

## COMPARISON BETWEEN SEISMIC RESPONSES AND FREE VIBRATIONS OF A UNIFORM SHEAR-BEAM ALLOWED TO UPLIFT

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### ABSTRACT

Uplift accompanied by rocking motion during earthquakes is often beneficial for reducing seismic forces and damage on structures. Authors have investigated the valuable effect of uplift through experiments and analyses. In previous studies, we analyzed the dynamic uplift behavior using a uniform shear-beam model on a rigid ground in order to grasp the general trend of the useful effect of uplift. Thus far, no study has directly compared or investigated the relationship between free vibration and seismic response accompanied by uplift.

In this paper, using a piecewise-linear modal analysis method, the seismic response and free vibration of a uniform shear-beam model allowed to uplift are compared and discussed. For free vibration, only the first mode in a full contact phase is excited, as done in the author's previous study. The initial velocities for free vibration are adjusted to correspond to the peak response values in seismic response analysis.

The results verify that the peak values of seismic response and free vibration are similar if the intensity of ground motion is moderate and that seismic response is larger than free vibration when the intensity of ground motion becomes severe. These differences between seismic response and free vibration are considered to be mainly due to the contribution of higher modes.

*Keywords: uplift, uniform shear beam, piece-wise linear system, classical modal analysis, higher mode*

### 1. INTRODUCTION

Uplift accompanied by rocking motion during earthquakes is often beneficial for reducing seismic forces and damage on structures. Following Housner's pioneering investigation on the survival of slender structures in the Great Chilean earthquake in 1960 (Housner 1963), many studies have been conducted thus far (e.g., Meek 1975, Oliveto et al. 2003, Acikgoz and DeJong 2012, 2016, Vassiliou et al. 2015, Wiebe and Christopoulos 2015). Authors have also investigated the valuable effect of uplift through experiments and analyses (e.g., Midorikawa et al. 2006, Azuhata et al. 2017). In previous studies, we analyzed the dynamic uplift behavior using a uniform shear-beam model on a rigid ground in order to grasp the general trend of the useful effect of uplift (Ishihara et al. 2008, 2010, 2012). The motion of the system was defined using two phases: the full contact phase (CP) and uplift phase (UP). In the analysis, the model at rest was excited by initial velocities proportional to the first mode in CP and analyzed as free vibration without damping. Using the results obtained therein, we clarified the effects of the height-to-width ratio and intensity of excitation on uplift behavior. Thus far, no study has directly compared or investigated the relationship between free vibration and seismic response accompanied by uplift.

In this paper, using a piecewise-linear modal analysis method, the seismic response and free vibration of a uniform shear-beam model allowed to uplift are compared and discussed.

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## 2. OUTLINE OF ANALYSIS METHOD

### 2.1 A uniform shear-beam model allowed to uplift

As described in our previous study, a multistory building is modeled as a uniform shear beam with a few parameters. Figure 1 shows a model used in this paper. The model is a uniform shear beam with the height  $H$  and the width  $B$ , initially at rest on a rigid ground. The base of the model is constrained against a horizontal motion, so no deformation and slippage are allowed between the base and the ground. The model is a simple representation of a multistory building allowed to uplift.  $V$  in the figure is the vertical reaction force which is assumed positive when it is a compression force.

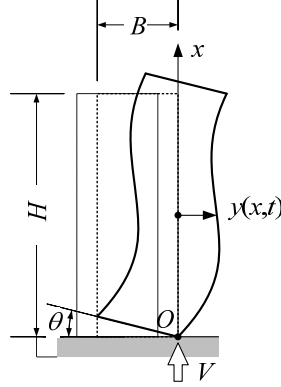


Figure 1. Uniform shear-beam model allowed to uplift

The structural parameters are as follows: (1) the first natural period in CP,  $T_1^{CP}$ , (2) damping ratio of the first mode in CP,  $h_1^{CP}$ , and (3) height-to-width ratio  $H/B$ .

In this paper, the first natural period in CP  $T_1^{CP}$  is 1 s, and the height-to-width ratio  $H/B$  is 4. Internal damping of shear-beam is assumed. The damping ratio of the first mode in CP  $h_1^{CP}$  is set to 2%.

### 2.2 A piece-wise linear modal analysis method

Small displacements are assumed, and equilibriums of forces/moments are considered based on the initial configuration of the structure. This assumption linearizes the governing equations of motions. Classical modal analysis is applied to calculate the free vibration and seismic response for each phase. Lateral displacement  $y$  at height  $x$  and at time  $t$  can be expressed as the summation of modes:

$$y(x, t) = \sum_j \underbrace{\beta_j^{CP} U_j^{CP}(x) q_j^{CP}(t)}_{\text{Contact phase(CP)}} = \sum_j \underbrace{\beta_j^{UP} U_j^{UP}(x) q_j^{UP}(t)}_{\text{Uplift phase(UP)}} \quad (1)$$

where superscript CP and UP denote the phases;  $\beta_j U_j(x)$  is a participation vector of  $j$ -th mode ( $\beta_j$  is a participation factor for horizontal input motion);  $q_j(t)$  is a response displacement of  $j$ -th mode. Similarly, rotational angle  $\theta(t)$  in UP can be written,

$$\theta(t) = \sum_j \underbrace{\beta_j^{UP} q_{0j}^{UP}(t)}_{\text{Uplift phase(UP)}} \quad (2)$$

In this paper, 10 modes are used for calculation for each phase.

Figure 2 shows the eigenmodes for both phases. The first mode in UP is a rigid body mode. Note that the damping ratio of the first mode in UP is zero because internal damping is assumed.

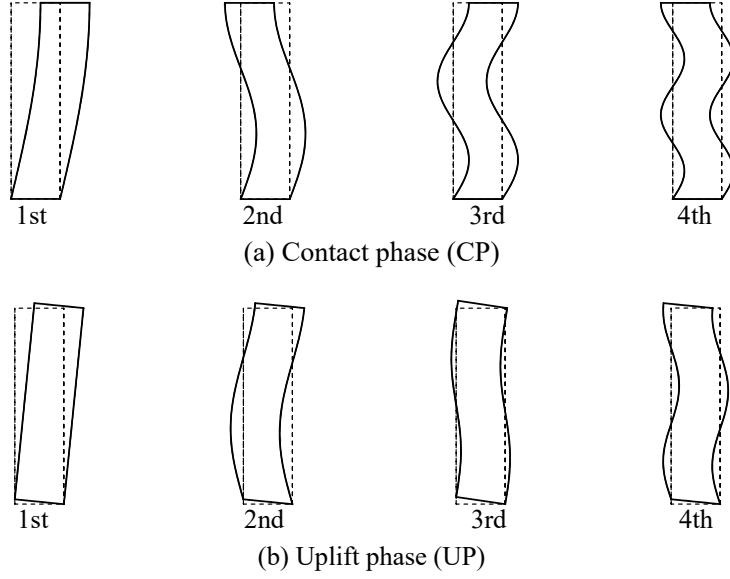


Figure 2. Mode shape ( $H/B=4$ )

Two sets of modes corresponding to the two phases are utilized to calculate the responses and converted to each other at the incipient of uplift and the instant of landing. The transition conditions are as follows:

At the incipient of uplift in CP:

$$|M_{ovt}| = M_{ovt,cr} = MgB/2 \quad (3)$$

At the instant of landing in UP:

$$\theta = 0 \quad (4)$$

where  $M_{ovt}$  is overturning moment;  $M$  is the total mass of the model;  $g$  is gravitational acceleration.

The assumption of complete plastic collision is adopted at landing instants to dissipate the vertical velocity in rocking motion. Although the details of the formulation are omitted in this paper, note that displacements in the system are not dependent on the scale of the model. This means that the rotational angular responses become larger as the scale decreases.

Figure 3 shows a flowchart of piece-wise linear modal analysis. At first in [A] in the chart, initial values etc. are set. ‘‘Upindex’’ is index of phases, i.e. Upindex=0 corresponds to CP and Upindex = sgn  $\theta = -1$  or  $+1$  corresponds to UP. In [B], the current phase is confirmed. In [C] or [D], modal responses for all modes during a small time step  $\Delta t$  are calculated as linear responses. At the end of incremental time step  $\Delta t$ , transition conditions for changing phases (Equation 3 and 4) are confirmed in [E] or [F]. If the condition is met, replace the current phase with the other in [G], where the calculation during the previous time step  $\Delta t$  is redone, and the instant of time which meets the condition is numerically identified. In [H], the vertical reaction force  $V$  (see Figure 1) is confirmed. The reason is that the negative reaction force (tension) means that the model jumps up in the air, and to deviate from the formulation of this paper. In [I], the calculation time is confirmed. When the time  $t$  does not exceed the specified time  $t_{cal}$ , the above calculation is repeated for the next time step  $\Delta t$  (return to [B]). Time increments  $\Delta t$  is set to 0.005 s.

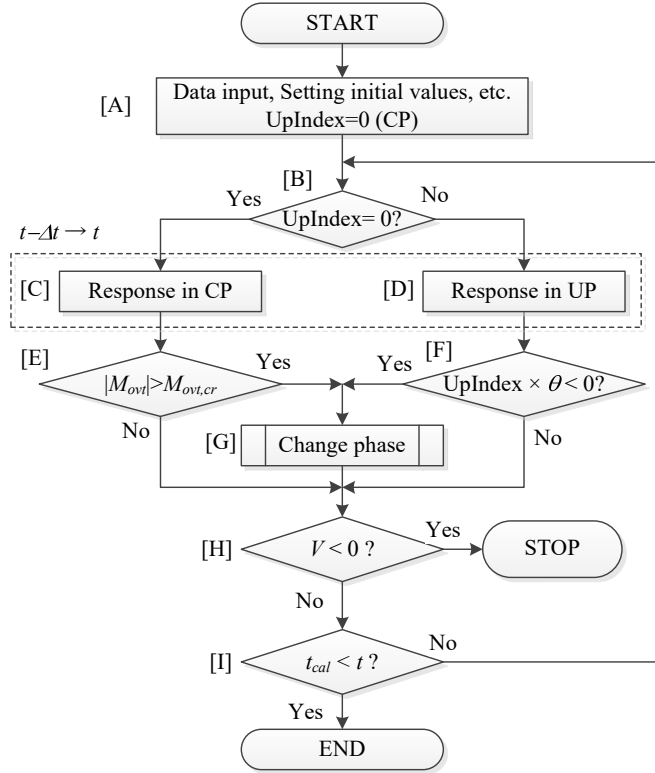


Figure 3. Flowchart of piece-wise linear modal analysis

### 2.3 How to excite the model in free vibration analysis

For free vibration, only the first mode in CP is excited, as done in the author's previous study (Ishihara et al. 2008, 2010, 2012). The initial velocities  $\dot{q}_1^{CP}(0)$  for free vibration are adjusted to correspond to the peak response values in seismic response analysis.

$$\underbrace{\dot{q}_1^{CP}(0)}_{\text{Initial velocity for free vibration}} = \max_t \underbrace{\sqrt{\left\{ \omega_1^{CP} q_1^{CP}(t) \right\}^2 + \left\{ \dot{q}_1^{CP}(t) \right\}^2}}_{\text{Seismic response}} \quad (5)$$

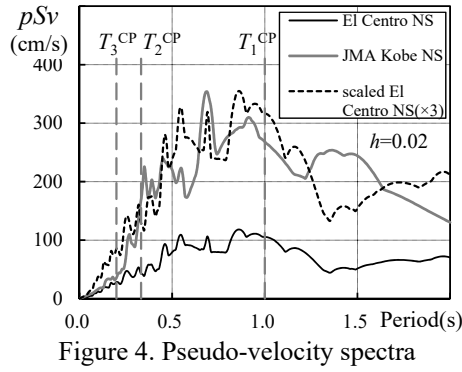
where  $\omega_1^{CP} = 2\pi/T_1^{CP}$  is the natural circular frequency of the first mode in CP. The dots signify differentiation with respect to time  $t$ . If the base of the model is fixed to the rigid ground (i.e. not allowed to uplift) and the damping is neglected, the peak base shear coefficient  $C_{Bf}$  in free vibration analysis can be calculated as follows:

$$C_{Bf} = \left( \overline{M}_1^{CP} / M \right) \omega_1^{CP} \dot{q}_1^{CP}(0) / g \quad (6)$$

where  $\overline{M}_1^{CP} / M$  ( $=8/\pi^2=0.81$ ) is the effective mass ratio of the first mode in CP. In this paper,  $C_{Bf}$  is also used as an expression of vibration intensity which is easy to understand.

### 2.4 Input motions for response history analysis

For seismic response analysis, NS component of El Centro 1940 and NS component recorded at Japan Meteorological Agency (JMA) Kobe station during Kobe earthquake in 1995 are used as input motions for 20 s. UD components are not applied. Figure 4 shows pseudo velocity spectra for these motions with scaled one (scaling factor = 3).



### 3. ANALYSIS RESULTS AND DISCUSSION

Figure 5 compares time histories of response history analysis (RHA) to El Centro motion and the corresponding free vibration analysis (FVA). In this case,  $\dot{q}_1^{CP}(0) = 82.6 \text{ cm/s}$  and  $C_{Bf} = 0.429$ . Figure (a) shows overturning moment coefficient  $C_m$  with critical one  $C_{m,cr}$  defined by Equations 7 and 8.

$$C_m \equiv M_{ovt} / (MgH/2) \quad (7)$$

$$C_{m,cr} \equiv (MgB/2) / (MgH/2) = B/H \quad (8)$$

When  $C_m$  exceeds  $C_{m,cr}$ , the model starts uplift (i.e., CP is replaced by UP). As shown in Figure (b), several uplift excursions occur in both analyses. Figure (c) shows responses of base shear coefficients in RHA and FVA. The influence of higher modes can be seen as ripples. Figure (d) shows horizontal displacements at the top. The dotted line represents rigid-body rotational displacement  $H\theta$  in UP in RHA. About the half or more of the displacement is caused due to the rigid-body rotation. Figure (e) shows horizontal absolute acceleration at the top. Influences of higher modes are clearly observed as ripples also in this figure. The last figure (f) shows vertical reaction forces  $V$  normalized by total weight  $Mg$ . The reaction force fluctuates in UP, but the range of fluctuations is not so large. Overall, the peak response value in RHA is almost equal to the corresponding FVA result.

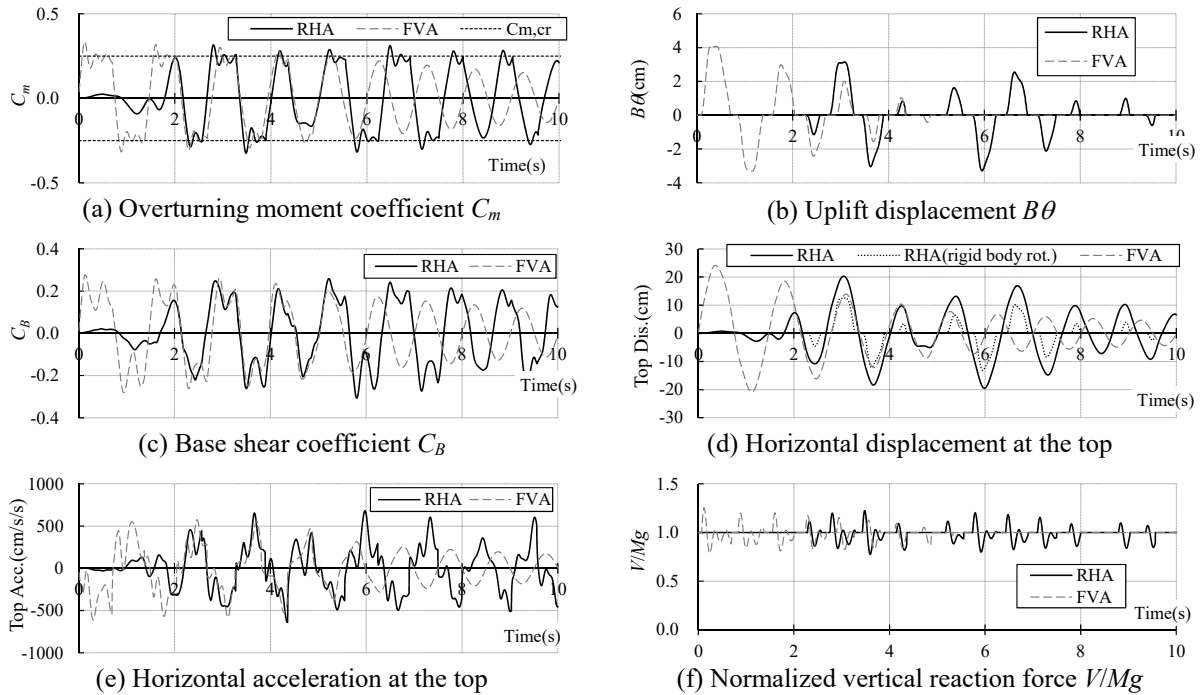


Figure 5. Response history to El Centro NS motion (RHA) and in the corresponding free vibration (FVA)

Figure 6 shows peak responses to El Centro motion along the height with the FVA results. Story shear coefficient and horizontal absolute acceleration in RHA are well consistent with those in FVA.

Figure 7 shows peak responses to JMA Kobe motion. In the corresponding FVA, initial velocity of the first mode was set to  $\dot{q}_1^{CP}(0) = 154.0 \text{ cm/s}$  ( $C_{Bf} = 0.800$ ). In contrast to Figure 6, peak responses in RHA are much larger than the FVA results.

In order to determine whether the difference between Figure 6 and Figure 7 is due to the ground motion characteristics, another calculation was made using the scaled El Centro motion (scale factor = 3). Figure 8 shows the results. In the corresponding FVA, initial velocity of the first mode was set to  $\dot{q}_1^{CP}(0) = 177.8 \text{ cm/s}$  ( $C_{Bf} = 0.923$ ). Similar to responses to JMA Kobe motion (Figure 7), peak responses in RHA are much larger than those in FVA. Peak response distributions along the height to JMA Kobe and scaled El Centro motions are very similar. From the shapes of the difference between RHA and FVA, higher modes, especially the second mode in UP, are considered to be more strongly excited in RHA.

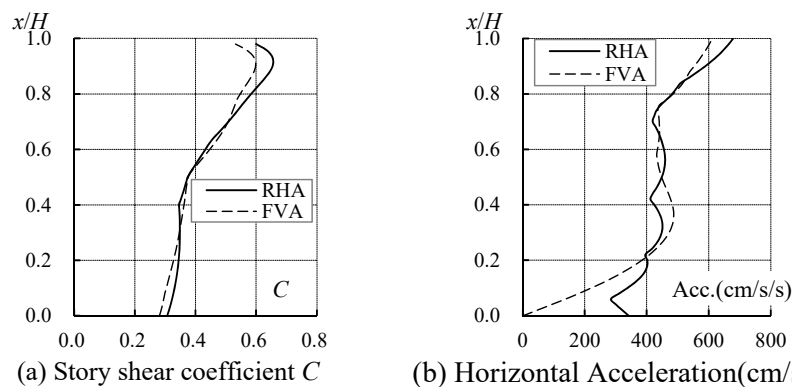


Figure 6. Peak responses to El Centro NS motion (RHA) and in the corresponding free vibration (FVA)

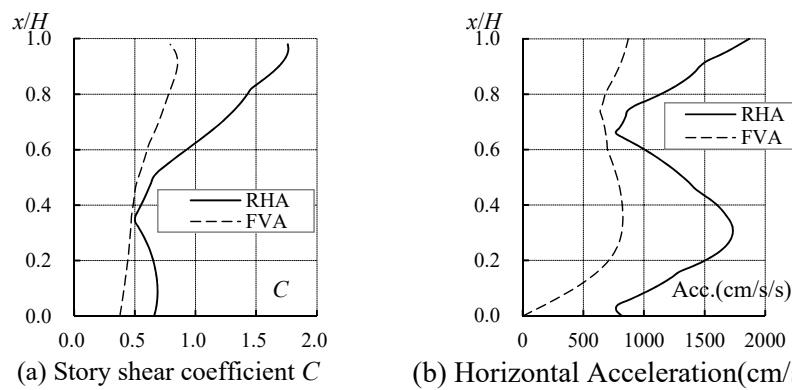


Figure 7. Peak responses to JMA Kobe NS motion (RHA) and in the corresponding free vibration (FVA)

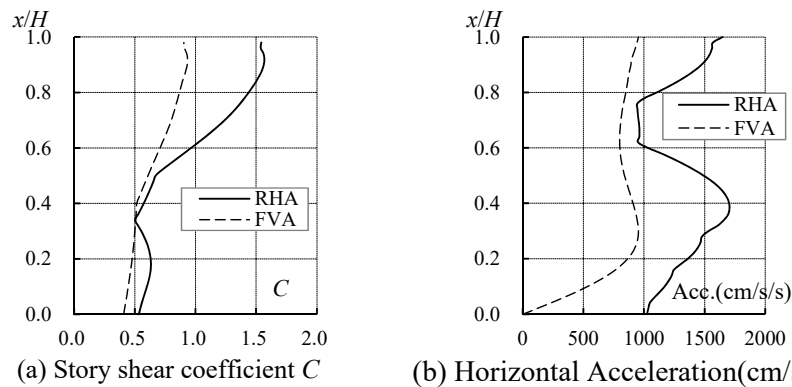


Figure 8. Peak responses to scaled El Centro NS motion (scaling factor = 3) (RHA) and in the corresponding free vibration (FVA)

## 4. CONCLUSIONS

In this paper, seismic responses and the corresponding free vibrations are compared using a uniform shear-beam model allowed to uplift. The results verify that the peak values of seismic response and free vibration are similar if the intensity of ground motion is moderate and that seismic response is larger than free vibration when the intensity of ground motion becomes severe. These differences between seismic response and free vibration are considered to be mainly due to the contribution of higher modes. The results of the free vibration in the author's previous studies is thought to be consistent with seismic responses when the ground motion is not so strong. Further study is necessary to quantitatively grasp the effect of higher modes.

## 5. ACKNOWLEDGMENTS

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