFA components

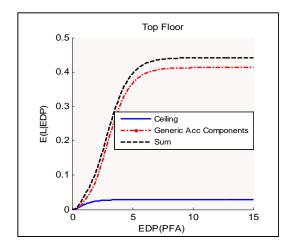


Figure 6. EDP-DV function in acceleration sensitive Components for the Top floor of SMF-V3 frame, that shows loss as a function of EDP

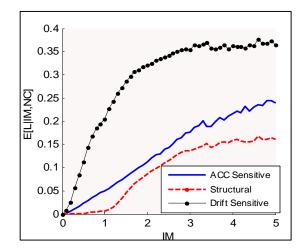


Figure 7. Loss curves (function of IM) assuming non-collapse (NC) condition, for the whole SMF-V3 frame

It can be concluded that the most contribution in loss is related to non-structural drift sensitive components. Additionally, for IM<0.5g, structural components' contribution in suffered loss is negligible in comparison with non-structural components.

2.2 Loss estimation results

Loss estimation under the non-collapse condition is calculated by multiplying EDP-DV function by probabilistic distribution of structural response (IDA results). The probability of collapse as in Figure 4, and non-collapse conditions (1.0-P(C|IM)) could be used to compute total loss, according to Equation 2. In addition, during the entire collapse of the frames, the rebuilding cost is considered as loss. Total loss curves as a function of IM are displayed as Figure 8.

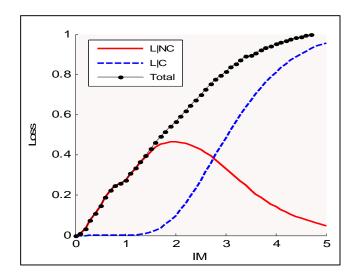


Figure 8. Loss curves (collapse, non-collapse, and total) in SMF-V3 frame, IM= Sa(T1,5%)

It is observed that by increasing the intensity measure of earthquake, non-collapse loss decreases due to its low collapse probability, while collapse loss surges.

In order to show a distinction of total loss among all six examined models, the mentioned approach is repeated for all frame models. Consequently, the estimated cost losses are presented in Figure 9, that indicates that, reduction in frame ductility, from SMF-V3 to OMF-V2 for instance, reduces the non-collapse loss, because of its direct relation with the amount of EDP, unlike the range of collapse loss increases. In addition, collapse initiates at Sa=1.5g in SMF-V3, while it starts for Sa<0.2g in OMF-V2 due to the ductility and capability of standing more drift in SMF-V3. According to collapse fragility (Figure 5), it is evident that SMF-V3 (e.g., R=10; the most ductile frame) is not only less probable to collapse in a certain Sa in comparison with others, but also tolerates a larger range of Sa. For this reason, this frame has the minimum total loss since P(C|IM) is involved in loss calculations as is presented in Equation 2 and Figure 9.

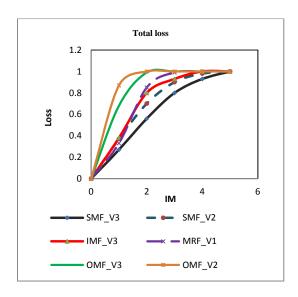


Figure 9. Comparison of total loss

3. CONCLUSION

In this paper, the PEER's overall story-based framework has been utilized for loss estimation of structures instead of using a component-based methodology, (Ramirez and Miranda (2009) [4]). It can be observed that the efficiency of computations is elevated because all components are categorized according to seismic sensitivities. Simultaneously, corresponding cost distribution has been considered for all of buildings' component inventory. Six steel moment-resisting frames, in the high level of relative seismic hazard zone, have been designed in compliance with Iranian seismic design code. Subsequently, the loss estimation of all six frames has been carried out using the story-based methodology. In the next stage, the results of the incremental dynamic analysis have been used to evaluate the collapse, fragility functions, and structural responses. The repair costs for each building have probabilistically been estimated using MATLAB to calculate the total loss stages. Although the results, presented herein, are based on a number of simplifying assumptions, they have been consistently applied to all buildings. Thus, the results of relative seismic performance, expressed in terms of economic losses, could effectively be compared to each other. To detect the effect of ductility on seismic loss, it has been considered as a variable. According to the set of buildings in this study, it has been observed that ductility leaves a positive impact on the expected losses. In other words, within frames, studied in this paper, the special moment-resisting frame, suffered the minimum rate of total loss in comparison to the other moment-resisting frames. Moreover, the contribution of non-structural components in non-collapse (NC) loss state is more impressive than other components. Finally, it can be observed that the effect of earthquake design code, from the first edition of Iranian earthquake design code toward the third edition, on seismic loss estimation goes toward reducing loss rates of frame by increasing frame ductility. In fact, the losses reduce in SMF-V3 and OMF-V3 samples, when compared to SMF-V2 and OMF-V2, respectively. It seems that the value of design base shear force, which has been presented in Table A-2, has a profound impact on the loss reduction.

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