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APPLICATION OF POLYNOMIAL ANALYTICAL MODEL FOR RUBBER BEARINGS IN SHAKING TABLE TEST SIMULATION

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ABSTRACT

Over the end of the last and beginning of this century, many isolation devices have been invented where rubber bearings are among the most frequently used isolators. The performed investigations confirmed that the actual behavior of rubber bearings under large shear strains is highly nonlinear. Therefore, in this investigation, a bilinear constitutive law is replaced by an advanced polynomial model in order to analytically model the bearing's nonlinear behavior. The polynomial model simulates the behavior of natural rubber bearings in case of small and large deformations. It is capable to cover the strengthening of the rubber during large deformations where the loading history effects are included. The model is based on polynomial function and eight additional parameters which are obtained from biaxial bearing tests.
The aim of this study is implementation of the polynomial model into finite element software package and its validation. Firstly, the experimental tests on the cylindrical rubber bearings were performed in order to verify the performances and the stability of the model. Then, a base isolated liquid storage tank was tested on earthquake simulation table to experimentally obtain the seismic response of a representative base isolated structure. Finally, the polynomial analytical model was verified by comparing results obtained from earthquake simulator tests to results calculated from nonlinear time history analysis. The obtained results showed a satisfactory accuracy of the polynomial analytical model in the case of the finite element dynamic analysis of the base isolated structure.

Keywords: Analytical model for rubber bearings; shaking table test; nonlinear dynamic analysis

1. INTRODUCTION

Over the end of the last and beginning of this century, seismic isolation as a technique for earthquake protection was used on many different types of structures. Many isolation devices have been invented where rubber bearings are among the most frequently used isolators (Skinner 1993 and Naeim 1999). The laboratory investigations performed on the rubber bearings confirmed that the actual behavior of the rubber bearings under large shear strains is highly nonlinear (Gjorgjiev 2013, Hwang et al. 2002, Jankowski 2003, Kikuchi 1997 and Tsai et al. 2003). The lateral behavior of the rubber bearings is characterized by high horizontal stiffness under low shear strains, low stiffness under moderate strains, and an increasing shear modulus under higher strains. Because, the generally accepted bilinear model is not enough capable to analytically simulate the realistic nonlinear rubber bearing behavior, in this investigation, an advanced polynomial analytical model is used (Gjorgjiev 2013). The polynomial model simulates the behavior of natural rubber bearings in case of small and large deformations where

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the loading history effects are included. The model is based on polynomial function and eight additional parameters which are obtained from biaxial bearing tests. A least-square regression was used to minimize the sum of the squares in an n’th order polynomial model. System property modification factors for rubber bearing defined in AASHTO, 2010 can also be included, if necessary.
The aim of this study is implementation of the advanced polynomial model into finite element software package and performing a nonlinear dynamic analysis of the representative base isolated structure. The first part of the paper deals with the experimental investigations performed on rubber bearings. The isolators were tested under vertical and horizontal dynamic loads, whereat a series of harmonic excitations with different amplitudes and frequencies were applied. The biaxial tests were performed with two sets of horizontal harmonic excitations: (1) with constant amplitude and (2) with linearly increasing amplitude. A database on the behavior of the isolators in different loading conditions was created which was used to calculate the parameters for the analytical model and to validate it. In this part, the results from the vertical and horizontal tests carried out on a few characteristic bearings are presented through force-displacement diagrams.
Then, the mathematical formulation of the polynomial analytical model is presented. The used model (Gjorgjiev 2013) can simulate the behavior of natural rubber bearings in conditions of small and large deformations. The model is defined by a polynomial function and eight parameters and it is able to cover the strengthening of the rubber in conditions of large deformations. It includes the loading history effect and enables adaptation to different shapes of loading/unloading. A least-square regression was used to determine the “best” coefficients in order to minimize the sum of the squares in an n’th order polynomial model. System property modification factors for the rubber bearing defined in AASHTO, 2010 can also be included, if necessary.
In the last part, the earthquake simulator testing of a small base isolated cylindrical steel reservoir was performed. The seismic response of the base isolated structure was experimentally obtained for the characteristic earthquake. Only the time history of the horizontal acceleration and displacement at the base and at the top of the model are presented in this paper. Next, the analytical simulation of the actual behavior of the base isolated structure was performed by comparing the experimental results to the results obtained from the three-dimensional finite element analysis. The behavior of the steel and the concrete structural elements was modeled as linear-elastic while the rubber bearings were modeled with the polynomial analytical model for rubber bearings (Gjorgjiev 2013).
The obtained results showed a satisfactory accuracy of the polynomial analytical model in the case of the finite element dynamic analysis of the base isolated structure. The model enables analytical modeling of the rubber bearings behavior with a sufficient accuracy.

2. TESTING OF CIRCULAR RUBBER BEARINGS

The purpose of the testing of the bearings on a two-component dynamic frame was to determine the force-displacement relationship under different loading conditions, which were used for developing of the analytical model and for simulation analysis. The test results for the square shaped rubber bearings were used for developing of the polynomial model while the cylindrical bearings in the simulation analysis. The isolators were produced by a small local company which also produced the steel mould for both shapes of bearings. Depending on the rubber compound, the technological process anticipates completing of the vulcanization of the rubber for a period of 50 to 60 minutes at temperature of $(140\div150)^{o}$C. In the course of the vulcanization, the rubber was constantly exposed to external pressure of 150 bars from the upper and the lower side (Figure 1a).

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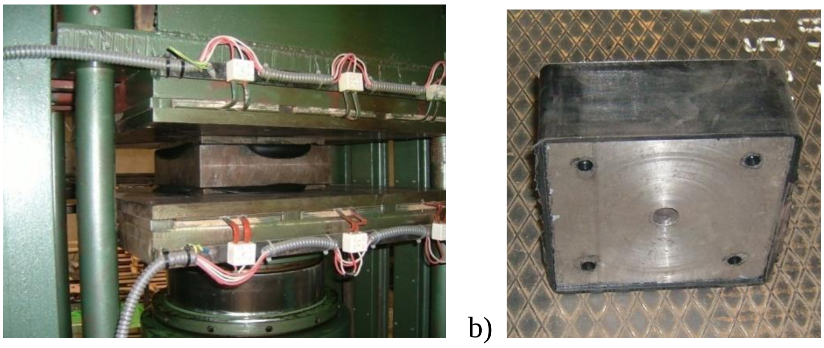

Figure 1. a) Vulcanization of the isolator b) View of the isolator

The tests were performed on the biaxial dynamic frame with a static capacity of 200kN and a dynamic capacity of 100kN. The actuators are capable to apply the static load in vertical direction and cyclic dynamic load in horizontal direction. The testing procedure includes: (1) thermal stabilization of the rubber by keeping it under room temperature of (22±2ºC) for 24 hours, (2) elimination of the Mullins' effect (Mullins, 1969) by applying three full sinusoid cycles at a low frequency and (3) application of the testing program. The testing program was divided into two separate tests (1) vertical and (2) biaxial (vertical and horizontal). The vertical tests consisted of loading the element with a vertical compressive force at different axial stress levels. First the element is loaded at predefined vertical force and then the three sinusoidal cycles with low force amplitudes are applied. The horizontal tests consisted of application of lateral load while the element was under axial load. The specimens were exposed to two different lateral loading histories: (1) sinusoid function with constant amplitudes and (2) sinusoid function with linear increasing amplitudes.
The results of the tests done in axial direction for squared bearings (Figure 1b) produced by hard and soft component of rubber are presented through vertical force-stiffness relationship (Figure 2a). Figure 2b shows the vertical force-displacement relationship for the rubber bearings produced by different number of internal steel plates. The results for the cylindrical bearings are given in the Table 1. The obtained vertical stiffness at different vertical loads leads to the conclusion that the stiffness increases with the increase of the load.
The biaxial tests were performed to define the behavior of the bearings under different loading conditions. The main aim of those test was to obtain the data which is used to develop the analytical model and to develop the model parameter for performing the finite element simulation analysis of the base isolated structure. The shear behavior of the isolators under the effect of harmonic excitation with the constant amplitude is presented in Figure 3. The presented graphs provide a clear insight into the difference in the damping abilities and the difference in shear behavior of the different rubber compounds. The lateral behavior of the cylindrical bearing is presented through horizontal force-displacement curves (Figure 4). The behavior under the effect of the sinusoidal excitation with constant amplitude is presented in Figure 4a, while Figure 4b shows the behavior under excitation with linear increasing amplitudes.

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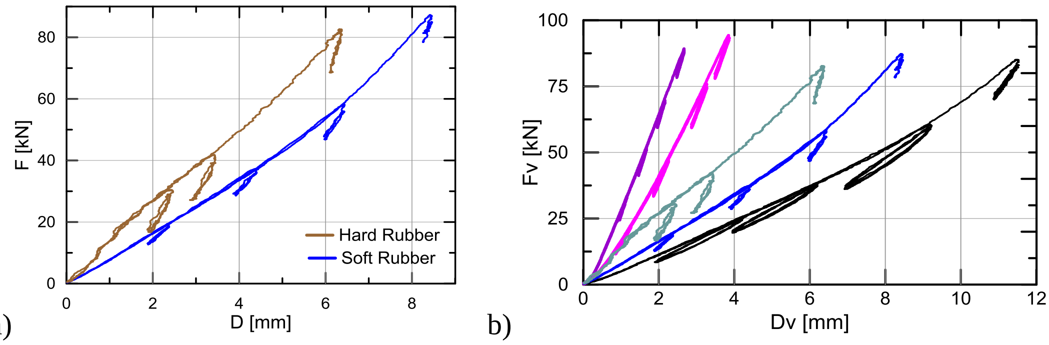

Figure 2. Relationship between vertical stiffness and vertical force 2) hard and soft compound b) different rubber compounds

Table 1. Vertical stiffness of rubber bearing at various vertical loads for the cylindrical bearing

F1 [kN]	K1 [kN/m]
12.0	13540.0
22.0	14100.0
32.6	15220.0
42.6	16700.0
55.0	19900.0
69.3	23700.0
81.4	28700.0

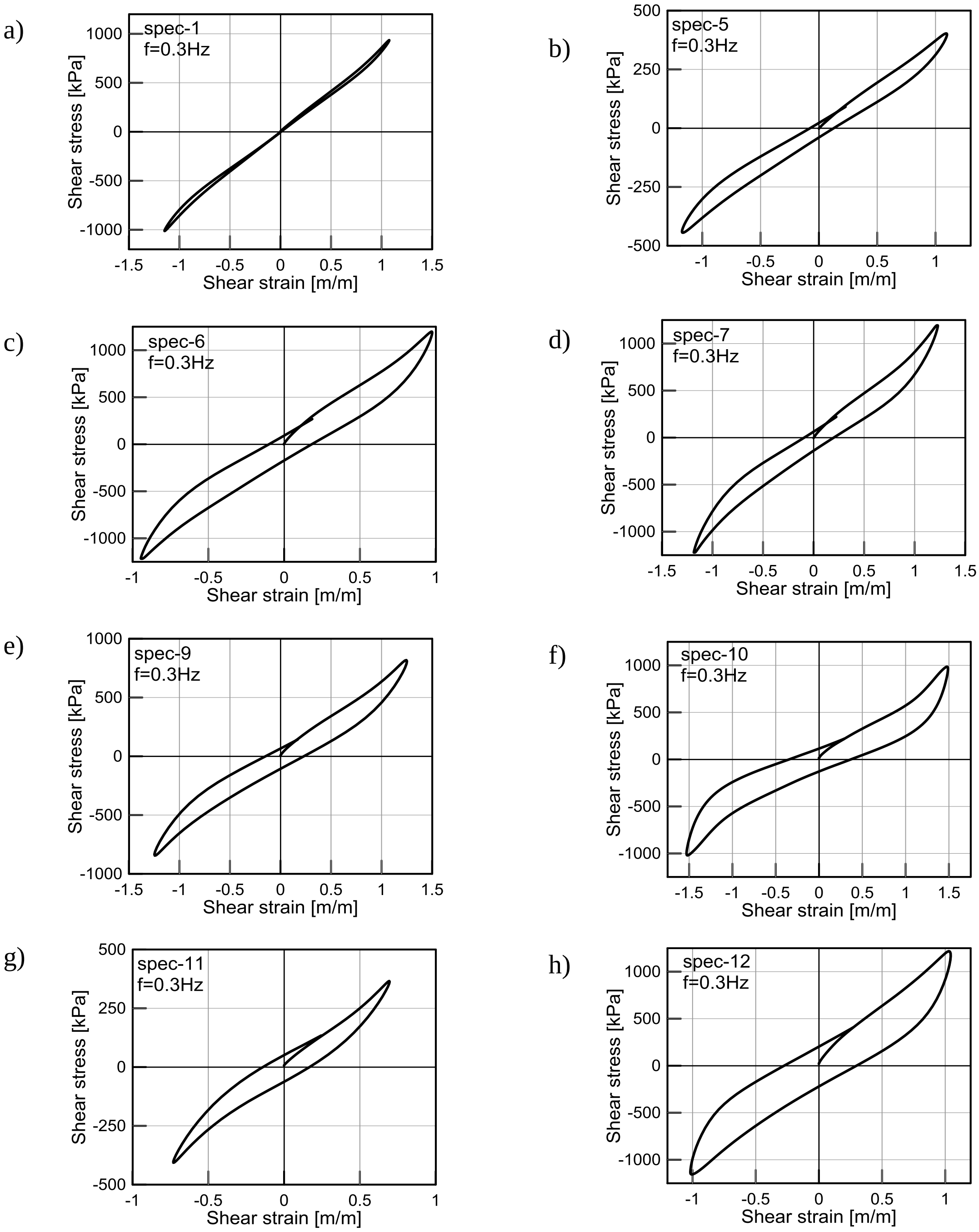

Figure 3. Comparison of horizontal behavior of bearings made of eight different rubber compounds

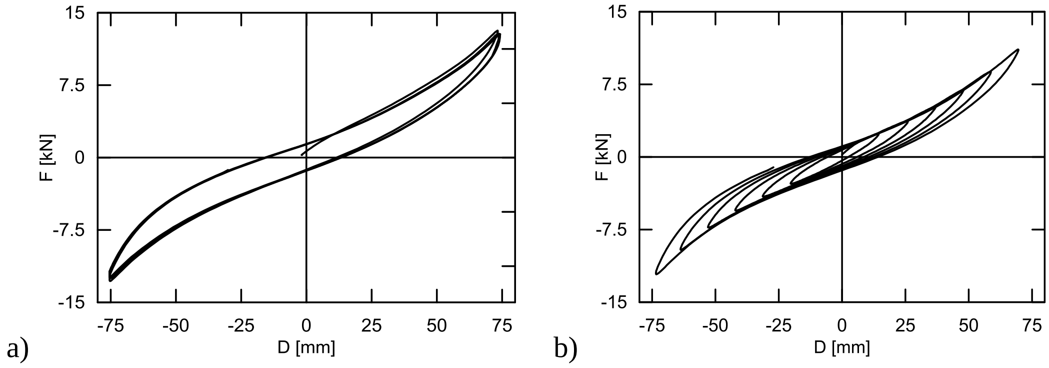

Figure 4. Relationship between horizontal displacement and horizontal force for cylindrical rubber bearings: a) constant amplitudes b) increasing amplitudes

3. MATHEMATICAL FORMULATION OF THE POLYNOMIAL ANALYTICAL MODEL

The polynomial analytical model was developed as a part of extensive study on behavior of bearings produced by 18 different natural rubber compounds (Gjorgjiev 2013). The aim of the polynomial model is to mathematically describe the behavior of the natural rubber bearings and in simple way to model the rubber behavior at large strains. The model is capable to simulate the behavior of natural rubber bearings in case of small and large deformations where the loading history effects are included. It includes the linear-elastic behavior, the post-elastic behavior at loading and the post-elastic behavior at unloading (Figure 5).

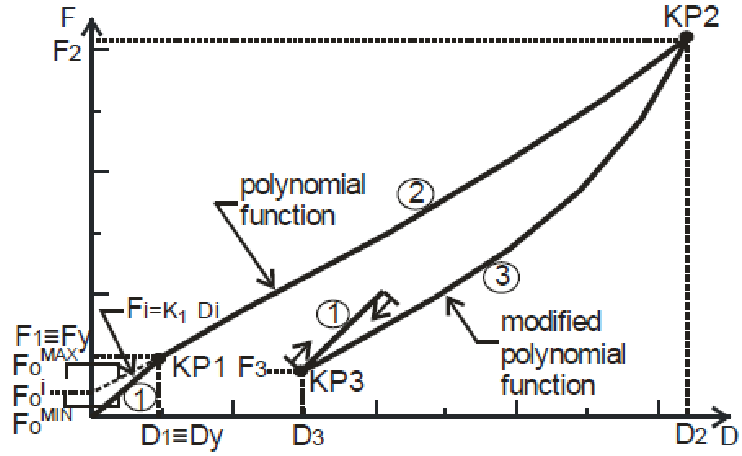

Figure 5. Polynomial analytical model of bearings made of rubber

The linear elastic state includes elastic behavior of the bearing at low strains and it is presented by the following expression:

$$F = K_1 \cdot D \tag{1}$$

where F is the horizontal force, D is the horizontal deformation and K1 is the elastic (initial) stiffness of the bearing. The linear elastic state of this model takes place at the beginning of loading of the bearing and at the transition from unloading into loading conditions (Figure 5).
The post-elastic state at loading involves the behavior of the bearing after the yielding point. Once the yielding point is exceeded, the force-displacement relationship is defined by the polynomial function given in Equation 2.

$$F = F_o + a_1 D + a_2 D^2 + a_3 D^3 + \dots \qquad (2)$$

where F is the current horizontal force, D – the current horizontal deformation and F_0, a_1, a_2, a_3,... are the coefficients of the polynomial function which are calculated for best fitting to the experimental curve.

The polynomial coefficient F_0 is defined by Equation 3 which is based on the current and previously experienced horizontal deformation.

$$\begin{aligned} F_o &= F_0^{min} && \text{when } D^{kp} < D_0^{min} \\ F_0 &= F_0^{min} + \left(F_0^{max} - F_0^{min}\right)\frac{D^{kp} - D_0^{min}}{D_0^{max} - D_0^{min}} && \text{when } D_o^{max} < D^{kp} < D_0^{min} \\ F_0 &= F_0^{max} && \text{when } D^{kp} > D_0^{max} \end{aligned} \qquad (3)$$

where $D^{kp} = \max(D^{kp}, D)$

Forces F_o^{min} and F_o^{max} and deformations D_o^{min} and D_o^{max} are parameters defined through tests on bearings.

The post-elastic state at unloading includes the behavior of the bearing when $|D_{i-1}|>|D_i|$, where $|D_{i-1}|$ is the previous horizontal deformation, $|D_i|$ is the current horizontal deformation of the bearing. The mathematical force-displacement relationship for post-elastic behavior at unloading is formulated by the same polynomial function for post-elastic behavior at loading which is given in Equation 2. To include different types of behavior of the bearings at unloading, modification is made in the computation of coefficient F_o^u (Equation 4).

$$F_o^u = F_o^{in}(2 \cdot kF_o - 1) \qquad (4)$$

where kF_0 is the decay coefficient and it is computed according to Equation 5.

$$kF_o = \frac{e^{\left(eD^{kp} \cdot \frac{D}{D^{kp}}\right)} - 1}{e^{eD^{kp}} - 1} \qquad (4)$$

where e is the natural logarithmic base (e=2.71828...), D^{KP} is the horizontal deformation of the isolator at the moment of beginning of unloading and eD^{KP} is the exponent at the moment of beginning of unloading.

The value of eD^{KP} is computed according to Equation 5 where eD^{KP} is linearly dependent on D^{KP}. The remaining variables are input parameters and they are defined on the basis of horizontal device tests.

$$eD^{kp} = eD_{min} + (eD_{max} - eD_{min}) \cdot \frac{D^{kp} - D_e^{min}}{D_e^{max} - D_e^{min}} \qquad (5)$$

where eD_{min} is exponent at deformation De_{min} and eD_{max} is exponent at deformation De^{max}.

This expression is valid only when deformation D^{KP} is within the range of [De^{min}, De^{max}]. In the case the deformation at the characteristic point is beyond the above domain, then it is assigned the limit values (De^{min} or De^{max}) of the interval (Figure 6). The graphical interpretation of the dependence of eD^{KP} on D^{KP} is shown in Figure 6.

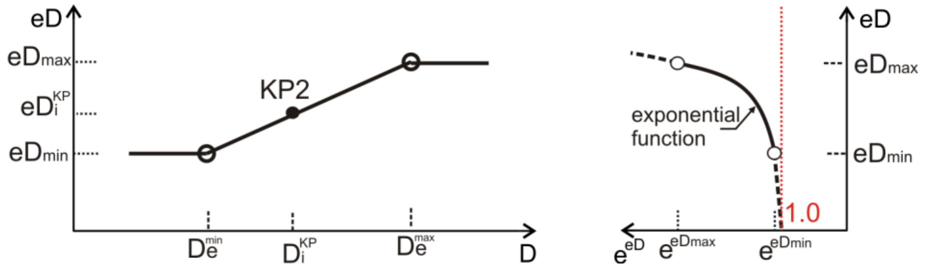

Figure 6. Dependence of the exponent on the horizontal deformation of the bearing

The computation of the decay coefficient kF_0 includes four parameters (D , D^{KP}, eD^{KP} and F_o^{in}) that directly depend on the current deformation of the bearing and the history of deformations. The current deformation is included in parameter D and its value at unloading ranges between D^{KP} and zero. The history of deformation of the bearing is included in parameters D^{KP} and eD^{KP}. Parameter D^{KP} is the maximum achieved deformation at loading, while parameter eD^{KP} directly depends on D^{KP} and it is calculated according Equation 5.
From equation (4), it can be seen that the value of the decay coefficient kFo ranges within the following limits:

for $$\begin{cases} D = 0 \;\rightarrow\; kF_o = 0 \\ D = D^{KP} \;\rightarrow\; kF_o = 1 \end{cases}$$

Accordingly:

when $$\begin{cases} D = 0 \quad then \quad F_o^{in} = -F_o^i \\ D = D^{KP} \quad then \quad F_o^{in} = F_o^i \end{cases}$$

With this, it is proved that force F_o^{in} satisfies both ultimate states:
When there is no deformation of the bearing, the force is equal to the value of coefficient F_o^i
At the beginning of unloading, the force at loading and that at unloading are equal.
To get an insight into the effect of eD^{KP} upon the horizontal force in the bearing, three curves with different values of eD^{KP} = 2, 5 and 10 were derived (Figure 7). Figure 7 shows that, in the case of curves with higher value of eD^{KP}, horizontal force F during unloading, decreases faster and more intensively. In all three curves, the force has an identical value at the beginning and at the end of unloading which points to the fact that the boundary conditions are satisfied.
The subsequently presented graph in Figure 7 shows the dependence of kF_o^{in} on the (D / D^{KP}) ratio. From the enclosed curves (eD^{KP}=2,5,10), it can be concluded that, at higher values of eD^{KP}, there is a considerable decay of the value of kF_o^{in} when the current displacement is within the limits of D=[0.80÷1.0]D^{KP}. This characteristic of kF_o enables control over the total restoring force at each deformed position.

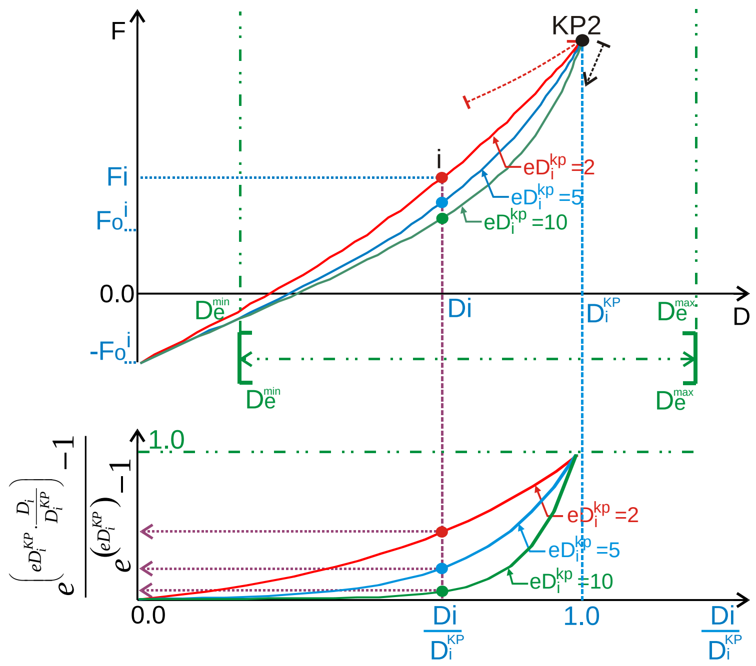

Figure 7. Dependence of the exponent on the horizontal deformation of the bearing

4. RESPONSE OF THE SMALL BASE ISOLATED CYLINDRICAL STEEL RESERVOIR OBTAINED FROM EARTHQUAKE SIMULATOR TESTS AND FINITE ELEMENT ANALYSIS

4.1 Earthquake Simulator Tests

One of the objectives of the performed seismic shaking table tests was to define the behavior of the base isolated structure. Therefore, the small steel storage reservoir was isolated by four cylindrical rubber bearings made of soft rubber component. The foundation structure consisted of a circular reinforced concrete plate which is connected to the seismic shaking table. Above the foundation plate, four isolators were installed. Over the isolators, there was a reinforced concrete base plate supporting the steel storage tank. The purpose of the base plate was to simulate a rigid structure that uniformly transfers the shear forces from the upper to the foundation. The steel storage tank had a diameter of 1700 mm and a total height of 2900 mm.

The instrumentation of the model arose from the need for obtaining data on the behavior of the structure during the testing. Three types of measuring instruments were used: (1) nine accelerometers, (2) seven linear potential transducers and (3) twelve strain gages. The displacement transducers and the accelerometers were placed at five places along the height. The strain gauges were placed at three levels along the height of the structure. At each level, there were four strain gauges placed orthogonally. The strain gauges were placed on the bottom of the steel storage tank to measure vertical strains in the cylinder, whereas the remaining strain gauges measured the circumferential strains.

Starting with the main goal of these experimental investigations, the testing program was conceptualized in such a way as to be able to respond to these needs. Because the main goal of this paper is to validate the polynomial model by finite element analysis, only the results from earthquake excitation tests are presented. One of the answers that needed to be obtained was the response of the base isolated structure exposed to earthquake excitation. Therefore, a characteristic real recorded earthquake was analyzed.

Firstly, the natural frequency of the small base isolated structure was obtained from the frequency response function curve which was defined from the random excitation tests. The first natural frequency of the structure in case of low shear deformation amounted to 1.95Hz (Figure 8), while for larger shear deformations the frequency of the model was decreased to range of (1.23÷1.28)Hz. The dependence of the model natural frequency on shear bearing deformation is presented in Figure 9. The presented graph indicates that structural natural frequency decreases as lateral deformation increases (region I & II). Then in region III an increasing in structural natural frequency is observed.

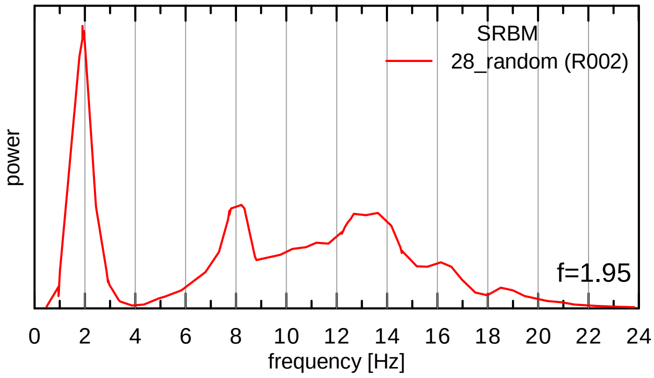

Figure 8. Frequency curve for base isolated structure

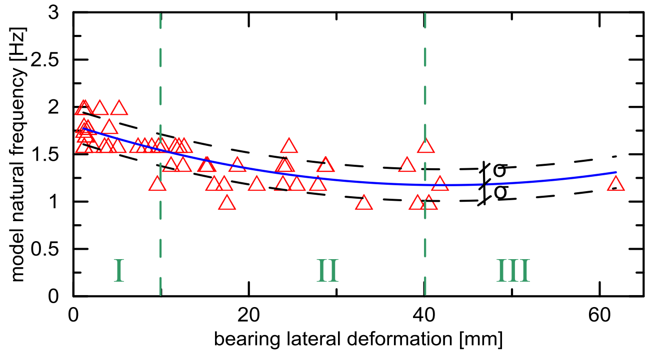

Figure 9. Shear deformation - model frequency relationship

Then, the influence of the earthquake frequency content on structural response was studied through analyzing the acceleration amplification factor calculated from a characteristic earthquake. The input earthquake was selected in such a way to match the structural natural reference and to produce the resonance effect. The acceleration amplification factor, which is defined as a ratio between the measured peak acceleration at the top and at the foundation level is graphically presented in Table 2. From the obtained results, it can be concluded that, under excitations with a frequency range from 1.0Hz to 1.5Hz, the amplification factor is in the range of 2.0 and higher. Actually, the maximum amplification factor is 2.95.

Table 2. Response of the base isolated reservoir

Label	Displacement (input/foundation) [mm]	Displacement (absolute)			Acceleration (absolute)			Amplification factor
		base [mm]	mid [mm]	top [mm]	base [m/s^2]	mid [m/s^2]	top [m/s^2]	top/base
EQ1	38.15	86.78	86.76	94.65	4.11	4.53	5.04	2.95

4.2 Finite Element Analysis

In the last phase, the behaviour of the the small base isolated cylindrical steel reservoir under a characteristic earthquake "EQ1" was analyzed through nonlinear dynamic analysis. Therefore, the polynomial analytical model was implemented into finite element software package. The base isolated structure was analytically analized by a three-dimensional finite element model. The steel structure and the reinforced concrete base plate were modeled by the shell finite element with both bending and membrane capabilities. The behaviour of the steel and concrete elements was considered as linear-elastic. The rubber isolators were modeled by link element with three degrees of freedom at each node: translations in the nodal x, y, and z directions. The behaviour of the rubber bearings in shear direction was modeled by a polinomial model in which case the hysteretic damping was included in the model itself (Gjorgjiev, 2013). In vertical direction the behavior is modeled as linear-elastic. The realistic behaviour of the bearing in horizontal direction was obtained by component testing and it was presented in the first part of this paper (Figure 4). This results were used to develop the model parameters. A least-square regression was used to determine the “best” coefficients in order to minimize the sum of the squares in the 3th order polynomial model. The model includes the effect of the large strain behavior and loading history dependence. In this study, the system property modification factors for the effects of aging, temperature and scragging of the rubber bearing which are defined in AASHTO (AASHTO 2010) were not considered in this analysis.

4.3 Validation of the polynomial model

The objective of the validation process is to prove the capabilities of the polynomial model and its usage in the finite element analysis. The model validation procedure is performed by comparison of the results from the nonlinear time history analysis of the base isolated structure to experimentally obtained results from the earthquake simulator test. The validation was performed for a characteristic earthquake which is listed in Table 1. Actually, the earthquake input in the finite element analysis was the measured acceleration at the foundation level during earthquake simulation tests of the small base isolated cylindrical steel reservoir.

The analytically calculated time histories of the accelerations at the isolation level and at the top of the structure were compared to the results obtained from the seismic shaking table tests (Figure 10, Figure 11). The enclosed graphs show the consistency between both time histories. Also, the frequency content of the both accelerations is in a good correlation between the analytical solution and the experimental data. The small difference is observed in the acceleration amplitudes which is acceptable error. This comparison indicates that the structural response was correctly modeled by the used polynomial analytical model.

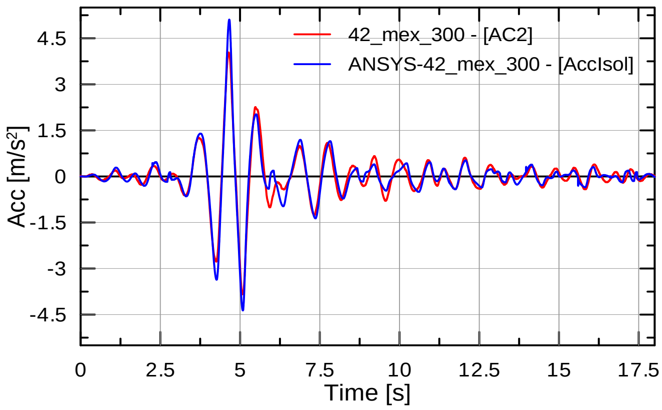

Figure 10. Comparison of time histories of acceleration at the isolation level under excitation EQ1

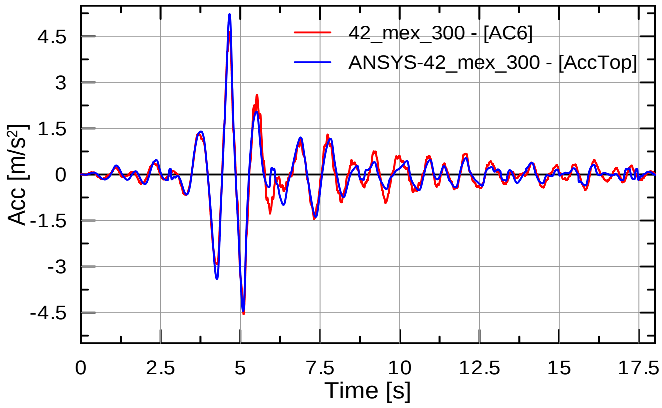

Figure 11. Comparison of time histories of acceleration at the top of the structure under excitation EQ1

The presented results showed a successful validation of the polynomial analytical model by comparing results from a nonlinear time history analysis to experimentally obtained results from earthquake simulator tests. There is a satisfactory accuracy of the polynomial analytical model in the case of the finite element dynamic analysis of the base isolated structure.

5. CONCLUSIONS

This study presents some of the results obtained from the realized experimental and analytical investigations of the rubber bearings and of the base isolated reservoir. The analytical and experimental research was realized by IZIIS in cooperation with several local companies. These investigations involved production of the squared and cylindrical rubber bearings, their testing, development of an analytical model for the rubber bearings and earthquake simulator test of a small base isolated cylindrical steel reservoir.

Firstly, the behavior of the isolators (force-displacement relationship) in axial and transverse direction was defined through dynamic tests on the isolators themselves. The obtained results confirmed that the actual horizontal behavior of the isolators depends on rubber compound and that under large shear strains it is nonlinear. Therefore, an advanced polynomial analytical model which supports nonlinearity at large horizontal deformations and includes loading history effects is used. The results from the vertical test showed the nonlinear relationship between the vertical stiffness and the vertical force.

The behavior of the representative base isolated structure was experimentally analyzed through seismic shaking table tests. The first natural frequency of the structure in case of low shear deformation amounted to 1.95Hz, while for larger deformations the frequency of the model was decreased to range of (1.23÷1.28)Hz. The obtained results showed that under earthquake excitations with a dominant frequency range from 1.0Hz to 1.5Hz, the amplification factor is in the range of 2.95. The high amplification response factors was obtained because the selected input dominant frequency was near to the structural natural frequency (resonant effect).

The analytical simulation of the base isolated structure was performed by nonlinear dynamic analyses of a three-dimensional model. The nonlinear behavior of the bearings was analytically modeled by an advanced polynomial model which was implemented into finite element software package. The analytical validation was performed for a characteristic earthquake. The comparison of the analytically calculated time histories of the accelerations at the isolation level and at the top of the structure to the results obtained from the seismic shaking table tests showed a satisfactory accuracy. Only a small difference is observed in the acceleration amplitudes which is in range of the acceptable error. It can be concluded that the model enables analytical modeling of the different hysteretic behaviors of the rubber bearings.

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