SEISMIC RETROFIT OF HIGH-RISE FRAME BUILDINGS USING SINGLE-INPUT-MULTIPLE-OUTPUT SYSTEM CONFIGURATION

Assaf SHMERLING¹, Robert LEVY²

ABSTRACT

This paper presents a novel approach for the optimal seismic retrofit of linearly-elastic frame buildings. A unique Single-Input-Multiple-Output (SIMO) general system interconnection (GSI) paradigm is formulated and used to attain structural changes in stiffness and mass in order to reduce the square root of the sum of squares (SRSS) of the absolute acceleration amplification factors. An analogy is drawn between the GSI closed-loop transfer function, having a first-order general-system-plant and a time-invariant structured controller, and the seismic upgrade at hand. The retrofit procedure itself applies $H_\infty$ synthesis to the GSI closed-loop transfer function, by using MATLAB's “hinstruct” function, to design the optimal structured controller. Then, the structural changes in stiffness and mass are extracted from the controller and applied to the linearly-elastic frame buildings. A case-study of a five-story shear-type building is examined. Optimal changes in floor-mass and horizontal-stiffness are obtained and show significant improvement in absolute story accelerations. The results indicate the efficiency of the proposed methodology that possesses the capability of attaining optimal changes in the structure’s physical characteristics of mass and stiffness, while catering for improved performance levels.

Keywords: Seismic Retrofit; $H_\infty$ synthesis; Passive Control; Stiffness changes; Mass Changes

1. INTRODUCTION

The acceleration response of buildings due to earthquakes is an important issue, especially when the safety of human lives and nonstructural components is concerned. Some structural components are exposed to damage from the inertial loading and, thus, considered acceleration-sensitive. According to FEMA 356 (2000), acceleration-sensitive components include architectural components (ceilings, stairs, chimneys, etc.) and mechanical equipment (boilers, storage vessels, fluid piping, etc.) which are very common in civil structures.

There exist quite a few seismic design methodologies that utilize structural optimization techniques in solving newly formulated problems that result in an upgraded structure of improved earthquake response. While not explicitly referring to the acceleration response in their problem formulation, some design procedures show a reduction in acceleration response which is merely a consequence of adding damping devices or weakening of the structure (see for example Agrawal and Yang 1998; Shukla and Datta 1999; Viti et al. 2006; Peng et al. 2013 and Shmerling et al. 2017). As of today, only a handful of seismic design methodologies address the acceleration response as a primary objective in their problem formulation. Cimellaro (2007), for example, suggested an algorithm for minimizing the objective function of the SRSS combination of displacements, story accelerations, and the base shear transfer functions, while subjected to a specified value of the sum of story stiffness coefficients, the sum of added damping coefficients, and constraints on the stiffness and damping coefficients. He solved by using a proposed systematic search algorithm. While such algorithms are useful in converging to a “local” optimal solution, the convergence rate is of low order. Leung et al. (2008) present their seismic design methodology for a tuned-mass-damper (TMD) attached to a viscously damped single-degree-of-freedom (SDOF) linearly-elastic system. Their objective function is an SRSS combination of either peak displacements, or peak accelerations, or both. They

¹Post-doctoral Fellow, Struct. Engrg., Ben-Gurion University of the Negev, Beer-Sheva, Israel, assafs@bgu.ac.il
²Professor, Struct. Engrg., Ben-Gurion University of the Negev, Beer-Sheva, Israel, levyrob@bgu.ac.il
apply the particle-swarm-optimization algorithm to yield three optimal values: (i) the ratio of the TMD mass to the system mass, (ii) the TMD damping ratio and (iii) the ratio of the TMD natural frequency to the system’s natural frequency. While the authors introduce an innovative optimization technique, it is limited to simple linearly-elastic SDOF systems. Shook et al. (2008) utilized the NSGA-II CE genetic algorithm of Deb et al. (2000) to minimize four objective functions (simultaneously) and to design fuzzy logic controllers for handling two magneto-rheological dampers fixed in the building. The four objective functions are: (i) the maximum peak inter-story drift, (ii) the maximum peak absolute acceleration, (iii) the RMS of peak inter-story drifts, and (iv) the RMS of peak story accelerations. The design procedure first chooses a randomized set of fuzzy logic controllers with uniformly distributed values. Then, the genetic algorithm is applied to yield an optimal combination. The algorithm is experimentally validated using a full-scale 3-story benchmark building placed on a shaking table with the input of four ground motion records. Results show that, in almost all the tests, the objective functions are indeed minimized. Nevertheless, there remain two problematic issues when using the genetic algorithm, which are the fine tuning of parameters and a lack of the application of side-constraints. Singh and Moreschi (2001) presented their method for optimally distributing and calculating the size of viscous and visco-elastic dampers in multistory buildings. The design methodology aims to achieve the best level of performance, defined as an SRSS modal combination of either inter-story drifts, or base shear, or story accelerations. Since a structure with added dampers is non-classically damped, the method by Singh (1980) is employed for modal analysis and calculating the consequent SRSS combination. The solution algorithm utilizes a gradient-based optimization approach, which was proved to be effective in reducing the peak story accelerations.

The methodologies described above address the acceleration response in the objective function of their optimization problem, and calculate their optimal structural changes quantity and added damping. While this is the case, it remains unclear if the improved performance is due to efficient structural changes or solely due to increasing the energy dissipation capability by the added damping. The developed methodology of this paper employs the $H_{\infty}$-synthesis algorithm by Apkarian and Noll (2006) for minimizing the SRSS combination of absolute acceleration amplification factors. The $H_{\infty}$-synthesis calculates the optimal changes in floor-mass and horizontal-stiffness with no added damping or control devices, which is similar to conventional design approaches. While the developed methodology is limited to the linear-elastic region, methods for equivalent linearization can be used in order to also address inelastic frame buildings.

2. OPTIMIZATION PROBLEM

Our retrofit approach aims to minimize the absolute acceleration response of a given shear-type building by assigning structural changes in floor-mass and story stiffness. Reducing the absolute acceleration response will reduce the damage risk to structural and nonstructural components that are sensitive to the acceleration response. In addition, the approach of changing the floor-mass and story stiffness is suitable for implementation in conventional building design. A general shear-type building scheme and deformations are depicted in Figure 1. In a different case, where a moment-resisting-frame building is addressed, the approach in Shmerling et al. (2017) for assessing an equivalent shear-type model and applying its structural changes may be utilized. The developed retrofit procedure deals with solving the following optimization problem:
The matrices $\mathbf{M}$, $\mathbf{DMA}$, $\mathbf{K}$ and $\Delta \mathbf{K}$ are the initial masses, mass change, initial stiffnesses and stiffness change of the shear-type building, respectively. $k_n$ and $m_n$ are the horizontal-stiffness and floor-mass of the $n^{th}$ story, respectively. $\Delta k_n$ and $\Delta m_n$ are the structural changes quantity, and $\Delta m^\text{max}_n$, $\Delta m^\text{min}_n$, $\Delta k^\text{max}_n$ and $\Delta k^\text{min}_n$ are the respective constraints on the structural changes. The matrix $\mathbf{C}$ is the classical Rayleigh damping matrix of the inherent damping ratio $\xi$ assigned to the two modal systems of lowest free-vibration angular frequencies ($\omega_1$ and $\omega_2$):

$$\mathbf{C} = a_0(\mathbf{M} + \Delta \mathbf{M}) + a_1(\mathbf{K} + \Delta \mathbf{K})$$

and:

$$a_0 = 2\xi \omega_1 \omega_2 / (\omega_1 + \omega_2)$$

$$a_1 = 2\xi / (\omega_1 + \omega_2)$$

$$\omega_1 < \omega_2 < \cdots < \omega_N$$

The vectors $\mathbf{x}(t)$, $\dot{\mathbf{x}}(t)$ and $\ddot{\mathbf{x}}(t)$ are the relative to ground displacement, velocity and acceleration in time, respectively, of the story floors, $\mathbf{a}^{\text{abs}}(t)$ is the absolute acceleration vector in time, $\mathbf{t}$ is the ground motion influence vector, $\mathbf{a}_g(t)$ is a harmonic ground acceleration of angular frequency $\omega_g$ and of peak ground acceleration “PGA”. The matrix $\mathbf{T}$ is used to clarify the structure of the stiffness and mass matrices and is the transformation matrix from displacements into drift coordinates:

$$\mathbf{T} = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
0 & 1 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
-1 & 0 & \cdots & 1
\end{bmatrix}$$

The objective function “$J$” is the SRSS of the stories’ absolute acceleration amplification factors “$\mathbf{A}^{\text{acc}}_1$, ..., $\mathbf{A}^{\text{acc}}_N$”. The time parameter $t_{\text{SSR}}$, in the term of $\mathbf{A}^{\text{acc}}_n$, stands for the starting time of the steady-state-response.
3. $H\infty$-SYNTHESIS

This section introduces a closed-loop control system subjected to $H\infty$-synthesis that correlates with the optimization problem of Equation 1. The $H\infty$-synthesis deals with minimizing the $H\infty$ norm of the GSI closed-loop process, depicted by the paradigm in Figure 2. The GSI paradigm consists of the general-system-plant transfer-function matrix and the controller transfer-function matrix. The general-system-plant has four partitions that characterize the linear connections between the two inputs of external load and controller force, and the two outputs of regulated response and simulated controller feedback.

In Figure 2, $G(s)$ and $Q(s)$ are the general-system-plant matrix and the controller transfer-function matrix, respectively. $G_{11}(s)$, $G_{12}(s)$, $G_{21}(s)$ and $G_{22}(s)$ are the four partitions of $G(s)$. The inputs $w_1(s)$ and $w_2(s)$ are the external load and the controller force, respectively, and the outputs $y_1(s)$ and $y_2(s)$ are the regulated response and the simulated controller feedback, respectively. The GSI connectivity is given by the following set of equations:
The variable “s” is a complex number in the Laplace domain (“S” domain). In our case, the controller is time-invariant and, therefore, the controller transfer-function matrix is henceforth denoted as Q.

We define the GSI closed-loop transfer-function matrix \( \text{CL}(Q, s) \) in the following relation:

\[
\begin{align*}
\dot{y}_1(s) &= G_{11}(s)w_1(s) + G_{12}(s)w_2(s) \\
\dot{y}_2(s) &= G_{21}(s)w_1(s) + G_{22}(s)w_2(s) \\
\dot{w}_2(s) &= Q(s)\dot{y}_2(s)
\end{align*}
\] (4)

Here, \( I \) is the identity matrix. The \( H\infty \) norm of \( \text{CL}(Q, s) \) is the supremum eigenvalue over all input frequencies:

\[
\|\text{CL}(Q, s)\|_\infty = \sup_{\omega_g \in [0, \infty)} \sqrt{\lambda_{\text{max}} \left( \text{CL}(Q, j\omega_g)^T \text{CL}(Q, j\omega_g) \right)}
\] (6)

where:
\[
j = \sqrt{-1}
\]

In Equation 6, \( \lambda_{\text{max}} \) is the maximal eigenvalue of \( \text{CL}(Q, j\omega_g) \) in squared form. Note that the input frequency, \( \omega_g \), is intentionally of the same notation as the angular frequency of the harmonic ground acceleration.

We now construct a unique GSI configuration of the following \( H\infty \)-synthesis:

\[
\begin{align*}
\text{minimize}_Q & \left\{ \|\text{CL}(Q, j\omega_g)\|_\infty = \sup_{\omega_g \in [0, \infty)} \left\{ \sqrt{\sum_{n=1}^{N} \left( \text{max}_{\omega \in [\omega_{\text{ss}}, \omega_{\infty}]} \left( |a_{n}(t)|/PGA \right)^2 \right)} \right\} \right\} \\
\text{s. t.} & \text{ closed-loop stability} \\
Q &= [\Delta M \quad \Delta K] \\
\Delta K &= T^T \text{diag}([\Delta k_1, \ldots, \Delta k_N])T \\
\Delta M &= \text{diag}([\Delta m_1, \ldots, \Delta m_N]) \\
\Delta m_{\text{max}}^n &\geq \Delta m_n \geq \Delta m_{\text{min}}^n \quad \forall n = 1, \ldots, N \\
\Delta k_{\text{max}}^n &\geq \Delta k_n \geq \Delta k_{\text{min}}^n \quad \forall n = 1, \ldots, N
\end{align*}
\] (7)

Note that the \( H\infty \)-synthesis formulation of Equation 7 has a similar objective function and structural changes constraints as in Equation 1. Such a system has external input and regulated output signals, \( w_1(s) \) and \( y_1(s) \), equal to the ground acceleration and absolute acceleration vector respectively:

\[
\begin{align*}
w_1(s) &= a_g(s) \\
y_1(s) &= a^{\text{abs}}(s)
\end{align*}
\] (8)

where \( a_g(s) \) and \( a^{\text{abs}}(s) \) are the Laplace-transform terms of \( a_g(t) \) and \( a^{\text{abs}}(t) \):

\[
\begin{align*}
a_g(s) &= \mathcal{L}[a_g(t)] \\
a^{\text{abs}}(s) &= \mathcal{L}[a^{\text{abs}}(t)]
\end{align*}
\]

Equation 8 leads to the objective function of Equation 7 and the closed-loop transfer matrix \( \text{CL}(Q, s) \) is a single-input-multiple-output (SIMO) transfer-function matrix of the following input-output relation:
\[ a_{\text{abs}}(s) = CL(Q, s) \, a_g(s) \]

and:

\[ |a_{\text{abs}}(j\omega_g)| = CL(Q, j\omega_g)|a_g(j\omega_g)| \]

In a case where the closed-loop transfer function matrix is SIMO, the H_{\infty} norm is the SRSS of the output gains:

\[ CL(Q_{\Delta}, j\omega_g)^T CL(Q_{\Delta}, j\omega_g) = \sum_{n=1}^{N} \left( \frac{|a_{\text{abs}}(j\omega_g)|}{|a_g(j\omega_g)|} \right)^2 \]

when represented in the time domain:

\[ CL(Q_{\Delta}, j\omega_g)^T CL(Q_{\Delta}, j\omega_g) = \sum_{n=1}^{N} \left( \max_{\tau \in [\ell_{\text{tssr}}]} \left( \frac{|a_{\text{abs}}(t)|}{PGA} \right) \right)^2 \]

and, finally:

\[ \| CL(Q_{\Delta}, s) \|_{\infty} = \sup_{\omega_g \in [0, \infty]} \sqrt{\sum_{n=1}^{N} \left( \max_{\tau \in [\ell_{\text{tssr}}]} \left( \frac{|a_{\text{abs}}(t)|}{PGA} \right) \right)^2} \]

We now formulate the terms for \( y_2(s) \), \( w_2(s) \) and the partitions \( G(s) \). Define the Laplace transformation of \( x(t) \), \( \dot{x}(t) \) and \( \ddot{x}(t) \):

\[ X(s) = \mathcal{L}[x(t)] \]
\[ sX(s) = \mathcal{L}[\dot{x}(t)] \]
\[ s^2X(s) = \mathcal{L}[\ddot{x}(t)] \]

Now, since \( Q \) consists of the changes in mass and stiffness, the simulated feedback to the controller \( y_2(s) \) is the absolute acceleration \( \ddot{x}(t) + t \, a_g(t) \) and relative-displacement \( x(t) \) respectively. Then, when formulated in the “S” domain:

\[ y_2(s) = - \left[ (s^2X(s) + ta_g(s)) \, X(s) \right]^T \quad (9) \]

Consequently, the applied control force vector, \( w_2(s) \), consists of the inertial forces and resisting forces by the structural changes (formulated in the “S” domain):

\[ w_2(s) = - \left[ \DeltaM \left( s^2X(s) + t \, a_g(s) \right) \, \DeltaK \, X(s) \right]^T \quad (10) \]

We define the transfer function matrix from the applied dynamic load to the displacement response of the initial shear-type building (before retrofit):

\[ P(s) = \begin{bmatrix} 1 & 0 \\ \begin{bmatrix} sI & -I \\ \begin{bmatrix} M^{-1}K & M^{-1}C \end{bmatrix}^{-1} & 0 \\ \end{bmatrix} \end{bmatrix} \]

and:

\[ C = a_0M + a_1K \]

and:

\[ a_0 = \frac{2\zeta_1 \omega_1}{(\omega_1 + \omega_2)} \]
\[ a_1 = \frac{2\zeta_1}{(\omega_1 + \omega_2)} \]
\[ \omega_1 < \omega_2 < \cdots < \omega_N \]

Given the above terms for \( w_1(s) \), \( y_1(s) \), \( y_2(s) \) and \( w_2(s) \), the four partitions of \( G(s) \) are formulated as follows:
Equations 7-12 present an H∞-synthesis and GSI configuration, which are the equivalent to the optimization problem of Equation 1 and the retrofitted shear-type building, respectively. The first-order steepest descent algorithm of Apkarian and Noll (2006) is applied to the H∞-synthesis of Equation 7, using the “hinfstruct” function in MATLAB® to calculate the structured passive controller. Then, the structural changes are extracted from the controller and applied to the initial shear-type building, which will yield the retrofitted shear-type building.

The main difference between the problem formulations in Equation 1 and Equation 7 is the inherent damping matrix “C”. Note that in Equation 1 “C” is proportional to the retrofitted building, as shown by Equation 2, while in Equation 7 “C” is proportional to the initial building, as shown by Equation 11. Consequently, H∞-synthesis will favor increasing the closed-loop angular free-vibration frequencies, so that the inherent damping ratio of the modal systems will increase, because:

$$\zeta_\ell = a_0/(2\omega_\ell) + a_1\omega_\ell/2$$

where $\zeta_\ell$ and $\omega_\ell$ are the $\ell^{th}$ modal damping ratio and modal angular free-vibration frequency. To resolve this issue, we utilize an option in the “hinfstruct” function that defines the maximum closed-loop angular free-vibration frequency, denoted henceforth as $\omega_{\text{max}}$ so that:

$$\omega_1 < \omega_2 < \cdots < \omega_N \leq \omega_{\text{max}}$$

The usefulness in defining $\omega_{\text{max}}$ is shown in the case study in this paper.

4. DESIGN PROCEDURE

The procedure for retrofitting shear-type buildings to result in reduced absolute accelerations is described below. The procedure consists of 4 stages and adopts the GSI configuration introduced in section 3 to solve the optimization problem of section 2. The procedure results in the optimal changes in floor-mass and horizontal-stiffness in all stories.

Stage 1. The structural properties and constraints are defined.

a. Determine the mass matrix $M$, the stiffness matrix $K$, and the inherent Rayleigh damping matrix $C$ so that:

$$C = a_0 M + a_1 K$$

and:

$$a_0 = 2\zeta_1 \omega_1/(\omega_1 + \omega_2)$$

$$a_1 = 2\zeta_1/(\omega_1 + \omega_2)$$

$$\omega_1 < \omega_2 < \cdots < \omega_N$$

b. Define the constraints of reduction/addition in floor-mass ($\Delta m_{\text{max}}^n$ and $\Delta m_{\text{min}}^n$) and reduction/addition in stiffness, ($\Delta k_{\text{max}}^n$ and $\Delta k_{\text{min}}^n$) for $n = 1, ..., N$.

Stage 2. Determine the transfer-function matrix $P(s)$ of the initial shear-type building from applied dynamic loading into the displacement response, as follows:

$$P(s) = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} sI & -I \\ M^{-1}K & sI + M^{-1}C \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$$

Stage 3. Determine the general-system-plant matrix, $G(s)$, as follows:

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \begin{bmatrix} -s^2 P(s)Mt + t & [s^2 P(s)] \\ -P(s)Mt & -P(s) \end{bmatrix}$$

Stage 4. Define the maximum closed-loop angular free-vibration frequency $\omega_{\text{max}}$, and apply the MATLAB® “hinfstruct” function to $G(s)$ to result in the optimal passive controller $Q$. 

$$G_{11}(s) = [-s^2 P(s)Mt + t]$$

$$G_{12}(s) = [s^2 P(s)]$$

$$G_{21}(s) = -[s^2 P(s)Mt]$$

$$G_{22}(s) = -[s^2 P(s)]$$
followed by extracting the optimal changes in floor-mass and horizontal-stiffness:

\[ Q = [\Delta M \quad \Delta K] \leftrightarrow \Delta M = \text{diag}([\Delta m_1, ..., \Delta m_N]) \]
\[ \Delta K = T^T \text{diag}(\Delta k_1, ..., \Delta k_N)T \]

Assign the changes to the initial shear-type building and result in the retrofitted building.

5. CASE-STUDY

The case-study of this paper attempts to retrofit a five-story shear-type building, by using the developed procedure to mitigate the absolute acceleration response in all stories. The scheme of the employed shear-type building is depicted in Figure 3, in which \( h = 3.0 \text{m} \). Following is a step-by-step description of the developed procedure.

The floor-mass and horizontal-stiffness quantities are indicated in Table 1(A) and construct the stiffness and mass matrices (K and M), as indicated in Stage 1a of the procedure. The inherent Rayleigh damping matrix, \( C \), is defined for a damping ratio of 5% assigned to the first and second modal systems as follows:

\[ C = a_0 M + a_1 K \]
where:
\[
a_0 = 2\zeta\omega_1\omega_2/(\omega_1 + \omega_2) = 0.8517 \text{ (1/sec)}
\]
\[
a_1 = 2\zeta/(\omega_1 + \omega_2) \approx 2.4 \times 10^{-3} \text{ (sec)}
\]
\[
\zeta = 5\%
\]
\[
\omega_1 \approx 11.97 \text{ (rad/sec)}
\]
\[
\omega_2 \approx 29.52 \text{ (rad/sec)}
\]

In Stage 1b, the constraints on the structural changes are prescribed as follows:

\[
\begin{align*}
\Delta m_{\max}^n &= 0.5 m_n \\
\Delta m_{\min}^n &= -0.5 m_n \\
\Delta k_{\max}^n &= 10.0 k_n \\
\Delta k_{\min}^n &= -0.5 k_n
\end{align*}
\]

\forall n = 1, ..., N

It might be worth noting that the drastic changes in floor-mass may be implemented in the retrofit of the building by changing the area load or floor load on the frame structure accordingly. In Stage 2, and in Stage 3 the transfer-function matrices \( P(s) \) and \( G(s) \) are defined, respectively.

The final stage, Stage 4, is now addressed and examined. The “hinstruct” function of MATLAB\textsuperscript{©} is applied to \( G(s) \) by using two different values for the maximum closed-loop angular free-vibration frequency \( \omega_{\max} \). The first is of \( \omega_{\max} = \infty \), and the second is of \( \omega_{\max} = 50 \text{ (rad/sec)} \), which was decided upon on the basis of a preliminary trial-and-error process.
The resultant building in the first case of $\omega_{\text{max}} = \infty$ is presented in Table 1(B). As shown, the synthesis has lowered all the floor-masses to the possible minimum (by 50%), unproportionally increased the horizontal-stiffness of the 1st story, and reduced the horizontal-stiffness for the rest of the stories. As expected, since the damping matrix remains fixed during the $H_\infty$-synthesis, the algorithm increases the modal frequency in the higher modes so that the damping ratio is also increased. More particularly, the modal frequencies are changed from $[12.0\ 29.5\ 46.6\ 63.5\ 80.0\ \text{rad/sec}]$ to $[28.1\ 37.7\ 43.5\ 47.0\ 148.0\ \text{rad/sec}]$, and the damping ratios are changed from $[5.0\%\ 5.0\%\ 6.53\%\ 8.32\%\ 10.17\%]$ to $[4.9\%\ 5.67\%\ 6.22\%\ 6.57\%\ 18.12\%]$ for the 1st to the 5th modes, respectively. Obviously this is not true, since the damping matrix is actually updated with respect to the 5% damping ratio assigned to the first and second modes, as follows:

$$C = a_0(M + \Delta M) + a_1(K + \Delta K)$$

where:

$$a_0 = 2\zeta \omega_1 \omega_2 / (\omega_1 + \omega_2) = 1.61 \ (1/\text{sec})$$
$$a_1 = 2\zeta / (\omega_1 + \omega_2) \cong 1.5 \cdot 10^{-3} \ (\text{sec})$$
$$\zeta = 5\%$$
$$\omega_1 \cong 28.1 \ (\text{rad/sec})$$
$$\omega_2 \cong 37.7 \ (\text{rad/sec})$$

The performance of the first case retrofitted building is examined for the Boston 20 ground motion records ensemble of 10% incidence in 50 years (http://civil.eng.buffalo.edu/Sac_records/boston.htm), referred to as “bo10in50” in short.

Table 2(A) and Table 2(B) present the peak absolute accelerations and inter-story drifts of the initial and retrofitted buildings, respectively. Note that the peak values remained roughly the same, besides the peak inter-story drift of the first-story, which reduced drastically due to great stiffening. This result exemplifies the ineffectiveness of the current case of $H_\infty$-synthesis with an unlimited frequencies-spectrum.

We now examine the case of $\omega_{\text{max}} = 50 \ (\text{rad/sec})$. The $H_\infty$-synthesis now yields the retrofitted building indicated in Table 1(C). In this case, the subsequent building has softened stories, reduced floor-mass in the
upper two stories, and increased floor-mass in the three lower stories. The modal frequencies are changed into [26.7 27.2 28.1 28.9 50.0 rad/sec], according to the H∞-synthesis, and the modal damping ratios are changed into [4.81% 4.84%, 4.90% 4.96% 6.90%] for the 1st to the 5th modes, respectively.

The peak responses of the retrofitted building (of 5% damping ratio assigned to the first and second modes), for the bo10in50 ensemble, are indicated in Table 2(C). Note that the second design solution is of lower peak absolute accelerations, while the inter-story drifts are about the same magnitude as in the initial building. Also, the SRSS of absolute acceleration has decreased by 25% when compared to the initial building.

Table 1. Floor-mass and horizontal-stiffness for all the designs of the five-story shear-type building

<table>
<thead>
<tr>
<th>n</th>
<th>m_n + Δm_n (kN sec²/m)</th>
<th>k_n + Δk_n (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Initial building (before retrofit)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.00</td>
<td>2,148.9</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>1,978.4</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>1,712.4</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>1,308.5</td>
</tr>
<tr>
<td>5</td>
<td>1.00</td>
<td>733.9</td>
</tr>
<tr>
<td>(B) Retrofitted building of ω_max = ∞</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.50</td>
<td>10,943.6</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>1,105.4</td>
</tr>
<tr>
<td>3</td>
<td>0.50</td>
<td>947.5</td>
</tr>
<tr>
<td>4</td>
<td>0.50</td>
<td>710.6</td>
</tr>
<tr>
<td>5</td>
<td>0.50</td>
<td>394.8</td>
</tr>
<tr>
<td>(C) Retrofitted building of ω_max = 50 (rad/sec)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.46</td>
<td>3,684.4</td>
</tr>
<tr>
<td>2</td>
<td>1.55</td>
<td>1,105.4</td>
</tr>
<tr>
<td>3</td>
<td>1.28</td>
<td>947.5</td>
</tr>
<tr>
<td>4</td>
<td>0.85</td>
<td>710.6</td>
</tr>
<tr>
<td>5</td>
<td>0.50</td>
<td>394.8</td>
</tr>
</tbody>
</table>
Table 2. Peak absolute acceleration and inter-story drift for all the designs of the five-story shear-type building

| n    | \( \max(|a^\text{abs}_n(t)|) \) (g) | \( \max(|\delta_n(t)/h|) \) (%) |
|------|----------------------------------|-------------------------------|

(A) Initial building/before retrofit

| 1    | 0.50                             | 0.38%                         |
| 2    | 0.71                             | 0.37%                         |
| 3    | 0.83                             | 0.38%                         |
| 4    | 0.88                             | 0.44%                         |
| 5    | 1.32                             | 0.59%                         |
| SRSS | 1.98                             |                               |

(B) Retrofitted building of \( \omega_{\text{max}} = \infty \)

| 1    | 0.63                             | 0.04%                         |
| 2    | 0.79                             | 0.36%                         |
| 3    | 0.95                             | 0.36%                         |
| 4    | 1.09                             | 0.42%                         |
| 5    | 1.13                             | 0.48%                         |
| SRSS | 2.10                             |                               |

(C) Retrofitted building of \( \omega_{\text{max}} = 50 \) (rad/sec)

| 1    | 0.44                             | 0.56%                         |
| 2    | 0.40                             | 0.50%                         |
| 3    | 0.53                             | 0.45%                         |
| 4    | 0.70                             | 0.47%                         |
| 5    | 1.01                             | 0.42%                         |
| SRSS | 1.46                             |                               |

6. CONCLUSIONS

This paper presents a new seismic design methodology for reducing peak absolute accelerations in high-rise shear-type buildings. The novelty of the developed methodology is the proposed general system interconnection (GSI) paradigm, for performing \( H_\infty \)-synthesis and attaining optimal changes in floor-mass and story stiffness (without adding damping devices) to result in an upgraded structural system. The developed design procedure is the solution for the formulated optimization problem and is composed of 4 stages. The first three stages regard the structural matrices and the transfer function matrices. The last stage applies the \( H_\infty \)-synthesis and results in the upgraded structure. While it might seem that the procedure is noniterative, the \( H_\infty \)-synthesis algorithm itself is of an iterative nature.

A five-story shear-type building is retrofitted in a case-study in this paper. Here, we learn of the importance of defining the maximum closed-loop angular free-vibration frequency when applying \( H_\infty \)-synthesis, due to a fixed inherent damping matrix, by examining two retrofit cases of unlimited frequency and of specified maximum frequency. In the second case, the maximum frequency was defined as 50 (rad/sec) in accordance with preliminary trial-and-error results. The peak earthquake responses of the two retrofitted buildings are determined for the bo10in50 ground motion ensemble. It was shown that in the second case, of specified maximum frequency, the resultant design is more realistic and more efficient in reducing the peak absolute accelerations, while maintaining about the same peak inter-story drifts as the original building design. On the other hand, in the first case, of unlimited frequency, the retrofit solution is inefficient in all realms. The case study of this paper proves that the developed methodology is successful in mitigating the peak absolute accelerations, while not requiring damping devices. In conclusion, further investigation is required on the issue of the fixed inherent damping matrix during the \( H_\infty \)-synthesis and on the maximum closed-loop angular free-vibration frequency. This will be addressed in future research work.
7. REFERENCES


