METHODOLOGY FOR THE REASSESSMENT OF MAGNITUDES ASSIGNED TO HISTORICAL EARTHQUAKES

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ABSTRACT

Since the end of the nineteenth century, France has encountered several large earthquakes, however, the country faces, (1) the small amount of data and (2) uncertainties related to the macro-seismic intensity, as well as those related to the epicenter location.

No considerable damage has been reported in historical documents found up to now. This work proposes a reassessment of the intensity based on probabilistic method and structural analysis. The method is built on an approach initially proposed by Ryu and Baker, who applied Bayesian updating to re-evaluate magnitudes of earthquakes based on the distribution of the damage states of the building stock. Several structures in the area considered for this study, still existing nowadays, have experienced historical earthquakes. Among them, a statue has been selected to develop and illustrate the global method. Despite the lack of precise information regarding this structure, this choice is motivated by the relative simplicity of the structure and the possibility to consider a model based on rigid-body motion. More particularly, this model of inverted pendulum proposed by Housner is used to investigate the response of the statue under the seismic loading. The different damage levels are defined in terms of amplitude of the rotation of the rigid-body, sliding or rocking. Despite its simplicity, this type of models gives sufficient information in order to compute fragility curve.

Keywords: historical earthquakes; fragility curves; Bayesian actualization; inverted pendulum.

1. INTRODUCTION

For about 40 years, having been started in support of the development of the nuclear industry in France, efforts have been jointly made by BRGM (Bureau de Recherche Géologique et Minière), EDF (Electricité de France) and IRSN (Institut de Radioprotection et Sûreté Nucléaire) to collect, compile, and distribute information related to historical events. The SISFRANCE macroseismic database contains 100,000 macroseismic observations (MSK intensity scale—Medvedev et al. (1967)) associated to 6000 earthquakes (AD463-2007). These Intensity Data Points (IDPs) are representative of earthquake effects in terms of damages and population perception at various localities. The descriptions of these effects, used to assess intensity values are collected from historical archives for each event. Epicentral location is determined and provided, together with the epicentral intensity value when possible (see Lambert et al. (2015) for epicentral location and intensity assessment explanations). IDPs are associated to quality factors that reflect confidence related to numerical value.
(quality A: certain intensity, quality B: fairly certain intensity, quality C: uncertain intensity). Epicentral intensity estimates are also associated to quality factors (quality A: certain epicentral intensity; quality B: fairly certain epicentral intensity; quality C: uncertain epicentral intensity, quality E: arbitrary epicentral intensity; quality K: fairly certain epicentral intensity, resulting from a calculation based on intensity attenuation).

According to the occurrence date of earthquakes and/or their location with respect to France borders, macroseismic field configuration can be observed in the SISFRANCE database as follows: (i) recent and largest events, located inside or close to the borders, exhibiting large and well distributed macroseismic field, (ii) off-shore or cross border events, characterized by a lack of information at short distances but with reliable data at greater distances, and (iii) old events associated with a poorly constrained macroseismic field, where either only the epicentral intensity is quantitatively known or no intensity value is available, expect for few felt testimonies.

Recent methodologies (Ryu et al., 2009) propose to introduce structural analyses in the process of definition of the macroseismic intensity of historical earthquakes. The main idea is to update the distribution of intensities or magnitudes of the considered earthquake by means of a Bayesian approach, combining the use of input data fragility curves and in-situ observed damages.

In the context of the French metropolitan territory, characterized by low to moderate seismic activity, and a large amount of old structures as well as cultural heritage, this methodology appears to be an interesting way to reanalyze historical earthquakes. The work exposed in this paper applies the methodology proposed in (Ryu et al., 2009) to a historical earthquake in France.

The first part of the paper is dedicated to the vulnerability analysis of the simple structure considered for the study and for testing the methodology. Different mechanical models are used for this purpose. More particularly, the process of defining and computing the response of the structure is exposed. The objective is to obtain a model capable of representing the main types of failure of the considered structure with a relatively short computational time. The second part recalls the main elements of the method proposed by (Ryu et al., 2009). In the last part of the paper, the methodology is applied to the site and the earthquake considered.

### 2. VULNERABILITY ANALYSIS OF A STATUE

In order to apply the methodology, a simple structure was chosen among the different structures which had been exposed to the earthquake on the considered site: the statue “La Madone” (Figure 1).

![Figure 1. The statue “La Madone”](image)
2.1 Main characteristics of the statue

This statue was built in 1858 in the Constant Baud’s workshop. The pedestal is made of Trept’s stone with a height of 5.6 meters. The statue is probably made of cast iron with a height of 3.2 meters. As no test of the material could be made, only an order of magnitude for the material characteristics has been considered: \( E = 100 \text{ GPa}, \rho = 7000 \text{ kg/m}^3 \).

At first a model with a continuous description of the statue has been considered not only to quantify the order of magnitude of the main eigenmode (Figure 2) but also to get global characteristics for the simplified model presented in the next part.

Figure 2. Main eigenmode for the 3D model of the statue

From this first analysis, the results obtained allow to define some hypotheses for a simplified modeling. The pedestal is sufficiently massive to directly transfer the signal to the bottom of the statue. As a consequence, the simplified model proposed in part 2.2 only considers the iron cast statue without the pedestal. Regarding the first eigenfrequency, the value obtained with this model is much higher than the one of the studied site. It is beyond the frequency where earthquake load contains considerable energy. As a consequence, it is sufficient to represent the statue by a rigid body. In this simplified model, the modes of failure of a rigid body (sliding, rocking and tilting) are investigated.

2.2 Simplified model of the statue: rigid body movement

Housner (1963) proposed a simple model of a pendulum to describe the rocking behavior of a rigid body. The structure studied is represented in 2D by a rectangle with a height of 2H and a width of 2b. This body has a moment of inertia equal to \( I_0 \) and a weight \( W \) with the corresponding mass \( m \).

Figure 3. Inverted pendulum model

The length \( R \) defines the distance between the point of contact O or O’ and the middle of the rectangle.
\( \alpha \) defines the angle between the side of the rectangle passing through O or O’ and the diagonal of the rectangle.

The model considers an infinite friction between the rigid body and the foundation in order to get only a rotating movement around the point of contact O or O’.

The movement is defined by the angle \( \theta \) (angle between the vertical passing through O or O’ and a side of the rectangle). During the oscillation around the point of contact O, the equilibrium of the rigid body reads:

\[
I_0 \times \frac{d^2\theta}{dt^2}(t) = -P \times R \times \sin[\alpha - \theta(t)]
\]  

(1)

There is dissipation during the movement each time the rigid body encounters the foundation with a coefficient of restitution \( r \), obtained by considering the kinetic momentum conservation.

\[
r = \left[ 1 - \frac{mR^2}{I_0} \times (1 - \cos(2\alpha)) \right]^2
\]  

(2)

To compute the response of the rigid body under seismic loadings, an implementation in Matlab© of the method proposed by (Yim et al., 1980) has been performed. The validation of this implementation has been done by comparing the results obtained with the ones given in (Makris and Konstandinis, 2001).

![Figure 4. Response of a rigid body submitted to sinusoidal pulse: a) Makris and Konstandinis, 2001 b) Implementation Matlab](image)

### 2.3 Definition of failure modes

In order to compute fragility curves, one has to define structural failure or damage criteria. In this case of a rigid body model, three criteria are considered: rocking, sliding and tilting.

The first two modes require only the knowledge of the maximum of amplitude of the input signal whereas the last one needs the computation of the whole time-response of the rigid body.

#### 2.3.1 Rocking
This mode of failure corresponds to a level of peak ground acceleration sufficiently high to counterpart the resisting moment obtained with the self-weight and the gravitational acceleration. According to the notations introduced earlier, this criterion leads:

\[
p_{ga} > \frac{B}{H} \times g \times \left(1 + \frac{a_{\text{ground}}}{g}\right)
\]  

\(a_{\text{ground}}\) corresponds to the vertical component of the ground acceleration. When only the horizontal component is considered, the criterion reduces to equation below

\[
p_{ga} > \frac{B}{H} \times g
\]  

This criterion of failure may be too hard to observe directly with in-situ investigations, as the rigid body should regain its initial position after the earthquake if it does not tilt. However, it has the advantage to be very simple to compute and to give a first level of damage. Furthermore, one can imagine by using this criterion for small objects, for which testimonies of rocking may be found.

2.3.2 Sliding

This criterion is based on a simple Coulomb friction model. It assumes a failure as soon as the peak ground acceleration generates an inertia force higher than the friction one. This criterion leads:

\[
p_{ga} > \mu \times g
\]  

with \(\mu\) the coefficient of friction. A coefficient of friction similar to the one between steel and concrete has been considered for the statue, due to the lack of data for this specific case studied. The sliding criterion also has the advantage to be very simple to compute, and could be used to investigate the response of small objects. Furthermore, one can observe with in-situ investigations if the structure has slid during the earthquake. This criterion can also be considered for low damage case.

2.3.3 Tilting

This criterion defines the complete failure of the structure. It corresponds to the state of the structure for which the center of mass is vertically aligned with the rotation center. It corresponds to an unstable state leading to the tilting of the structure with a small amount of energy. According to the geometrical characteristics of the simple model, this criterion leads:

\[
\max_{t} \theta(t) \geq \alpha
\]  

This criterion is more complex to obtain, as a time-history analysis needs to be performed. Furthermore, the amount of energy needed to reach this state is relatively large, so this criterion corresponds to the highest level of damage (i.e. complete damage).

2.4 Fragility curves

This part provides the main information and tools that are used to compute the fragility curves for the studied structure.

2.4.1 Generation of synthetic earthquakes accelerograms

In order to compute the response and to determine the potential damage of the structure subject to seismic loadings, one has to define a set of earthquakes. In this study only very slight natural
earthquakes are available, that is why synthetic accelerograms are used. This database is generated by considering the frequency and the dissipation of the studied site. These data can be obtained thanks to in-situ tests using ambient noise vibrations. The synthetic accelerograms are generated thanks to a stochastic process using the power spectral density (PSD) representation of the earthquake. This PSD is computed according to the model of (Kanai, 1957, Tajimi, 1960) and modulated using a gamma function $q(t)$:

$$q(t) = \alpha_2 \times t^{(\alpha_2-1)} \exp(-\alpha_3 \times t)$$ (7)

$\alpha_2$ and $\alpha_3$ respectively drive the shape and the duration of the strong part of the signal. $\alpha_3$ defines the energy of the signal. The methodology used to generate the database is described in details in (Zentner, code_Aster).

2.4.2 Numerical computation of fragility curves

Fragility curves express the conditional probability of failure $P_f$ of the structure for a given seismic Intensity Measure IM denoted $\alpha$. In this study, the failure is defined by the criteria expressed in section 2.3, and the Intensity Measure is the peak ground acceleration – PGA. From the fragility curve, one can define a probability of non-failure, according to a damage level $d$ for an intensity level $\alpha$.

$$P(d|\alpha) = 1 - P_f(\alpha)$$ (8)

To reduce the number of calculations of the structural response under seismic loadings, the classical lognormal fragility model is used (Reed and Kennedy, 1994). The computation of the two parameters defining the fragility curve (Equation 9) is performed thanks to the maximum likehood methodology (Shinozuka, 2000).

$$P_f(\alpha) = \phi \left( \ln \left( \frac{\alpha}{A_m} \right) \right)$$ (9)

$A_m$ is the median capacity and $\beta$ is the logarithmic standard deviation. Figure 4 shows the fragility curve and the probability of no damage for the case corresponding to the sliding failure mode. The parameter $\alpha$ considered here is the PGA.
3. MAGNITUDE ESTIMATION OF A HISTORICAL EARTHQUAKE

The analysis presented here to estimate the magnitude of an historical earthquake is based on the methodology proposed by (Ryu et al., 2009). In the present work, the approach is slightly modified so as to investigate the magnitude of “small” earthquakes. Indeed, the structures considered are the ones that have not encountered damage or only relatively small degradations. The main objective is to analyze if the magnitude currently considered is consistent with the state of the structures that have experienced the historical earthquake. The methodology proposed by (Ryu et al., 2009) is briefly recalled and is then applied to the case studied.

3.1 Methodology

The estimation of a posteriori magnitude is obtained by Bayesian updating of the a priori magnitude $f_M(m)$ of the earthquake. This realization yields the knowledges $d$ of damaged or undamaged structures. The magnitude posteriori $f(m|d)$ is computed according to:

$$ f(m|d) \propto P(d|m)f_M(m) $$

This actualization needs the probability $P(d|m)$ of damage observation $d$ for a given magnitude $m$.

$$ P(d|m) = \int \int f_R(r)f_s(s)drds \int f_{im}(\alpha|m,r,s)d\alpha $$

The functions $f_R(r)$ and $f_s(s)$ allow to introduce respectively the uncertainties relative to the localization of the earthquake and to the site proxies (e.g. resonant frequency, VS30). The function $f_{im}(\alpha|m,r,s)$ corresponds to a Ground Motion Prediction Equation (GMPE) using site proxies.

3.2 Characterization of the historical earthquake

According to FCAT-17 (Manchuel et al, 2017), the considered earthquake has an average moment magnitude of $M_w=4.4$ with a standard deviation equal to 0.42. This information is used to define the a priori magnitude distribution. Different choices are investigated (Figure 5):
- Gaussian distribution centered on the average $M_w$ with the standard deviation given by the catalogue,
- Gaussian distribution centered on the average $M_w$ with a higher standard deviation in order to include a possible error on the macro-seismic initial classification
- Uniform distribution
Currently, as no GMPE including site proxies for the specific studied region exists, the GMPE developed by (Derras et al., 2012) for Japanese data has been considered. This GMPE has been established with the Japanese KiK-net data. It has the advantage of depending on the fundamental frequency of the site, which is used as a site proxy in this work. The GMPE has been derived by means of Artificial Neural Network. The parameters defining the neural network are given in (Derras et al., 2012).

For this study, no uncertainty is considered for the site proxies. For the localization of the earthquake, an uncertainty regarding the depth is introduced through a lognormal distribution (Figure 7).

3.3 Reassessment of the magnitude

The methodology presented in this section is used to reanalyze the magnitude of the considered historical earthquake. As expected, the first result is the fact that a too large capacity of the structure with respect to the seismic load (for example in terms of PGA) does not allow to obtain any new information on the magnitude distribution. The structure does not experience any damage in the magnitude range where the seismic load is close to the structural capacity values. This magnitude range is much higher than the orders of magnitude of the considered earthquake.

In this study, the PGA needed to tilt the statue is much too large. As a consequence, only the first two criteria are used.

3.3.1 Reassessment of the magnitude according to the rocking criterion

For the rocking criterion, Figure 8 gives the fragility curve and the probability of no damage.
Figure 8. Fragility curve and probability of no damage for rocking criterion

Thanks to a Bayesian updating process, considering the probability of no damage curve, as no damage has been observed on the structure studied, Figure 9 gives the a priori and a posteriori magnitude distribution.

Figure 9. A priori and a posteriori magnitude considering the rocking criterion

The introduction of knowledge in the quantification of the magnitude allows to increase the confidence in the expected value of magnitude while its standard deviation decreases. The median capacity of the structure regarding the rocking criterion is relatively close to the PGA that could be observed at the a priori median magnitude using the GMPE of (Derras et al., 2012). As a consequence, the modification of the expected a posteriori magnitude is minor compared to the expected a priori magnitude.

3.3.2 Reassessment of the magnitude with the sliding criterion

For the sliding criterion, Figure 10 gives the fragility curve and the probability of no damage.
Figure 10. Fragility curve and probability of no damage for sliding criterion

Thanks to a Bayesian updating process, considering the probability of no damage curve, as no damage has been observed on the structure studied, Figure 11 gives the a priori and a posteriori magnitude distribution.

Figure 11. A priori and a posteriori magnitude considering the sliding criterion

The same tendencies as the ones observed for the rocking criterion are obtained for sliding (i.e. increasing confidence in the expected value of magnitude and decreasing standard deviation). For this criterion, as the median capacity is smaller than that for the rocking, the analyses are more significant for a reassessment of the distribution of the magnitude of the historical earthquake. This criterion seems to be interesting to reassess the magnitude, but the friction coefficient as well as the weight of the statue are uncertain parameters. These uncertainties need to be considered in the analysis, to further improve the reassessment.

4. CONCLUSIONS

A methodology to reassess the magnitude of historical earthquakes has been applied in this work. The information relative to the studied historical earthquake has been extracted from a large French database. As the level of uncertainty is relatively high for some earthquakes, the introduction of knowledge to reassess the magnitude is considered thanks to a Bayesian process. For the assessment of the feasibility of the method, the structural analysis has been limited in this study to a statue. Three
criteria have been defined for the fragility curve and the no damage distribution. In this case where no damage has been observed, the criterion leading to a median capacity much larger or lower than the one corresponding to the a priori expected magnitude tends to be useless. By using criteria more in accordance with the a priori magnitude, it has been shown that this method helps to increase the confidence in the expected magnitude and to decrease the standard deviation of the magnitude distribution. This method tends to be promising to reassess historical earthquakes. The work presented here is the first step of a more ambitious work program where it is intended to further analyze the damage observed after an earthquake. By including experimental assessment and numerical modelling, the magnitude assumed in the earthquake catalogue for some historical earthquakes will be analyzed.

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6. REFERENCES

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